Competing Through Information Provision*

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Abstract

This paper studies the symmetric equilibria of a two-buyer, two-seller model of directed search in which sellers commit to information provision. More informed buyers have better differentiated private valuations and extract higher rents from trade. When sellers cannot commit to sale mechanisms, information provision is higher under competition than under monopoly. In contrast, when sellers commit to both information provision and sale mechanisms, I identify simple conditions under which sellers post auctions and provide full information in every equilibrium, ensuring that all equilibrium outcomes are constrained efficient. Sellers capture the efficiency gains from increased information and compete only over non-distortionary rents offered to buyers.

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> *(Christie's and Sotheby's) embarked on cutthroat competition to get goods for sale (... and) provide ever more luxurious services. Catalogues became ever fatter, printed in colour, on glossy art paper. (...) On the inside page of Sotheby's catalogue of the Old Master paintings sale held in London on Dec. 13 (2001), six "specialists in charge" are listed. (...) They identify the paintings, research them, know which world specialist on this or that painter needs to be contacted, and, more mundanely, which client is most likely to be interested in what painting, etc.*[1]

What leads buyers to visit particular sellers is more than simply the terms of trade on offer. In particular, since the quality of buyers' information about goods affects their gains from trade, sellers may try to attract buyers by offering better information. This paper considers a market in which sellers post levels of information provision and buyers sort into selling sites

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[1] International Herald Tribune, 12/01/2002.

ex ante, drawn by promises of being better informed once on-site. A buyer's information about his private valuation for a good consists of (a) private knowledge of some personal attributes, along with (b) an understanding of how these characteristics relate to the good's properties. By controlling the information about their goods through, say, the quality and knowledge of their sales staff, sellers do not affect or acquire information about the buyers' private tastes. Instead, they shape the precision of buyers' understanding of how the good matches these tastes. In the art auction market described in the quote above, the clients of Christie's and Sotheby's know their own tastes and would know how they value the objects on offer at these firms were they to have all relevant information about them. However, as this information is specialised and difficult to acquire, these buyers rely on the information provided by the firms' experts.

Privately informed buyers gain informational rents through trade and, as noted by Bergemann and Pesendorfer (2007), by providing less information to buyers *before* trading, sellers give out fewer informational rents *during* the exchange process. A monopolist's choice of information provision trades off informational rents against efficiency, since more information provision better identifies the buyers that most value the goods. However, and this is the novel insight of this paper, if sellers compete for buyers, the latter may shun low-information selling sites. Competing sellers still face the post-sorting efficiency-rents trade-off but also face a pre-sorting trade-off between market share and the rents promised to buyers.

I present a model of directed search in which two sellers with unit supplies compete for the unit demands of two buyers. Sellers commit to information structures and may or may not commit to (ex post) incentive compatible and individually rational sale mechanisms, buyers choose which seller to visit and sales take place.[2] With information provision interpreted as quality of customer service, my assumption that sellers can credibly commit to information structures captures the fact that the number, training and availability of sales staffs is observed by potential buyers. Terms of trade, on the other hand, can either be proposed by sellers after buyers have interacted with their sales staff or credibly posted beforehand. As in Peters and Severinov (1997), sorting occurs ex ante; buyers obtain their private information only once they choose a seller. Following the sellers' announcements, buyers sort into sale sites according to that subgame's (in most cases) unique symmetric mixed strategy equilibrium. Buyers compete for the good when both visit the same seller and this selection, common in directed search, rules out equilibrium coordination among buyers. In equilibrium, sellers face a random demand, whose distribution they affect through their choice of information provision and sale mechanisms. In both cases, I restrict attention to symmetric equilibria of the game between the sellers.

[2]For models of directed search with price posting, see Burdett et al. (2001), Coles and Eeckhout (2003), Moen (1997), Peters (2010), Shi (2001) and Shimer (2005), as well as Delacroix and Shi (2007) for a model in which prices provide information about good quality. For models of directed search with competing auctioneers, see Burguet and Sákovics (1999), McAfee (1993), Hernando-Veciana (2005), Pai (2009), Peters and Severinov (1997) and Virág (2010).

Once at a selling site, buyers' information is mediated by the information structures offered by sellers, which, as described by Bergemann and Pesendorfer (2007), map signals controlled by sellers into buyers' private inferences about their valuations for goods. If fully informed, buyers either have (independent and private) high or low valuations for both sellers' objects. As in Damiano and Li (2007), Ganuza and Penalva (2010), Johnson and Myatt (2006) and Ivanov (2008), I consider information structures ordered by the precision with which they allow buyers to access their true private valuations. For tractability, I assume that information structures have a symmetric correlated structure; sellers commit to a randomisation between two information states for their site: informed or uninformed. The realisation of this information state is commonly known. While ex post all buyers visiting a particular seller are informed or uninformed, ex ante sale sites are differentiated by the probability with which all buyers get access to their private valuations for the goods upon visiting.

I show that the effect of information provision depends on its role in competition. First, when sellers cannot commit to sale mechanisms and propose ex post optimal terms of trade after buyers have made their visit decisions, I show that the unique symmetric equilibrium in information provision has both sellers commit to full information. By setting ex post optimal mechanisms once buyers have sorted, the sellers' fix their ex ante trade-off between information provision and buyer visits. The full information result highlights that sellers' incentives for traffic-stealing are high since ex post optimal mechanisms (a) maximise allocative efficiency, so that information provision increases site surplus, and (b) minimise buyers' rents, so that their visit decisions are more sensitive to information provision.

Second, when sellers commit to both sale mechanisms and information provision, they can disentangle their rent and information provision decisions and they channel competition away from inefficient restrictions on information and into redistributive rent transfers to buyers. Under a condition guaranteeing that a monopolist seller would serve low-valuation buyers, I fully characterise the model's symmetric equilibria. In these equilibria, sellers provide full information, hold auctions and compete over the rents offered to buyers by setting appropriate (non-exclusionary) reserve prices. Since sellers provide full information and allocate goods efficiently based on that information, equilibrium outcomes are constrained efficient. Closely related to Coles and Eeckhout (2003), who present a two-buyer, two-seller model of directed search with sale mechanisms under perfect information, a continuum of symmetric equilibria exist that are differentiated by the sharing of a fixed level of surplus between buyers and sellers. In all equilibria, competition equates the marginal cost in rents of attracting additional visit probability with its marginal benefit in additional site surplus. The full information result exploits the fact that sellers post their offers of information provision and sale mechanisms before buyers sort into selling sites. I show that profiles in which sellers do not offer full information are vulnerable to deviations in which they provide more information, adjust buyers' rents through transfers to keep their visit decisions fixed and pocket the extra surplus generated by the additional information. The intuition that a seller can exploit efficiency gains through ex ante offers is very general. The key to my result is that this arises as a competitive outcome. Sellers endogenously harness the complementarity between information provision and efficiency by channelling all competition for buyers through non-distortionary transfers.

Recent work in mechanism design, auctions and optimal pricing has found that monopolists have incentives to manipulate their customers' access to information about their private valuations. In a model in which a seller designs a sale mechanism ex post, Bergemann and Pesendorfer (2007) characterise optimal information structures, which take a discrete monotone partitional form. Ganuza and Penalva (2010) study information provision in second-price auctions when buyers' ex post distributions of valuations are ordered by dispersion and show that the seller's incentive to limit buyers' information vanishes as the number of buyers grows and the competition between them for the good wipes out their informational rents (see also Board (2009)).[3] In a model of monopoly pricing, Johnson and Myatt (2006) have information provision order buyers' ex post distributions of valuations by sequences of rotations and in a result recalling that of Lewis and Sappington (1994), they find conditions under which a seller will always optimally release either all or none of the available signals.[4]

In contrast, when a monopolist designs a mechanism ex ante and can 'sell' information to buyers, Esö and Szentes (2007) show that the seller can capture all rents accruing from the information it controls by setting appropriate entry fees and hence provides full information. Their result shows that sellers will have an incentive to manipulate information only in those environments in which they cannot charge entry fees before any information about the goods is revealed. I impose that all buyer participation decisions are made ex post and hence my full information result when sellers can commit to mechanisms does not rely on entry fees but on sellers' ability to channel rents to buyers through means other than information provision. The interpretation of information provision as quality of customer service is consistent with ex post participation constraints as buyers typically discuss terms of trade only after they have received the sales staff's input about a product.

The question of how the incentives to provide information extend to a competitive market has received little attention to date. A later paper by Valverde (2011) studies a model related to mine in which sale mechanisms are restricted to auctions, but in which sellers provide information prior to buyers making their sorting decisions. In that case, while information provision can reduce visits from low-valuation bidders, Valverde (2011) provides conditions that guarantee the existence of a full-information equilibrium. Damiano and Li (2007) present a model of two-seller competition with information provision and ex post price competition which generalises that of Moscarini and Ottaviani (2001) (see also Huang (2010)). With a

[3] For random variables X and Y with distribution functions F and G, Y is said to be *more dispersed* than X if $F^{-1}(\beta) - F^{-1}(\alpha) \leq G^{-1}(\beta) - G^{-1}(\alpha)$ for all $0 < \alpha \leq \beta < 1$. See Shaked and Shanthikumar (2007).

[4] Continuous distribution function G is said to obtained from distribution F *by (clockwise) rotation around* z if $F(x) \leq G(x)$ for all $x \leq z$ and $F(x) \geq G(x)$ for all $x \geq z$.

single buyer and price competition, information does not enhance surplus and in equilibrium sellers provide information to differentiate goods ex post and soften competition. Ivanov (2008) studies a related model with any number of sellers and continuous type distributions and shows that as the number of sellers increases there is a unique symmetric equilibrium with full information provision.

1 Model

1.1 Setup, Information and Timing

Two sellers, A and B, have a single good for sale and two buyers, 1 and 2, have unit demands. Informed buyers have private valuations for both sellers' goods of either θ_H or θ_L, with $\theta_H > \theta_L$. Valuations are independently distributed across buyers and across sellers' goods. Buyers are initially uninformed about their private valuations for both sellers' goods, which are ex ante identical. The prior distribution of buyers' valuations for either good is (p_H, p_L) and their expected valuation is $\bar{\theta} = p_L\theta_L + p_H\theta_H$.

At all selling sites, information provision consists of private signals received by buyers about their valuation for the good at that site. Information provision is symmetric across buyers that visit a particular seller, that is, it cannot be tailored to individual buyers. As a tractable parametrisation, I assume that sellers commit to a randomisation between two information regimes, full information and no information. That is, seller $K \in \{A, B\}$ posts a probability π_K with which all buyers that attend site K learn their private valuations for the good. The realisation of the information regime, although not of the private signals received by the buyers, is commonly known once buyers visit a selling site. Ex post, either all buyers at site K are informed and have private valuations in $\{\theta_H, \theta_L\}$, or all are uninformed, and have a known expected valuation of $\bar{\theta}$.

In the first stage of the game, sellers commit to information provision and may simultaneously commit to ex post individually rational and incentive compatible sale mechanisms, although I also consider the case without commitment to mechanisms in which sellers propose ex post optimal terms of trade. In the second stage of the game, buyers simultaneously sort into selling sites, either receive information about the good or not (according to (π_A, π_B)), learn how many other buyers are also present and take part in the sale mechanism at that site. In Section 5, I present a fuller discussion of my main modelling assumptions.

1.2 Sale Mechanisms

Allowing for maximal flexibility in the terms of trade sellers can post to attract buyers is important to focus on the interaction between sellers' information and pricing policies. To this end, at the cost of additional (but familiar) notation, this section describes general direct sale mechanisms in my framework. Let $\eta \in \{1,2\}$ denote the *demand state* of a sale site, that is, whether the seller attracts one or two buyers (the seller may attract no buyers, although in this case no sale mechanism needs to be specified). Similarly, let $\tau \in \{i,u\}$ denote the *information state* of a sale site, where i stands for informed and u for uninformed. Let $\Theta^{\eta,\tau}$ denote the set of *report profiles* that can be received by the seller in state (η,τ), where

$$\Theta^{\eta,\tau} = \begin{cases} \{(\theta_m,\theta_n)\}_{(m,n)\in\{L,H\}^2} & \text{if } \eta = 2 \text{ and } \tau = i, \\ \{\theta_m\}_{m\in\{L,H\}} & \text{if } \eta = 1 \text{ and } \tau = i, \\ \emptyset & \text{if } \tau = u. \end{cases}$$

An *anonymous direct mechanism* is a collection of pairs of functions $(x^{\eta,\tau}, y^{\eta,\tau})$ where $x^{\eta,\tau}: \Theta^{\eta,\tau} \to [0,1]$ and $y^{\eta,\tau}: \Theta^{\eta,\tau} \to \mathbb{R}$ are, respectively, the probability a buyer obtains the good and the transfer he must pay to a seller when the report profile is $\theta \in \Theta^{\eta,\tau}$ in state (η,τ). Since no report is necessary when buyers are uninformed, I write probabilities and transfers as $x^{\eta,u}$ and $y^{\eta,u}$ for $\eta \in \{1,2\}$. Also, since mechanisms are anonymous, define $x^{2,i}(\theta_m,\theta_n)$ as the probability that a buyer reporting θ_m obtains the good when the other buyer reports θ_n. A similar remark holds for the transfer $y^{2,i}(\theta_m,\theta_n)$. The allocation probabilities satisfy feasibility restrictions

$$\begin{aligned} &x^{1,\tau}(\theta) \leq 1 \quad \text{for } \theta \in \Theta^{1,\tau} \text{ and } \tau \in \{i,u\}, \\ &x^{2,u} \leq \frac{1}{2}, \\ &x^{2,i}(\theta_m,\theta_n) + x^{2,i}(\theta_n,\theta_m) \leq 1 \quad \text{for } (m,n) \in \{L,H\}^2. \end{aligned}$$

In principle, sellers can condition sale mechanisms on the number of buyers that visit them. For example, a seller can offer a low price to any single buyer that visits its site, while hosting an auction with an exclusionary reserve price when both buyers visit. Here, since the information state at a sale site is known once buyers have sorted, sellers also condition sale mechanisms on whether or not buyers have private information. For example, sellers can set prices when buyers are uninformed and host auctions when they are informed.

Sale mechanisms must be ex post individually rational and incentive compatible. Let Γ denote the set of such mechanisms, which respect the standard constraints reproduced below. Note that since buyers will be willing to participate in sellers' mechanisms for all realisations of demand and information states, their ex ante participation is also assured. Fix any mechanism $\gamma \in \Gamma$. When buyers are uninformed, no incentive constraints apply. The relevant individual rationality constraints are

$$x^{1,u}\bar{\theta} - y^{1,u} \geq 0, \tag{IR^{1,u}}$$

$$x^{2,u}\bar{\theta} - y^{2,u} \geq 0. \tag{IR^{2,u}}$$

In state $(1,i)$, γ must satisfy

$$x^{1,i}(\theta_H)\theta_H - y^{1,i}(\theta_H) \geq x^{1,i}(\theta_L)\theta_H - y^{1,i}(\theta_L), \tag{IC$^{1,i}(\theta_H)$}$$
$$x^{1,i}(\theta_L)\theta_L - y^{1,i}(\theta_L) \geq x^{1,i}(\theta_H)\theta_L - y^{1,i}(\theta_H), \tag{IC$^{1,i}(\theta_L)$}$$
$$x^{1,i}(\theta_L)\theta_L - y^{1,i}(\theta_L) \geq 0. \tag{IR$^{1,i}(\theta_L)$}$$

As is well known, the individual rationality of the θ_H-type, (IR$^{1,i}(\theta_H)$), is satisfied whenever (IC$^{1,i}(\theta_H)$) and (IR$^{1,i}(\theta_L)$) hold.

For $m \in \{H, L\}$, let $X^{2,i}(\theta_m) = \mathbb{E}_{\theta_{-m}} x^{2,i}(\theta_m, \theta_{-m})$ and $Y^{2,i}(\theta_m) = \mathbb{E}_{\theta_{-m}} y^{2,i}(\theta_m, \theta_{-m})$. In state $(2,i)$, γ must satisfy

$$X^{2,i}(\theta_H)\theta_H - Y^{2,i}(\theta_H) \geq X^{2,i}(\theta_L)\theta_H - Y^{2,i}(\theta_L), \tag{IC$^{2,i}(\theta_H)$}$$
$$X^{2,i}(\theta_L)\theta_L - Y^{2,i}(\theta_L) \geq X^{2,i}(\theta_H)\theta_L - Y^{2,i}(\theta_H), \tag{IC$^{2,i}(\theta_L)$}$$
$$X^{2,i}(\theta_L)\theta_L - Y^{2,i}(\theta_L) \geq 0. \tag{IR$^{2,i}(\theta_L)$}$$

Again, the individual rationality constraint of the θ_H-type, (IR$^{2,i}(\theta_H)$), is satisfied whenever (IC$^{2,i}(\theta_H)$) and (IR$^{2,i}(\theta_L)$) hold.

Given that buyers sort into selling sites before receiving any information about their valuations, the ex ante properties of posted sale mechanisms will determine their visit decisions. Any mechanism $\gamma \in \Gamma$ induces *ex ante rents for buyers in state* (η, τ), $R^{\eta,\tau}(\gamma)$, which are given by

$$\begin{aligned}
R^{\eta,u}(\gamma) &= x^{\eta,u}\bar{\theta} - y^{\eta,u} \qquad \text{for } \eta \in \{1,2\}, \\
R^{1,i}(\gamma) &= \mathbb{E}_\theta \left[x^{1,i}(\theta)\theta - y^{1,i}(\theta)\right], \\
R^{2,i}(\gamma) &= \mathbb{E}_\theta \left[X^{2,i}(\theta)\theta - Y^{2,i}(\theta)\right].
\end{aligned}$$

A mechanism γ also induces an *ex ante surplus in state* (η, τ), denoted $S^{\eta,\tau}(\gamma)$, and given by

$$\begin{aligned}
S^{1,u}(\gamma) &= x^{1,u}\bar{\theta}, \\
S^{2,u}(\gamma) &= 2x^{2,u}\bar{\theta}, \\
S^{1,i}(\gamma) &= \mathbb{E}_\theta \left[x^{1,i}(\theta)\theta\right], \\
S^{2,i}(\gamma) &= 2\mathbb{E}_\theta \left[X^{2,i}(\theta)\theta\right].
\end{aligned}$$

1.3 Strategies and Equilibrium

A *posting strategy* for seller $K \in \{A, B\}$ is the choice of a probability with which information is provided to buyers and a sale mechanism, formally $(\pi_K, \gamma_K) \in [0,1] \times \Gamma$. A *visiting stragegy* for buyer $L \in \{1,2\}$ is $q_L : ([0,1] \times \Gamma)^2 \to [0,1]$, where $q_L((\pi_A, \gamma_A), (\pi_B, \gamma_B))$ denotes the probability with which the buyer visits seller A when, in the first stage, sellers posted $((\pi_A, \gamma_A), (\pi_B, \gamma_B))$. I restrict attention to *symmetric* profiles of visiting strategies, which are such that both buyers use the same visiting strategy $q((\pi_A, \gamma_A), (\pi_B, \gamma_B))$. The restriction to symmetric mixed strategy equilibria of the buyers' subgame is common in the directed search and competing auctions literature, in which the possibility of coordination among the buyers gives rise to equilibrium multiplicity.[5] Symmetry introduces search frictions by requiring a plausible anonymity in buyers' visit decisions.

Given posting strategy (π_A, γ_A) for seller A and a common probability $\hat{q} \in [0,1]$ that each buyer visits seller A, the expected payoffs to a buyer when attending site A are

$$\begin{aligned}\mathcal{R}_A((\pi_A, \gamma_A), \hat{q}) &\equiv \mathbb{E}_\eta \mathbb{E}_\tau R^{\eta,\tau}(\gamma_A) \\ &= \hat{q}\Big[\pi_A R^{2,i}(\gamma_A) + (1-\pi_A) R^{2,u}(\gamma_A)\Big] \\ &\quad + (1-\hat{q})\Big[\pi_A R^{1,i}(\gamma_A) + (1-\pi_A) R^{1,u}(\gamma_A)\Big]. \end{aligned} \tag{1}$$

Similarly to (1), given strategy (π_B, γ_B) for seller B and a probability $\hat{q}$ that buyers visit seller A, the expected payoffs to a buyer when attending site B are given by $\mathcal{R}_B((\pi_B, \gamma_B), \hat{q})$. Given posting strategies $((\pi_A, \gamma_A), (\pi_B, \gamma_B))$ and symmetric visiting strategy q, the profits of seller K can be expressed as surplus less rents as

$$\mathcal{P}_K((\pi_A, \gamma_A), (\pi_B, \gamma_B), q) = \mathbb{E}_\eta \mathbb{E}_\tau \left[S^{\eta,\tau}(\gamma_K) - \eta R^{\eta,\tau}(\gamma_K)\right],$$

where the expectations are taken with respect to the distributions of demand states, determined by the visiting probability $q((\pi_A, \gamma_A), (\pi_B, \gamma_B))$, and information states, determined by probability π_K, at site K.

Definition 1. A *symmetric (subgame perfect Nash) equilibrium* consists of a posting strategy (π^*, γ^*) and a visiting strategy q^* such that

(i). For any $((\pi_A, \gamma_B), (\pi_B, \gamma_B))$, q^* satisfies

$$q^*((\pi_A, \gamma_B), (\pi_B, \gamma_B)) = \begin{cases} 1 & \text{if } \mathcal{R}_A((\pi_A, \gamma_A), 0) \geq \mathcal{R}_B((\pi_B, \gamma_B), 0) \\ & \quad \text{and } \mathcal{R}_A((\pi_A, \gamma_A), 1) \geq \mathcal{R}_B((\pi_B, \gamma_B), 1) \\ & \quad \text{with one inequality strict,} \\ 0 & \text{if } \mathcal{R}_A((\pi_A, \gamma_A), 0) \leq \mathcal{R}_B((\pi_B, \gamma_B), 0) \\ & \quad \text{and } \mathcal{R}_A((\pi_A, \gamma_A), 1) \leq \mathcal{R}_B((\pi_B, \gamma_B), 1) \\ & \quad \text{with one inequality strict,} \\ \frac{1}{2} & \text{if } \mathcal{R}_A((\pi_A, \gamma_A), 0) = \mathcal{R}_B((\pi_B, \gamma_B), 0) \\ & \quad \text{and } \mathcal{R}_A((\pi_A, \gamma_A), 1) = \mathcal{R}_B((\pi_B, \gamma_B), 1), \end{cases}$$

while otherwise $q^*((\pi_A, \gamma_B), (\pi_B, \gamma_B)) = \hat{q} \in (0,1)$ is the unique solution to

$$\mathcal{R}_A((\pi_A, \gamma_A), \hat{q}) = \mathcal{R}_B((\pi_B, \gamma_B), \hat{q}). \tag{2}$$

[5] See Burdett et al. (2001), and also Levin and Smith (1994) in the context of a single auction with buyer entry.

(ii). Given q^*, (π^*, γ^*) satisfies

$$(\pi^*, \gamma^*) \in \underset{(\pi',\gamma')\in[0,1]\times\Gamma}{\arg\max} \mathcal{P}_A((\pi', \gamma'), (\pi^*, \gamma^*), q^*).$$

When sellers post general sale mechanisms, the restriction to symmetric visiting strategies does not ensure the existence of a unique equilibrium of the buyers' subgame, so that condition (i) embeds a selection of these equilibria. In particular, if posting strategies $((\pi_A, \gamma_A), (\pi_B, \gamma_B))$ are such that $\mathcal{R}_A((\pi_A, \gamma_A), 0) \leq \mathcal{R}_B((\pi_B, \gamma_B), 0)$ and $\mathcal{R}_A((\pi_A, \gamma_A), 1) \geq \mathcal{R}_B((\pi_B, \gamma_B), 1)$, then both $q((\pi_A, \gamma_A), (\pi_B, \gamma_B)) = 1$ and $q'((\pi_A, \gamma_A), (\pi_B, \gamma_B)) = 0$ are symmetric equilibria of the buyers' subgame, along with any $q''((\pi_A, \gamma_A), (\pi_B, \gamma_B))$ satisfying (2). In this case, the selection (i) retains the solution to (2) if it is unique, and sets $q((\pi_A, \gamma_A), (\pi_B, \gamma_B)) = \frac{1}{2}$ otherwise. As an example of posting profiles generating multiple equilibria of the buyers' subgame, suppose that both sellers have posted $(0, \gamma)$, where the sale mechanism γ is such that in state $(2, u)$, the seller delivers the good to either buyer with equal probability at a price of 0, while in state $(1, u)$ the seller delivers the good to the single visiting buyer at a price of $\frac{\bar{\theta}}{2}$. Hence, $\mathcal{R}_A((0, \gamma), \hat{q}) = \mathcal{R}_B((0, \gamma), \hat{q}) = \frac{\bar{\theta}}{2}$ for any $\hat{q} \in [0, 1]$, and there exists a continuum of symmetric visiting equilibria following this symmetric posting profile. The equilibrium selection in (i) retains the symmetric equilibrium that generates symmetric outcomes across sellers.

Posting profiles that do not induce a unique symmetric visiting equilibrium have the counter-intuitive property that in the buyers' subgame the buyers do not compete for a given seller's good: both buyers are better off if they visit the same seller. I say that that sellers' mechanisms generate *congestion effects* if buyers' rents at a given site decrease when the other buyer visits it more frequently, specifically, if $\mathcal{R}_A((\pi_A, \gamma_A), \hat{q})$ is decreasing in $\hat{q}$ and $\mathcal{R}_B((\pi_B, \gamma_B), \hat{q})$ is increasing in $\hat{q}$. Natural sale mechanisms, such as posted prices and auctions, generate congestion effects, as do the mechanisms considered in this paper: ex post optimal mechanisms in the absence of commitment and the equilibrium mechanisms with commitment. Note that, if the posting strategies of both sellers generate congestion effects, then $\mathcal{R}_A((\pi_A, \gamma_A), 1) \geq \mathcal{R}_B((\pi_B, \gamma_B), 1)$ implies that $\mathcal{R}_A((\pi_A, \gamma_A), 0) > \mathcal{R}_B((\pi_B, \gamma_B), 0)$ and $\mathcal{R}_A((\pi_A, \gamma_A), 0) \leq \mathcal{R}_B((\pi_B, \gamma_B), 0)$ implies that $\mathcal{R}_A((\pi_A, \gamma_A), 1) < \mathcal{R}_B((\pi_B, \gamma_B), 1)$. Hence, for such posting profiles, there is a unique symmetric equilibrium of the buyers' subgame.

1.4 A Characterisation of Incentive Compatible Mechanisms

A technical remark that is of considerable usefulness for establishing my results is that buyers' sorting decisions, as expressed by (2), depend only on information provision (π_A, π_B) and expected rents $(R^{\eta,\tau}(\gamma_A), R^{\eta,\tau}(\gamma_B))_{\eta,\tau}$. In particular, buyers' decisions are not affected by how rents are shared between types conditional on being informed. This ex ante feature of sellers' rent promises allows a simple characterisation of incentive-compatible mechanisms, which simplifies sellers' strategy sets. In Appendix A.1, I show that it is without loss of generality to restrict sellers to posting individually rational and incentive compatible mechanisms in which the incentive constraints for θ_H-types in informed states, $IC^{1,i}(\theta_H)$ and $IC^{2,i}(\theta_H)$, are binding. However, contrary to the case of monopoly, the individual rationality constraint of θ_L-types need not bind, as competition can force sellers to deliver more rents to buyers.

2 Example: Second-Price Auctions

I start with an example of competitive information provision by second-price auctioneers. More formally, I restrict seller $K \in \{A, B\}$ to posting strategies (π_K, γ^{SP}) in which γ^{SP} specifies a second-price auction without reserve price in all demand and information states. As Board (2009) and Ganuza and Penalva (2010) derive the optimal information structures for monopolists in a second-price auction with two buyers, this example constitutes a useful benchmark to gauge the effects of competition on information provision.

With second-price auctions, buyers obtain the good for free in the one-buyer state, and capture the full surplus $\bar{\theta}$. In the two-buyer state, to bid their best estimate of their true value is a weakly dominant strategy for buyers. When uninformed, this best estimate is $\bar{\theta}$. A buyer that attends site A, given π_A and $\hat{q}$, expects rents

$$\mathcal{R}_A((\pi_A, \gamma^{SP}), \hat{q}) = \hat{q}\pi_A p_H p_L(\theta_H - \theta_L) + (1 - \hat{q})\bar{\theta},$$

while a buyer attending site B, given π_B and $\hat{q}$, expects rents

$$\mathcal{R}_B((\pi_B, \gamma^{SP}), \hat{q}) = (1 - \hat{q})\pi_B p_H p_L(\theta_H - \theta_L) + \hat{q}\bar{\theta}.$$

In the buyers' subgame, the probability with which buyers visit site A is given by

$$q^*((\pi_A, \gamma^{SP}), (\pi_B, \gamma^{SP})) = \frac{\bar{\theta} - \pi_B p_H p_L(\theta_H - \theta_L)}{\bar{\theta} - \pi_A p_H p_L(\theta_H - \theta_L) + \bar{\theta} - \pi_B p_H p_L(\theta_H - \theta_L)}. \tag{3}$$

Given (π_A, π_B) and q^*, the profits of seller A are

$$\begin{aligned}\mathcal{P}_A((\pi_A, \gamma^{SP}), (\pi_B, \gamma^{SP}), q^*) &= q^{*2}\left[\pi_A\Big(p_H^2\theta_H + (1 - p_H^2)\theta_L\Big) + (1 - \pi_A)\bar{\theta}\right] \\ &= q^{*2}\big[\bar{\theta} - \pi_A p_H p_L(\theta_H - \theta_L)\big]. \\ &\equiv q^{*2} w(\pi_A)\end{aligned} \tag{4}$$

The term $w(\pi_A)$ is the expected price paid by the buyer who obtains the good in the two-buyer state, which is also the revenue of a monopolist second-price auctioneer facing a fixed set of two buyers. This price decreases in π_A, since the seller then gives away a higher share of the surplus as informational rents. The following result is known from Board (2009) and Ganuza and Penalva (2010).

Remark 1. *When a monopolist second-price auctioneer with no reserve price faces two buyers, providing no information is optimal.*

Returning the model with competition, note that (3) can be rewritten as

$$q^*((\pi_A, \gamma^{SP}), (\pi_B, \gamma^{SP})) = \frac{w(\pi_B)}{w(\pi_A) + w(\pi_B)}. \tag{5}$$

Since buyers get all the surplus if alone, equilibrium visit probabilities depend only on how much profits sellers get from demand states with two buyers. Thus (4) becomes

$$\begin{aligned} \mathcal{P}_A((\pi_A, \gamma^{SP}), (\pi_B, \gamma^{SP}), q^*) &= \left[\frac{w(\pi_B)}{w(\pi_A) + w(\pi_B)}\right]^2 w(\pi_A) \\ &= w(\pi_B)\left[\frac{w(\pi_B)}{w(\pi_A) + w(\pi_B)} \cdot \frac{w(\pi_A)}{w(\pi_A) + w(\pi_B)}\right] \\ &= w(\pi_B) q^*(1 - q^*). \end{aligned} \tag{6}$$

Seller A's strategy affects profits (6) only through its effect on $q^*(1-q^*)$, which attains a maximum when $q^*((\pi_A, \gamma^{SP}), (\pi_B, \gamma^{SP})) = \frac{1}{2}$, which seller A can attain by posting $\pi_A = \pi_B$.

Remark 2. *When the sale mechanism is a second-price auction with no reserve price,* (π^*, γ^{SP}) *is a symmetric equilibrium posting strategy for any* $\pi^* \in [0, 1]$.

This result, which follows by extending a standard framework to duopoly, is noteworthy in itself, but two features warrant broader discussion. First, competition allows for more information provision than monopoly. Second, when sellers cannot commit to sale mechanisms, equilibrium information provision depends on the trade-off between information and buyer visits, which depends critically on the details of the mechanisms that buyers expect once on-site. In other words, sale mechanisms constrain the rent offers sellers can extend to buyers through their choice of information provision. Under second-price auctions with no reserve price, this trade-off generates a surprisingly inclusive set of levels of equilibrium information provision.

3 No Commitment to Sale Mechanisms

In this section, sellers commit to levels of information provision but cannot commit to sale mechanisms. Once demand and information states have been realised, sellers deliver their good through each state's ex post optimal mechanism. More formally, I restrict seller $K \in \{A, B\}$ to posting strategies (π_K, γ^{EP}) in which γ^{EP} is an ex post optimal mechanism. This extends the framework of Bergemann and Pesendorfer (2007) to competing sellers.

Under ex post optimal mechanism γ^{EP}, it must be that $R^{1,u}(\gamma^{EP}) = R^{2,u}(\gamma^{EP}) = 0$. That is, in uninformed states sellers make take-it-or-leave-it offers of $\bar{\theta}$ and capture all gains from trade. When buyers are informed, the ex post optimal mechanisms for both the one and two-buyer states depend on whether or not sellers prefer to exclude θ_L-types and sell only to θ_H-types. When θ_L-types are excluded from trade, sellers extract all informational rents from θ_H-types. In that case, buyers expect no rents from any demand state regardless of the level of information provision and their sorting decisions are trivial. The interesting case is when informed θ_H-types obtain rents.

Assumption 1. $\frac{\theta_H}{\theta_L} < \frac{1}{p_H}$.

Under an ex post optimal mechanism, sellers prefer to sell to θ_L-types in both informed demand states whenever Assumption 1 is satisfied. In state $(1, i)$, this mechanism calls for sellers to set a price of θ_L, so that a monopolist facing a single buyer would not provide any information. In state $(2, i)$, this mechanism yields expected payoff $\bar{\theta}$, so that a monopolist facing two buyers is indifferent between all levels of information provision.[6] In particular, providing no information is optimal. Furthermore, no symmetric equilibrium with posting strategy (π^*, γ^{EP}) such that $\pi^* = 0$ exists, as uninformed buyers get no rents under γ^{EP} and any deviation by some seller from this profile to any $\pi' > 0$ would attract all buyers. Hence relative to monopoly, competition can improve informational efficiency. The following result shows that when sellers cannot commit to sale mechanisms, they achieve their favoured ex post outcomes, yet competition leads them to make their most costly ex ante information commitments.

Proposition 1. *Suppose that sellers cannot commit to sale mechanisms and that Assumption 1 is satisfied. Then there exists a unique symmetric equilibrium with posting strategy* (π^*, γ^{EP}) *such that* $\pi^* = 1$.

An alternative interpretation of the model with no commitment to sale mechanisms is that the sellers compete through information provision only, with the mechanisms at their sites exogenously set to γ^{EP}. A by-product of the proof of Proposition 1 is a necessary condition on the level of information provision in a symmetric equilibrium in which sale mechanisms at both selling sites are exogenously set to some mechanism γ such that (*a*) $R^{1,u}(\gamma) = R^{2,u}(\gamma) = 0$ (exploit uninformed buyers), (*b*) $R^{1,i}(\gamma) > R^{2,i}(\gamma)$ (congestion effects) and (*c*) $S^{2,i}(\gamma) - \bar{\theta} \leq 2R^{2,i}(\gamma)$ (sellers' payoffs in state $(2, i)$ are decreasing in information provision).[7] Under such mechanisms, of which γ^{EP} is a special case, equilibrium information provision is decreasing in the rents $R^{1,i}(\gamma)$ and $R^{2,i}(\gamma)$ offered to buyers in informed states and increasing in the surplus $S^{2,i}(\gamma)$ in the two-buyer informed state. Under γ^{EP}, the buyers' rents $R^{1,i}(\gamma^{EP})$ and $R^{2,i}(\gamma^{EP})$ are minimised, while the surplus $S^{2,i}(\gamma^{EP})$ is maximised, leading to the full-information result.

[6] See the proof of Proposition 1 in Appendix A.2.
[7] See Lemma 3 in Appendix A.2.

If sellers' mechanisms are fixed and buyers expect low rents at both sites, their visit decisions are more sensitive to shifts in information provision, which enhances sellers' traffic-stealing incentives. Hence, mechanisms offering higher rents dampen the competition between sellers. Also, when mechanisms generate a high surplus in the two-buyer state, information provision increases efficiency, giving the sellers more incentive to provide information and intensifying the competition between them. This highlights a more general principle: information provision is more attractive to sellers under mechanisms that better exploit the information that is generated. The complementarity that competition generates between mechanisms' allocative efficiency and information provision has a more negative implication: if sellers could collude and commit to sales mechanisms while anticipating future competition in information they would choose mechanisms with inefficient allocations.

4 Commitment to Sale Mechanisms

In this section, sellers commit jointly to information provision and sale mechanisms. The main result of the paper, Proposition 2, is a characterisation of symmetric equilibria under the no-exclusion Assumption 1 that shows that they all have full information. First, I require the following definition.

Definition 2. A mechanism $\gamma \in \Gamma$ is *quasi-efficient* if

$$x^{1,i}(\theta_H) = x^{1,u} = 1,$$
$$x^{2,u} = \frac{1}{2},$$
$$x^{2,i}(\theta_H, \theta_L) = 1, \text{ and } x^{2,i}(\theta_H, \theta_H) = \frac{1}{2}.$$

Furthermore, a mechanism $\gamma \in \Gamma$ is *efficient* if it is quasi-efficient and also

$$x^{1,i}(\theta_L) = 1,$$
$$x^{2,i}(\theta_L, \theta_L) = \frac{1}{2}.$$

A mechanism is quasi-efficient whenever the good is always sold to some buyer in uninformed states, and to a θ_H-type in informed states if such a type is present, but it may exclude θ_L-types.[8] A mechanism is efficient whenever it is quasi-efficient and the good is always sold to a θ_L-type in informed states if no θ_H-type is present. Given an efficient mechanism γ, the surplus in state $(2, i)$ is maximized and denoted by $\bar{S}^{2,i} \equiv S^{2,1}(\gamma)$.

Proposition 2. *Suppose that sellers can commit to both information provision and sale mechanisms and that Assumption 1 is satisfied. Then (π^*, γ^*) is a symmetric equilibrium posting strategy if and only if $\pi^* = 1$, γ^* is efficient, $R^{2,i}(\gamma^*) \leq R^{1,i}(\gamma^*)$ and $R^{1,i}(\gamma^*) = \frac{\bar{S}^{2,i}}{2}$.*

[8]This follows a corresponding definition in Burguet and Sákovics (1999).

All equilibrium outcomes under commitment are efficient subject to the constraints imposed by symmetric visiting strategies. Specifically, constrained efficiency requires that (a) there is full information provision, so that θ_H-type buyers, if present at a selling site, can be identified, that (b) the sale mechanisms are efficient, so that goods are allocated first to θ_H-types and then to θ_L-types, and that (c) the equilibrium in sellers' posting strategies is symmetric. To see the last point, note that any unconstrained efficient distribution of buyers across sale sites assigns each buyer to a single seller, so that no buyer is left unserved and no good is left unsold. When constrained by the absence of coordination, efficiency requires maximising the likelihood of having one buyer visit each seller, which is guaranteed under symmetry given that $q^*((\pi^*, \gamma^*), (\pi^*, \gamma^*)) = \frac{1}{2}$.

Under Assumption 1, ex post optimal mechanisms are efficient and information provision increases buyers' expected informational rents. When, as in Section 3, sellers cannot commit to sale mechanisms, ex post optimal mechanisms fix how information provision trades off buyer visits and informational rents per buyer and full information provision obtains in equilibrium since this trade-off favours traffic-stealing. Full information provision also obtains in all equilibria identified by Proposition 2, although the intuition is quite different. When sellers commit to both information provision and sale mechanisms, they disentangle the efficiency effects of information provision and the need to deliver rents to buyers to encourage visits. Both sellers post non-exclusionary auctions and take advantage of their allocative efficiency by providing full information, which maximises the pre-sorting surplus available at both sites. Competition determines how this surplus is shared between buyers and sellers, and, in equilibrium, rents are delivered to buyers through non-distortionary transfers.

There is a continuum of symmetric equilibria, differentiated only by the rents provided to buyers in state $(2, i)$. For all equilibrium mechanisms γ^*, buyers' expected rents in state $(1, i)$ are $R^{1,i}(\gamma^*) = \frac{\bar{S}^{2,i}}{2}$. In the equilibrium most favourable to the sellers, $R^{2,i}(\gamma^*) = R^{2,i}(\gamma^{EP}) = \frac{p_H p_L}{2}(\theta_H - \theta_L)$, as the rents delivered to buyers under ex post optimal mechanisms correspond to the minimal amount of rents delivered in any efficient mechanism. All equilibrium mechanisms have congestion effects, so that in the equilibrium most favourable to buyers, $R^{2,i}(\gamma^*) = R^{1,i}(\gamma^*) = \frac{\bar{S}^{2,i}}{2}$. The continuum of rent levels supported in equilibrium is closely related to a corresponding result in Coles and Eeckhout (2003) who assume, however, that buyers are homogenous. In their model, equilibrium mechanisms consist of demand state-dependent prices and are all efficient. In my model, the benefits of screening buyer types imply that auctions have an efficiency advantage.

Competition does not drive sellers' payoffs to zero in any equilibrium. In the one-buyer state, profits are positive since they are given by $\bar{\theta} - \frac{\bar{S}^{2,i}}{2}$ and it is the case that $2\bar{\theta} > \bar{S}^{2,i}$. In the two-buyer state, profits are $\bar{S}^{2,i} - 2R^{2,i}(\gamma^*)$, which is positive except in the equilibrium most favourable to buyers. That sellers do not compete away all profits in the presence of traffic effects has been noted in the literature on competing auctions (see Peters and Severinov (1997) and Burguet and Sákovics (1999)). This is due to the fact that sellers have inelastic supplies, so that under mechanisms with congestion effects, the absence of coordination imposed by symmetric visit strategies hurts buyers and ensures that sellers' payoffs are continuous in their posting strategies. Hence, the discontinuities in demand functions that drive Bertrand outcomes are not present. As in models of Cournot competition, sellers retain some profits, although these vanish as the number of sellers increase (see Peters and Severinov (1997)). That competition leads to full information provision bears some resemblance to the outcomes of Bertrand competition. However, as noted above and explained in detail in Section 4.1, full information do not arise because competition leads sellers to maximise the amount rents delivered to buyers, but because providing full information maximises the amount of profits they can retain.

The key to my results under full commitment lies in sellers' ability to design the efficiency-rents trade-off generated by information provision through an appropriate choice of mechanism. This point is reinforced by focusing on the version of my model with commitment to sale mechanisms but not to information provision. In this case, sellers choose the ex post optimal level of information provision once buyers have sorted into sites. It can be shown that all the symmetric equilibrium outcomes under full commitment identified in Proposition 2 are also symmetric equilibrium outcomes in the model with no commitment to information provision.[9] In that case, the sellers commit to the same mechanisms as in Proposition 2 in informed states but commit to giving away the good to buyers in all uninformed states, so that, ex post, sellers have incentives to provide full information. Hence, commitment to sale mechanisms gives sellers sufficient flexibility to mimic commitment in information provision.

It is unclear if symmetric equilibrium outcomes other than those identified by Proposition 2 could be supported in the absence of the selection assumption in part (i) of Definition 1. Any such additional outcomes would have sellers post mechanisms without congestion effects, and, since in any equilibrium buyers must visit each seller with positive probability, these posting profiles would have to induce a continuum of symmetric equilibria in the buyers' subgame. Note that any such additional equilibrium outcomes would not be constrained efficient. It could also be that some of the equilibrium outcomes identified in Proposition 2 are not supported under alternative selections of visiting equilibria. For example, in the visiting subgame of the equilibrium most favourable to the buyers, there exists a continuum of symmetric equilibria. If the equilibrium in which all buyers visit seller A was selected in this case, seller B's posting strategy would no longer be a best-response. A full characterisation of equilibrium outcomes under arbitrary selections of equilibria in the buyers' subgame does not appear to be straightforward.

The proof of Proposition 2 proceeds in two steps. First, necessary conditions for symmetric equilibria under Assumption 1 characterise information provision, the efficiency of sale mechanisms and buyers' rents. Second, these conditions are shown to be sufficient for equilibrium. In the remainder of the paper, I provide detailed intuition for the necessary

[9]See Proposition 3 in Appendix A.2.

conditions, leaving their precise statement and the sufficiency argument to Appendix A.2.

4.1 Equilibrium Information Provision

Under commitment, any symmetric equilibrium posting strategy must have full information.[10] To establish this, the key remark is that, as information provision increases efficiency, any symmetric posting profile with less than full information allows a Pareto-improving deviation for seller A in which it provides more information. The difficulty with constructing such a deviation is that additional information provision affects the sharing of the surplus between buyers and sellers, which can affect buyers' visit decisions in ways that depend on the original posting profile. However, since sellers commit ex ante to state-contingent rents and to information, seller A can always offset the effect of more information provision on buyers' rents by suitably adjusting transfers. In this way, a deviation is constructed in which buyers' visit decisions are unchanged relative to the original profile and seller A pockets the additional surplus. There are two provisos to this argument. First, the allocations specified by the original posting profile must be such that more information actually increases the expected surplus. This will hold in equilibrium since the sellers post efficient auctions. Second, it cannot be that the original posting profile involves ex post optimal mechanisms. In that case, the buyers' rents are minimised in all demand and information states and seller A need not have the flexibility to adjust rents as required by the construction. Again, Proposition 2 shows that the set of equilibrium mechanisms excludes ex post optimal mechanisms. This discussion makes it clear that the logic of the full information result is more general than my simple model: if sellers (a) compete ex ante through both information provision and sale mechanisms, and (b) equilibrium mechanisms generate higher surplus when buyers have better information, then sellers must provide full information.

4.2 Equilibrium Allocations

For both sellers, posting a quasi-efficient mechanism, i.e., an auction, is weakly dominant.[11] More specifically, given any posting strategy by seller A with a mechanism that excludes buyers of type θ_H and any posting strategy by seller B, there exists a quasi-efficient mechanism for seller A that, mirroring analogous arguments in the monopoly case, keeps θ_H-types at the same level of rents and does not alter the allocation and transfer of θ_L-types. Hence, this alternative mechanism leaves buyers' visit decisions unchanged and yields higher profits to seller A. A similar argument shows that any posting strategy with a sale mechanism in which uninformed buyers are excluded with positive probability is weakly dominated. Virág (2007) provides a related result with two buyer types, but shows that, with a continuum of types, equilibria in mechanisms that are not quasi-efficient can exist, while Virág (2007) and

[10]See Lemma 4 in Appendix A.2.
[11]See Lemma 5 in Appendix A.2.

Pai (2009) provide conditions under which equilibria in quasi-efficient mechanisms exist. In these models, buyers sort ex post, and posting mechanisms that are not quasi-efficient can allow sellers to deliver rents more profitably to the types that visit them. When buyers sort ex ante, sellers' mechanisms affect the level of buyer visits, but not the ex post distribution of types conditional on visits. Hence, a seller cannot benefit from offering a mechanism that is not quasi-efficient whenever there exists a quasi-efficient mechanism that offers the same expected rents to buyers, since the seller claims the additional surplus.

In the monopoly case, Assumption 1 ensures that sellers serve θ_L-types in both informed demand states, since given any mechanism in which θ_L-types are excluded with some probability, the seller can increase profits by posting an efficient mechanism in that keeps θ_L-types' rents unchanged, even if this increases θ_H-type rents. This alternative mechanism increases rents expected over informed types. In the competitive case, an increase in rents in any state increases traffic but may decrease the likelihood of the one-buyer state (when $q^*((\pi^*, \gamma^*), (\pi^*, \gamma^*)) > \frac{1}{2}$), and hence its effect on total profits may depend on the relation between profits in the one-buyer and two-buyer informed states. However, in symmetric equilibria, the probability of demand state $(1, i)$ is maximised, so that marginal changes in traffic have a negligible effect on this probability. Hence, applying the argument for the monopoly case outlined above to a symmetric posting profile with a mechanism that is not efficient allows the construction of a profitable deviation for seller A.[12] When Assumption 1 fails, a monopolist excludes θ_L-types from trade to depress θ_H-type rents. When sellers compete, whether this is profitable depends on whether the increased profits from θ_H-types compensate the drop in traffic in the two-buyer state. In this case, it is difficult to derive a simple necessary condition on equilibrium θ_L-type allocations which, as when Assumption 1 holds, does not depend on information provision.

4.3 Equilibrium Rents

When sellers commit to sale mechanisms, all symmetric equilibrium mechanisms have congestion effects.[13] To see this, rewrite a buyer's expected rents at site A from a symmetric posting strategy (π^*, γ^*) with $\pi^* = 1$ as

$$R^{1,i}(\gamma^*) + \frac{1}{2}(R^{2,i}(\gamma^*) - R^{1,i}(\gamma^*)). \tag{7}$$

That is, it is as though a buyer is charged an 'attendance fee' of $R^{1,i}(\gamma^*)$ along with a 'bonus' ('congestion charge') of $R^{2,i}(\gamma^*) - R^{1,i}(\gamma^*)$ when another buyer attends and $R^{2,i}(\gamma^*) > R^{1,i}(\gamma^*)$ ($R^{2,i}(\gamma^*) \leq R^{1,i}(\gamma^*)$), which, given symmetry, is paid with probability $\frac{1}{2}$. Suppose that $R^{2,i}(\gamma^*) > R^{1,i}(\gamma^*)$, and consider an alternative mechanism $\hat{\gamma}$ for seller A such that $R^{2,i}(\hat{\gamma}) < R^{2,i}(\gamma^*)$. That is, while $\hat{\gamma}$ pays out a lower congestion bonus than γ, buyers are

[12] See Lemma 6 in Appendix A.2.

[13] See Lemma 7 in Appendix A.2.

indifferent between attending sites A and B only if this bonus is handed out more often, i.e., if $q((\pi^*, \hat{\gamma}), (\pi^*, \gamma^*)) > q((\pi^*, \gamma^*), (\pi^*, \gamma^*))$. As sellers can decrease rents while increasing traffic, posting strategies with $R^{2,i}(\gamma^*) > R^{1,i}(\gamma^*)$ admit a profitable deviation.

The condition that $2R^{1,i}(\gamma^*) = \bar{S}^{2,i}$ ensures that, in equilibrium, the marginal cost of attracting additional visit probabilities equates its marginal contribution to site surplus. From (7), we can interpret the rents $R^{1,i}(\gamma^*)$ as an attendance fee paid by the seller to any buyer visiting its site, with the corresponding congestion charge being applied when both buyers visit. Hence, the marginal cost of attracting additional visit probability from both buyers is the attendance fees $2R^{1,i}(\gamma^*)$ paid out to them, since the additional congestion charges are negligible. Meanwhile, the marginal contribution to site surplus generated by additional visit probability is $\bar{S}^{2,i}$, the surplus in the two-buyer informed state. Note that we neglect the additional surplus generated in one-buyer informed state since under symmetry the probability of this state is maximised.

5 Discussion and Conclusion

This paper studies the strategic interactions of sellers who compete for buyers by committing to information provision. When sellers cannot commit to sale mechanisms and compete solely through offers of information, they may prefer to compete in environments in which the established terms of trade offer higher rents to buyers than those of ex post optimal mechanisms, as the former lessen the intensity of competition and lead to lower information provision. Furthermore, as higher surplus mechanisms increase sellers' competitive incentives to provide information, they prefer to compete in environments with low allocative efficiency, and hence low information provision. When sellers commit to both information provision and mechanisms, all symmetric equilibria have full information provision and are constrained efficient. One interpretation of this result is that sellers prefer to channel competition through sale mechanisms rather than through restrictions on information provision. By doing so they maximize the available surplus, while competition determines the equilibrium share of this surplus going to buyers.

The model I present is stylised, but it is tailored specifically to study the question of competition through information provision in a tractable way. While many of the insights from my model are more general, I make strong simplifying assumptions to get exhaustive and tractable results. The two-seller, two-buyer setup counters well-known equilibrium existence and tractability issues in finite directed search and competing auctions.[14] The central difficulty is characterising buyers' visit decisions for any posting profile by sellers, which is one reason why Peters and Severinov (1997), following McAfee (1993), focus on large economies

[14] Burguet and Sákovics (1999) prove existence of a symmetric equilibrium in a 2-seller, n-buyer model. See also Hernando-Veciana (2005) and Virág (2010) for existence results in finite competing auctions, and Galenianos and Kircher (2012) along with Galenianos et al. (2011) for price-posting equilibria in finite markets.

in which a seller's impact on market conditions vanishes. In these models, buyers sort ex post, so that visit strategies are conditioned on types. In my model, buyers sort ex ante and under symmetry buyers' visit strategies are identical. However, in finite markets these simpler visit strategies are still intractable, as condition (2), which determines how buyers' visit decisions vary with sellers' offers, is poorly behaved with more than two buyers. In these more complicated alternatives, obtaining a complete characterisation of all symmetric equilibria, as I do, would be unlikely. On the other hand, the addition of more sellers does not lead to any pronounced tractability problems or alter qualitative results, but increasing the competition faced by a given seller will definitely affect the level of the rents delivered to buyers in equilibrium.

Information structures are generally modelled as signals that map buyers' ex ante into ex post distributions of types, where the latter are ordered by a suitable notion of precision. In my model, ex ante types are either high or low, and information provision is correlated across buyers at any given sale site, so that I allow only two ex-post distributions of types: the informed and uninformed distributions. Note that since I focus on ex ante competition, buyers make their sorting decisions before any information is provided and sellers' information policies allow for a continuous differentiation of the selling sites with respect to information ex ante.[15] However, two directions for generalisation suggest themselves: allowing for more than two ex ante types and allowing for a seller's information provision to be independent across buyers. Both alternatives are related in terms of the complexity added to my analysis. In particular, adding independent information disclosure to my model generates a model with three ex post types: high, low and 'middle' uninformed types. In this model, as with a model with a continuum of ex ante types (with or without independent disclosure), a complete characterisation of equilibrium sale mechanisms is much harder to achieve. This is where my results draw extensively on there being at most two ex post types in all information states, so that Assumption 1 is sufficient to derive equilibrium allocations. On the other hand, as noted in the text, if it could be established that equilibrium mechanisms are such that more information generates higher surplus, then my result that sellers provide full information should be expected.

Finally, while sale mechanisms are both demand and information state-contingent, I require that information provision be independent of demand states. If one interprets information provision as a pre-match investment by sellers in, say, the training of its sales staff, then this assumption is natural. However, my results are qualitatively robust to allowing sellers to condition their information provision on the number of buyers attending their site. Under full commitment, this would be almost immediate. In fact, my results on the neces-

[15] Furthermore, the information structures of my model can be seen to be discrete examples of those of Johnson and Myatt (2006). Consider ex post distribution of valuations F^π for a single buyer over valuation space $\{\theta_L, \bar{\theta}, \theta_H\}$ generated by the information structure of my model with probability π. $\bar{\theta}$ is a rotation point for the family of distributions $\{F^\pi\}$ since for $\pi > \pi'$, $F^\pi(x) \geq F^{\pi'}(x)$ for all $x < \bar{\theta}$ and $F^\pi(x) \leq F^{\pi'}(x)$ for all $x \geq \bar{\theta}$.

sity of full information for symmetric equilibrium are easier to establish in this case, since the sellers face less constraints in shifting rents from informed to uninformed states. When sellers cannot commit to sale mechanisms, sellers' strategy sets, and hence equilibrium analysis, would be more complex. However, the key feature of competition through information under ex post optimal mechanisms is that the efficiency-rents trade-off, which determines the intensity of competition between the sellers, is exogenous. This remains the case under demand state-contingent information policies.

A Appendix

A.1 Characterisation of Incentive-Compatible Mechanisms

Any incentive-compatible mechanism γ that achieves rents $(R^{\eta,\tau}(\gamma))_{\eta,i}$ with non-binding θ_H-type incentive constraints can be replaced by an incentive compatible mechanism that achieves the same levels of expected rents with the same allocations, but in which these constraints bind. Under this new mechanism, profits are unchanged and all traffic and information provision incentives are preserved.

Lemma 1. *Given any posting profile* $((\pi_A, \gamma_A), (\pi_B, \gamma_B))$*, there exists incentive compatible mechanisms* $(\tilde{\gamma}_A, \tilde{\gamma}_B)$ *in which incentive constraints of* θ_H*-types in states* $(1, i)$ *and* $(2, i)$ *are binding and allocations are as in* (γ_A, γ_B)*. Furthermore, under profile* $((\pi_A, \tilde{\gamma}_A), (\pi_B, \gamma_B))$*, buyers' rents and sellers' profits are the same as under profile* $((\pi_A, \gamma_A), (\pi_B, \gamma_B))$*.*

Proof of Lemma 1. Consider an incentive compatible mechanism γ_A at site A (the proof for seller B is symmetric) such that $(\mathrm{IC}^{1,i}(\theta_H))$ is slack. In particular, say

$$x^{1,i}(\theta_H)\theta_H - y^{1,i}(\theta_H) = x^{1,i}(\theta_L)\theta_H - y^{1,i}(\theta_L) + C,$$

with $C > 0$. Consider an alternative mechanism $\tilde{\gamma}_A$ identical to γ_A except that

$$\begin{aligned}\tilde{y}^{1,i}(\theta_H) &= y^{1,i}(\theta_H) + p_L C \\ \tilde{y}^{1,i}(\theta_L) &= y^{1,i}(\theta_L) - p_H C.\end{aligned}$$

In that case,

$$\begin{aligned}\tilde{x}^{1,i}(\theta_H)\theta_H - \tilde{y}^{1,i}(\theta_H) &= x^{1,i}(\theta_H)\theta_H - y^{1,i}(\theta_H) - p_L C \\ &= x^{1,i}(\theta_H)\theta_H - y^{1,i}(\theta_H) - C + p_H C \\ &= x^{1,i}(\theta_L)\theta_H - y^{1,i}(\theta_L) + p_H C \\ &= \tilde{x}^{1,i}(\theta_L)\theta_H - \tilde{y}^{1,i}(\theta_L).\end{aligned}$$

Thus, $\widetilde{IC}^{1,i}(\theta_H)$ binds. Since under $\tilde{\gamma}_A$ the transfer of type θ_L has been decreased, $\widetilde{PC}^{1,i}(\theta_L)$ is satisfied. Since both $\widetilde{IC}^{1,i}(\theta_H)$ and $\widetilde{PC}^{1,i}(\theta_L)$ hold, then so does $\widetilde{PC}^{1,i}(\theta_H)$. Finally, under

$\tilde{\gamma}_A$ θ_H-types are worse off and θ_L-types are better off, so that $\widetilde{IC}^{1,i}(\theta_L)$ holds. Hence $\tilde{\gamma}_A$ is incentive compatible.

Profits for seller A in state $(1, i)$ under mechanism $\tilde{\gamma}_A$ are given by

$$\begin{aligned} p_H\tilde{y}^{1,i}(\theta_H) + p_L\tilde{y}^{1,i}(\theta_L) &= p_H y^{1,i}(\theta_H) + p_L y^{1,i}(\theta_L) + p_H p_L C - p_L p_H C \\ &= p_H y^{1,i}(\theta_H) + p_L y^{1,i}(\theta_L), \end{aligned}$$

where the last line is profits under γ_A in state $(1, i)$. Profits in other states are also unaffected. The proof for the case in which $IC^{2,i}(\theta_H)$ is slack is identical, with reduced-form mechanisms replacing the mechanisms. To that end, note that in state $(2, i)$, profits under mechanism γ_A are given by

$$\begin{aligned} &p_H^2 \left[2y^{2,i}(\theta_H, \theta_H)\right] + 2p_L p_H \left[y^{2,i}(\theta_H, \theta_L) + y^{2,i}(\theta_L, \theta_H)\right] + p_L^2 \left[2y^{2,i}(\theta_L, \theta_L)\right] \\ &= 2 \left[p_H Y^{1,i}(\theta_H) + p_L Y^{1,i}(\theta_L)\right]. \end{aligned}$$

As the proof manipulates mechanisms in different demand states independently, given an original profile where the incentive compatibility constraints of θ_H-types in both demand states are slack, one could find a rent and profit-equivalent mechanism with incentive constraints binding in both states by the same procedure. □

Denote $\tilde{\gamma}$ as the *$IC(\theta_H)$-equivalent* of γ. Similarly, denote by $\tilde{\Gamma}$ the set of IC(θ_H)-equivalent mechanisms. Given information provision (π_A, π_B), a game with mechanisms $(\gamma_A, \gamma_B) \in (\Gamma \setminus \tilde{\Gamma})^2$ generates the same distribution over outcomes as a game with mechanisms $(\tilde{\gamma}_A, \tilde{\gamma}_B)$. That is, excluding mechanisms in $\Gamma \setminus \tilde{\Gamma}$ does not reduce the set of equilibria in terms of information provision. On the other hand, when sellers also choose mechanisms, it is not the case that equilibrium mechanisms must belong to $\tilde{\Gamma}$. However, Lemma 1 states that excluding mechanisms in $\Gamma \setminus \tilde{\Gamma}$ does not reduce the set of equilibrium allocations, traffic levels and payoffs. In what follows, incentive compatible mechanisms refers to mechanisms in $\tilde{\Gamma}$.

Denote *low-type rents under mechanism γ in state (η, τ)* by $r^{\eta,\tau}(\gamma)$. These are the rents offered to θ_L-types in informed states and to the uninformed otherwise. Lemma 1 justifies the use of the well-known result that mechanisms $\gamma \in \tilde{\Gamma}$ are characterised by monotone allocation probabilities, low-type rents $r^{\eta,\tau}(\gamma) \geq 0$ for all states (η, τ) and the 'envelope' condition for high-type rents. The proof of the following lemma is standard and omitted.

Lemma 2. *$\gamma \in \tilde{\Gamma}$ if and only if $x^{1,i}(\theta_H) \geq x^{1,i}(\theta_L)$, $X^{1,i}(\theta_H) \geq X^{1,i}(\theta_L)$, $r^{\eta,\tau}(\gamma) \geq 0$ for all $\eta \in \{1, 2\}$ and $\tau \in \{i, u\}$, and*

$$\begin{aligned} &R^{\eta,u}(\gamma) = r^{\eta,u}(\gamma) \text{ for } \eta \in \{1, 2\}, \\ &R^{1,i}(\gamma) = r^{1,i}(\gamma) + p_H x^{1,i}(\theta_L)(\theta_H - \theta_L), \\ &R^{2,i}(\gamma) = r^{2,i}(\gamma) + p_H X^{2,i}(\theta_L)(\theta_H - \theta_L). \end{aligned}$$

A.2 Proofs of Main Results

Proof of Proposition 1. The following lemma characterises candidates for optimal information provision in symmetric equilibria with mechanisms that are exogenous.

Lemma 3. *Assume that sellers can commit only to information provision and that, at both sites, sale mechanisms are exogenously set to* γ *such that (a)* $R^{1,u}(\gamma) = R^{2,u}(\gamma) = 0$*, (b)* $R^{1,i}(\gamma) > R^{2,i}(\gamma)$ *and (c)* $S^{2,i}(\gamma) - \bar{\theta} \leq 2R^{2,i}(\gamma)$*. Then there is a unique candidate profile* (π^*, π^*) *for symmetric equilibrium in information provision, where*

$$\pi^* \equiv \begin{cases} \frac{-(R^{1,i}(\gamma)+R^{2,i}(\gamma))\bar{\theta}}{2R^{1,i}(\gamma)(S^{2,i}(\gamma)-\bar{\theta}-(R^{1,i}(\gamma)+R^{2,i}(\gamma)))} & \text{if } 2R^{1,i}(\gamma) > \bar{\theta} \text{ and } R^{1,i}(\gamma) + R^{2,i}(\gamma) > \frac{2R^{1,i}(\gamma)(S^{2,i}(\gamma)-\bar{\theta})}{2R^{1,i}(\gamma)-\bar{\theta}}, \\ 1 & \text{otherwise.} \end{cases}$$

Proof. Given any mechanism γ satisfying the conditions in the statement, seller A's profits can be written as

$$\begin{aligned} \mathcal{P}_A((\pi_A, \gamma), (\pi_B, \gamma), q^*) = q^{*2} \left[\pi_A S^{2,i}(\gamma) + (1-\pi_A)\bar{\theta} - 2\pi_A R^{2,i}(\gamma)\right] \\ + 2q^*(1-q^*)\left[\bar{\theta} - \pi_A R^{1,i}(\gamma)\right]. \end{aligned} \tag{8}$$

At symmetric profiles, the market is shared equally between the two sellers, which maximises the probability that a seller is visited by a single buyer ($2q^*(1-q^*)$). Hence, marginal shifts in information provision at symmetric profiles have no effect on this probability.[16] This simplifies the expression for seller A's marginal payoff at a symmetric profile (π, π) under mechanism γ, which is given by

$$\begin{aligned} \left.\frac{\partial \mathcal{P}_A((\pi_A, \gamma), (\pi_B, \gamma), q^*)}{\partial \pi_A}\right|_{\pi_A=\pi_B=\pi} = \left.\frac{\partial q^*}{\partial \pi_A}\right|_{\pi_A=\pi_B=\pi} \left[\pi S^{2,i}(\gamma) + (1-\pi)\bar{\theta} - 2\pi R^{2,i}(\gamma)\right] \\ + \frac{1}{4}\left[S^{2,i}(\gamma) - \bar{\theta} - 2R^{2,i}(\gamma)\right] - \frac{1}{2}R^{1,i}(\gamma). \end{aligned} \tag{9}$$

Setting (9) equal to zero and checking the conditions for which $\pi < 1$, we obtain the expression for π^*. Note that when $\pi^* < 1$, its comparative statics with respect to $S^{2,i}(\gamma)$, $R^{1,i}(\gamma)$ and $R^{2,i}(\gamma)$ are as claimed in the text.

To show uniqueness of the symmetric equilibrium candidate, I establish that both $\pi S^{2,i}(\gamma) + (1-\pi)\bar{\theta} - 2\pi R^{2,i}(\gamma)$ and $\frac{\partial q^*}{\partial \pi_A}\big|_{\pi_A=\pi_B=\pi}$ are decreasing in π. First, seller A's profits in the two-buyer state decrease in π_A, since the term in the first brackets of (8) is linear in π_A and $\mathcal{S}^{2,i} - \bar{\theta} - 2R^{2,i} \leq 0$ by assumption. Second, by (2) and using the fact that $R^{1,u}(\gamma) = R^{2,u}(\gamma) = 0$, we have that

$$q^*((\pi_A, \gamma), (\pi_B, \gamma)) = \frac{\pi_A R^{1,i}(\gamma) - \pi_B R^{2,i}(\gamma)}{(R^{1,i}(\gamma) - R^{2,i}(\gamma))(\pi_A + \pi_B)},$$

[16] This observation, often useful in the the rest of the paper, is due to the binomial distribution of demand at sale sites. That is, if $X \sim B(n, q)$ then $\frac{\partial \Pr(X=k)}{\partial q} > 0$ whenever $k > qn$, where qn is the mean state of X. If qn is an integer, then $\frac{\partial \Pr(X=qn)}{\partial q} = 0$. That is, if q is increased marginally, states above the mean state become more likely and states below the mean less likely, while the probability of the mean state is unchanged.

and it can be verified that

$$\left.\frac{\partial q^*}{\partial \pi_A}\right|_{\pi_A=\pi_B=\pi} = \frac{R^{1,i}(\gamma)+R^{2,i}(\gamma)}{4\pi(R^{1,i}(\gamma)-R^{2,i}(\gamma))},$$

which is decreasing in π since $R^{1,i}(\gamma) > R^{2,i}(\gamma)$.

□

Since the profit function in (8) does not have convenient properties in π (e.g., it is not concave), Lemma 3 alone is not sufficient to establish the existence of a symmetric equilibrium. To complete the proof of Proposition 1, the first step is to show that under ex post optimal mechanisms, the conclusion of Lemma 3 is valid. To do this, it suffices to show that under Assumption 1, mechanism γ^{EP} satisfies the properties of those mechanisms in the lemma. For (a), as noted in the text, $R^{1,u}(\gamma^{EP}) = R^{2,u}(\gamma^{EP}) = 0$. Given Assumption 1, under ex post optimal mechanisms $R^{1,i}(\gamma^{EP}) = p_H(\theta_H - \theta_L)$ and $R^{2,i}(\gamma^{EP}) = X^{2,i}(\theta_L)p_H(\theta_H-\theta_L)$. For (b), note that since $X^{2,i}(\theta_L) = \frac{p_L}{2} < 1$, we have that $R^{1,i}(\gamma^{EP}) > R^{2,i}(\gamma^{EP})$. For (c), note that

$$\begin{aligned}
S^{2,i}(\gamma^{EP}) - \bar{\theta} - 2R^{2,i}(\gamma^{EP}) &= S^{2,i}(\gamma^{EP}) - \bar{\theta} - 2X^{2,i}(\theta_L)p_H(\theta_H-\theta_L) \\
&= \theta_H\left[1-p_L^2\right] + \theta_L p_L^2 - p_L p_H(\theta_H-\theta_L) \\
&= 0,
\end{aligned}$$

where the second equality follows from Assumption 1. Hence, the conclusion of Lemma 3 applies to γ^{EP}, and furthermore

$$\begin{aligned}
2R^{1,i}(\gamma^{EP}) - \bar{\theta} &= p_H\theta_H - p_L\theta_L - 2p_H\theta_L \\
&= p_H\theta_H + p_L\theta_L - 2\theta_L \\
&< \theta_L(p_L-1) \\
&< 0,
\end{aligned}$$

where the first inequality follows from Assumption 1. Hence, by Lemma 3 the only candidate for symmetric equilibrium is $\pi^* = 1$. To establish existence of equilibrium, I show that under Assumption 1, $\mathcal{P}_A((\pi_A,\gamma^{EP}),(1,\gamma^{EP}),q^*)$ is increasing in π_A. Given $\pi_A \le 1$, $q^* \le \frac{1}{2}$, and if π_A is such that $q^* > 0$, then

$$\begin{aligned}
\mathcal{P}_A((\pi_A,\gamma^{EP}),(1,\gamma^{EP}),q^*) &= \left(\frac{\pi_A - \frac{p_L}{2}}{(1+\pi_A)(1-\frac{p_L}{2})}\right)^2 \bar{\theta} \\
&\quad + 2\left(\frac{(\pi_A - \frac{p_L}{2})(1-\frac{\pi_A p_L}{2})}{((1+\pi_A)(1-\frac{p_L}{2}))^2}\right)\big(\bar{\theta} - \pi_A p_H(\theta_H-\theta_L)\big) \\
&= \left(\frac{\pi_A - \frac{p_L}{2}}{((1+\pi_A)(1-\frac{p_L}{2}))^2}\right) \\
&\qquad \cdot \left(\bar{\theta}(p_H\pi_A + 2 - \frac{p_L}{2}) + 2(1-\frac{\pi_A p_L}{2})\pi_A p_H(\theta_H-\theta_L)\right) \\
&\equiv A(\pi_A)\big(B(\pi_A)+C(\pi_A)\big)
\end{aligned}$$

Where $B(\pi_A)$ is clearly increasing in π_A, while it can be verified that $A(\pi_A)$ and $C(\pi_A)$ are increasing whenever $\pi_A \leq 1 + p_L$ and $\pi_A \leq \frac{1}{p_L}$, respectively, which is always true. □

Proof of Proposision 2. Lemmas 4-7 provide necessary conditions for symmetric equilibrium information provision, allocations and rents.

Lemma 4. *Suppose that (π^*, γ^*) is a symmetric equilibrium posting strategy, that $\gamma^* \neq \gamma^{EP}$ and that $2S^{1,i}(\gamma^*) + S^{2,i}(\gamma^*) > 2S^{1,u}(\gamma^*) + S^{2,u}(\gamma^*)$. Then $\pi^* = 1$.*

Proof of Lemma 4. Lemma 4, stated in terms of $IC(\theta_H)$-equivalent mechanisms, requires that it not be the case that $\gamma^* \in \tilde{\Gamma}$ is such that $r^{2,i}(\gamma^*) = r^{2,u}(\gamma^*) = r^{1,i}(\gamma^*) = r^{1,u}(\gamma^*) = 0$. This condition ensures that it is always possible, for at least one state, to decrease the buyers' rents without violating incentive compatibility. The argument establishes the claim of the lemma for seller A and the argument for seller B is symmetric.

Suppose that (π^*, γ^*) is a symmetric equilibrium posting strategy, that $\gamma^* \neq \gamma^{EP}$ and that $2S^{1,i}(\gamma^*) + S^{2,i}(\gamma^*) > 2S^{1,u}(\gamma^*) + S^{2,u}(\gamma^*)$. Suppose also, towards a contradiction, that $\pi^* < 1$. Consider a deviation by seller A to posting strategy $(\hat{\pi}, \hat{\gamma})$, identical to (π^*, γ^*) except that

$$\begin{aligned} \hat{\pi} &= \pi^* + \lambda \\ r^{\eta,\tau}(\hat{\gamma}) &= r^{\eta,\tau}(\gamma^*) - \delta^{\eta,\tau}, \end{aligned}$$

where $\lambda \in [0, 1 - \pi^*]$ and $\delta^{\eta,\tau} \leq r^{\eta,\tau}(\gamma^*)$ for all (η, τ). For this deviant profile not to affect buyers' visit decisions (or expected rents), we need

$$\begin{aligned} q &\left[(\pi^* + \lambda)\left[r^{2,i}(\gamma^*) - \delta^{2,i} + z^{2,i}(\gamma^*)\right] + (1 - \pi^* - \lambda)\left[r^{2,u}(\gamma^*) - \delta^{2,u}\right]\right] \\ &+ (1 - q)\left[(\pi^* + \lambda)\left[r^{1,i}(\gamma^*) - \delta^{1,i} + z^{1,i}(\gamma^*)\right] + (1 - \pi^* - \lambda)\left[r^{1,u}(\gamma^*) - \delta^{1,u}\right]\right] \\ &= q\left[\pi^*\left[r^{2,i}(\gamma^*) + z^{2,i}(\gamma^*)\right] + (1 - \pi^*)r^{2,u}(\gamma^*)\right] \\ &+ (1 - q)\left[\pi^*\left[r^{1,i}(\gamma^*) + z^{1,i}(\gamma^*)\right] + (1 - \pi^*)r^{1,u}(\gamma^*)\right], \end{aligned}$$

where $z^{1,i}(\gamma^*) = p_H x^{1,i}(\theta_L)(\theta_H - \theta_L) \geq 0$ and $z^{2,i}(\gamma^*) = p_H X^{2,i}(\theta_L)(\theta_H - \theta_L) \geq 0$ are the expected informational rents given the allocations of the original mechanism γ^*. By rearranging, since $q((\pi^*\gamma^*), (\pi^*, \gamma^*)) = \frac{1}{2}$ and $\lambda > 0$, we have

$$\begin{aligned} &\frac{\pi^* + \lambda}{\lambda}\left[\delta^{2,i} + \delta^{1,i}\right] + \frac{1 - \pi^* - \lambda}{\lambda}\left[\delta^{2,u} + \delta^{1,u}\right] \\ &\qquad = \left[r^{2,i}(\gamma^*) + z^{2,i}(\gamma^*) - r^{2,u}(\gamma^*)\right] + \left[r^{1,i}(\gamma^*) + z^{1,i}(\gamma^*) - r^{1,u}(\gamma^*)\right]. \end{aligned} \tag{10}$$

The sign of the right-hand side (RHS) of (10) is given by the properties of the mechanism γ^*. It is positive if buyers prefer, on average, to be informed at site A, and negative if buyers prefer, on average, to be uninformed.

Suppose that $RHS > 0$. First, suppose also that $\pi^* > 0$. By assumption, there exists some $r^{\hat{\eta},\hat{\tau}}(\gamma^*) > 0$. Set $\delta^{\eta,\tau} = 0$ for all $(\eta,\tau) \neq (\hat{\eta},\hat{\tau})$ and $\delta^{\hat{\eta},\hat{\tau}} \in (0,(1-\pi^*)RHS)$, which is well-defined since $\pi^* < 1$. Furthermore, $\pi^* \in (0,1)$, implies that

$$\begin{aligned}\lim_{\lambda\to 0} LHS((\delta^{\eta,\tau}),\lambda) &= \infty \\ &> RHS,\end{aligned}$$

as well as

$$\begin{aligned}LHS((\delta^{\eta,\tau}),1-\pi_A) &\leq \frac{\delta^{\hat{\eta},\hat{\tau}}}{1-\pi^*} \\ &< RHS.\end{aligned}$$

Hence there exists $\hat{\lambda} \in (0,1-\pi_A)$ such that $LHS((\delta^{\eta,\tau}),\hat{\lambda}) = RHS$. Second, suppose that $\pi^* = 0$. If either $r^{1,u}(\lambda^*) > 0$ or $r^{2,u}(\lambda^*) > 0$, then the previous argument can be applied without change. Otherwise, towards a contradiction, suppose that $r^{1,u}(\lambda^*) = r^{2,u}(\lambda^*) = 0$. Then, by symmetry, buyers get no rents in equilibrium. But in this case, a deviation by seller A to posting strategy $(\hat{\pi},\hat{\gamma})$ with $\lambda = 0$ and $\delta^{2,u} < 0$ sufficiently small would lead to all buyers visiting site A and increased profits for seller A, yielding the desired contradiction.

Now suppose that $RHS < 0$. Set $\delta^{\eta,\tau} = 0$ for all $(\eta,\tau) \neq (2,u)$ and $\delta^{2,u} < 0$. Then, since $\pi^* < 1$,

$$\begin{aligned}\lim_{\lambda\to 0} LHS((\delta^{\eta,\tau}),\lambda) &= -\infty \\ &< RHS,\end{aligned}$$

as well as

$$\begin{aligned}LHS((\delta^{\eta,\tau}),1-\pi_A) &= 0 \\ &> RHS.\end{aligned}$$

Hence, again, there exists $\hat{\lambda} \in (0,1-\pi_A)$ such that $LHS((\delta^{\eta,\tau}),\hat{\lambda}) = RHS$

Finally, if $RHS = 0$, then buyers are indifferent between informed and uninformed states at site A under λ^* and the seller can increase information provision without shifting traffic by setting $\delta^{\eta,\tau} = 0$ for all $\eta \in \{1,2\}, \tau \in \{i,u\}$.

In all cases, the arguments above yield a deviation for seller A which keeps buyers' visiting decisions and rent payouts unchanged and, since $q^*((\pi^*,\gamma^*),(\pi^*,\gamma^*)) = \frac{1}{2}$ and $2S^{1,i}(\gamma^*) + S^{2,i}(\gamma^*) > 2S^{1,u}(\gamma^*) + S^{2,u}(\gamma^*)$, strictly increases the surplus available at site A, yielding the desired contradiction to the optimality of equilibrium posting strategy (π^*,γ^*). □

Lemma 5. *For any seller, a posting strategy (π,γ) in which γ is not quasi-efficient is weakly dominated.*

Proof of Lemma 5. Consider a posting strategy (π, γ) or seller A, with mechanism $\gamma \in \tilde{\Gamma}$ such that $x^{1,i}(\theta_H) < 1$, with the argument for seller B being symmetric. Consider an alternative mechanism $\hat{\gamma}$ identical to γ except that

$$\hat{x}^{1,i}(\theta_H) = 1$$
$$\hat{y}^{1,i}(\theta_H) = y^{1,i}(\theta_H) + (1 - x^{1,i}(\theta_H))\theta_H.$$

We have $\hat{x}^{1,i}(\theta_H) > x^{1,i}(\theta_H) \geq \hat{x}^{1,i}(\theta_L) > x^{1,i}(\theta_L)$ and $r^{1,i}(\hat{\gamma}) = r^{1,i}(\gamma) \geq 0$ since $\gamma \in \tilde{\Gamma}$, and so $\hat{\gamma} \in \tilde{\Gamma}$. Note that $R^{1,i}(\hat{\gamma}) = R^{1,i}(\gamma)$ and hence buyer rents and visit decisions are identical under γ and $\hat{\gamma}$. However, since θ_H-types make larger transfers in state $(1, i)$, seller A's profits are strictly higher under $\hat{\gamma}$ than under γ as long as (a) seller B's posting strategy is such that buyers sometimes visit seller A and/or (b) $\pi > 0$. Otherwise, seller A's profits are identical under γ and $\hat{\gamma}$.

If the mechanism γ is such that $X_k^{2,i}(\theta_H) < p_L + \frac{1}{2}p_H$ and $x^{2,i}(\theta_H, \theta_L) + x^{2,i}(\theta_L, \theta_H) < 1$, then the previous argument for state $(1, i)$ applies directly to the reduced-form mechanisms in state $(2, i)$. Now suppose that under γ, $X_k^{2,i}(\theta_H) < p_L + \frac{1}{2}p_H$ and $x^{2,i}(\theta_H, \theta_L) + x^{2,i}(\theta_L, \theta_H) = 1$. Hence, it must be that $x^{2,i}(\theta_L, \theta_H) > 0$, that is, a θ_L-type buyer is sometimes allocated the good in the presence of a θ_H-type. Consider an alternative mechanism $\hat{\gamma}$ identical to γ except that

(i). θ_L-types never get preference over θ_H-types, $\hat{x}^{2,i}(\theta_H, \theta_L) = 1$ and $\hat{x}^{2,i}(\theta_L, \theta_H) = 0$, so that

$$\hat{X}^{2,i}(\theta_L) = X^{2,i}(\theta_L) - p_H x^{2,i}(\theta_L, \theta_H)$$
$$\hat{X}^{2,i}(\theta_H) = X^{2,i}(\theta_H) + p_L x^{2,i}(\theta_L, \theta_H).$$

(ii). Transfers are adjusted so that rents to both types are unchanged

$$\hat{Y}^{2,i}(\theta_L) = Y^{2,i}(\theta_L) - \theta_L(X^{2,i}(\theta_L) - \hat{X}^{2,i}(\theta_L))$$
$$\hat{Y}^{2,i}(\theta_H) = Y^{2,i}(\theta_H) + \theta_H(\hat{X}^{2,i}(\theta_H) - X^{2,i}(\theta_H)).$$

Note that (i) implies that under $\hat{\gamma}$, $\hat{X}^{2,i}(\theta_H) = p_L + \frac{1}{2}p_H$. Hence, since $\gamma \in \tilde{\Gamma}$, we have that $\hat{X}^{2,i}(\theta_H) > X^{2,i}(\theta_H) \geq X^{2,i}(\theta_L) > \hat{X}^{2,i}(\theta_L)$. Along with condition (ii), this implies that $\hat{\gamma} \in \tilde{\Gamma}$.

Profits to seller A in the state $(2, i)$ under $\hat{\gamma}$ are given by

$$2\left[p_L \hat{Y}^{2,i}(\theta_L) + p_H \hat{Y}^{2,i}(\theta_H)\right] = 2\left[p_L Y^{2,i}(\theta_L) + p_H Y^{2,i}(\theta_H) + p_H p_L(\theta_H - \theta_L)x^{2,i}(\theta_L, \theta_H)\right]$$
$$> 2\left[p_L Y^{2,i}(\theta_L) + p_H Y^{2,i}(\theta_H)\right],$$

where the last expression is profits to seller A in state $(2, i)$ under γ. The inequality follows since by hypothesis $x^{2,i}(\theta_L, \theta_H) > 0$. Thus seller A's profits are strictly higher under $\hat{\gamma}$ than under γ as long as (a) seller B's posting strategy is such that buyers sometimes visit seller A and/or (b) $\pi > 0$. Otherwise, seller A's profits are identical under γ and $\hat{\gamma}$.

Similarly, if the mechanism γ is such that $x^{\eta,u} < 1$ for some $\eta \in \{1, 2\}$, then consider an alternative mechanism $\hat{\gamma}$, identical to γ except that in state (η, u)

$$\hat{x}^{\eta,u} = 1$$
$$\hat{y}^{\eta,u} = y^{\eta,u} + (1 - x^{\eta,u})\bar{\theta},$$

and the arguments from above apply. $\square$

Lemma 6. *Suppose that Assumption 1 is satisfied and that (π^*, γ^*) is a symmetric equilibrium posting strategy. Then γ is efficient.*

Proof of Lemma 6. Suppose that (π^*, γ^*), with $\gamma^* \in \tilde{\Gamma}$, is a symmetric equilibrium posting strategy and suppose, towards a contradiction, that $x^{1,i}(\theta_L) < 1$. Then

$$y^{1,i}(\theta_L) = \theta_L x^{1,i}(\theta_L) - r^{1,i}(\gamma^*), \tag{11}$$

and, by Lemma 5,

$$y^{1,i}(\theta_H) = \theta_H - x^{1,i}(\theta_L)(\theta_H - \theta_L) - r^{1,i}(\gamma^*). \tag{12}$$

By (11) and (12), write seller A's profits (the argument for seller B is symmetric) conditional on ($\mathrm{IC}^{1,i}(\theta_H)$) binding and type θ_L receiving rents $r^{1,i}(\gamma^*)$ as

$$x^{1,i}(\theta_L)(\theta_L - p_H\theta_H) + p_H\theta_H - r^{1,i}(\gamma^*). \tag{13}$$

These are increasing in $x^{1,i}(\theta_L)$ whenever $\theta_L > p_H\theta_H$. Since $x^{1,i}(\theta_H) = 1$ by Lemma 5, an increase in $x^{1,i}(\theta_L)$ maintains incentives compatibility so seller A can increase profits in state $(1, i)$ by doing so. This increases traffic to site A (since rents to θ_H-types increase). But at a symmetric equilibrium $q^*((\pi^*, \gamma^*), (\pi^*, \gamma^*)) = \frac{1}{2}$ and marginal changes in traffic have negligible effects on the probability of state $(1, i)$ (which is $2q^*(1 - q^*)$), so that the profits of seller A increase with marginal changes in $x^{1,i}(\theta_L)$ if profits in state $(2, i)$ are nonnegative. However, note that the previous argument ensures that profits state $(2, i)$ must be nonnegative in a symmetric equilibrium. If not, a seller could marginally increase transfers in state $(2, i)$ without affecting traffic significantly in state $(1, i)$, while both traffic and losses per buyer would decrease in state $(2, i)$. $\square$

Lemma 7. *Suppose that Assumption 1 is satisfied and that (π^*, γ^*) is a symmetric equilibrium. Then $R^{2,i}(\gamma^*) \leq R^{1,i}(\gamma^*)$ and $R^{1,i}(\gamma^*) = \frac{\bar{S}^{2,i}}{2}$.*

Proof of Lemma 7. To show that $R^{2,i}(\gamma^*) \leq R^{1,i}(\gamma^*)$, consider a symmetric equilibrium posting strategy (π^*, γ^*) with $\pi^* = 1$, and an efficient mechanism $\gamma^* \in \tilde{\Gamma}$ such that $R^{1,i}(\gamma^*) < R^{2,i}(\gamma^*)$. This last fact implies that $r^{2,i}(\gamma^*) > 0$. Consider a mechanism $\hat{\gamma}$ for seller A (the case for seller B is symmetric) identical to γ except that $r^{2,i}(\hat{\gamma}) = r^{2,i}(\gamma^*) - \Delta$, where $\Delta > 0$. By the argument in the text, $\hat{\gamma}$ leads to an increase in the visit probabilities of buyers to site A. For Δ sufficiently small, the difference in the probability of the one-buyer state under γ^* and $\hat{\gamma}$ is negligible, while the probability of the two-buyer state, where rents are lower, is higher under $\hat{\gamma}$ than under γ^*. This deviation is thus profitable given that profits in the two-buyer state are nonnegative (see the proof of Lemma 6).

To show that $R^{1,i}(\gamma^*) = \frac{\bar{S}^{2,i}}{2}$, consider alternative mechanisms $\hat{\gamma}$ that lead to marginal changes in $R^{1,i}(\gamma^*)$ and $R^{2,i}(\gamma^*)$ that leave $\pi^* = 1$ and allocative efficiency unchanged. Assume for now that $r^{1,i}(\gamma^*) > 0$ and $r^{2,i}(\gamma^*) > 0$ to ensure that it is always possible to construct such mechanisms $\hat{\gamma}$ by varying the transfers under γ^*. The equilibrium profits for seller A are given by

$$\mathcal{P}_A((\pi^*, \gamma^*), (\pi^*, \gamma^*), q^*) = q^{*2}[\bar{S}^{2,i} - 2R^{2,i}(\gamma^*)] + 2q^*(1 - q^*)[\bar{\theta} - R^{1,i}(\gamma^*)].$$

At a symmetric profile, the marginal changes in the term $q^*(1 - q^*)$ can be ignored and thus

$$\frac{\partial \mathcal{P}_A((\pi^*, \gamma^*), (\pi^*, \gamma^*), q^*)}{\partial R^{1,i}} = 2q^* \left[\frac{\partial q^*}{\partial R^{1,i}} (\bar{S}^{2,i} - 2R^{2,i}(\gamma^*)) - (1 - q^*) \right],$$

where, at a symmetric profile with $\pi^* = 1$ we have $q^* = \frac{1}{2}$ and $\frac{\partial q^*}{\partial R^{1,i}} = \frac{1}{4(R^{1,i}(\gamma^*) - R^{2,i}(\gamma^*))}$. Thus

$$\begin{aligned} \frac{\partial \mathcal{P}_A((\pi^*, \gamma^*), (\pi^*, \gamma^*), q^*)}{\partial R^{1,i}} &= \left(\frac{1}{4}\right) \frac{\bar{S}^{2,i} - 2R^{2,i}(\gamma^*)}{R^{1,i}(\gamma^*) - R^{2,i}(\gamma^*)} - \frac{1}{2} \\ &= 0 \quad \text{only when } R^{1,i}(\gamma^*) = \frac{\bar{S}^{2,i}}{2}. \end{aligned}$$

In the same way, it can be computed that $\frac{\partial \mathcal{P}_A((\pi^*, \gamma^*), (\pi^*, \gamma^*), q^*)}{\partial R^{2,i}} = 0$ only when $R^{1,i}(\gamma^*) = \frac{\bar{S}^{2,i}}{2}$. That is, the same condition holds for marginal changes in expected rents in both one-buyer and two-buyer informed states. Since $\frac{\partial \mathcal{P}_A((\pi^*, \gamma^*), (\pi^*, \gamma^*), q^*)}{\partial R^{2,i}} = 0$ and $\frac{\partial \mathcal{P}_A((\pi^*, \gamma^*), (\pi^*, \gamma^*))}{\partial R^{1,i}, q^*} = 0$ yield the same condition, we need to worry about the existence of derivatives only when $r^{1,i}(\gamma^*) = r^{2,i}(\gamma^*) = 0$. But then an argument considering deviations $R^{1,i}(\gamma^*) + \Delta$ or $R^{2,i}(\gamma^*) + \Delta$ yields the result. □

The proof of Proposition 2 follows from Lemmas 4-7. The necessity of mechanism efficiency for symmetric equilibrium has been established in Lemma 6. Under an efficient mechanism γ, information provision increases the surplus available at a selling site since two buyers generate more surplus when informed than when uninformed, as $S^{2,i}(\gamma) = \bar{S}^{2,i} > \bar{\theta}$ and $S^{1,i}(\gamma) = \bar{\theta}$, and hence Lemma 4 states that $\pi = 1$ is necessary for symmetric equilibrium unless both sellers commit to the ex post optimal mechanisms. The necessity of full information under Assumption 1 for ex post optimal mechanisms follows from Proposition 1. Lemma 7 provides the necessary conditions for equilibrium rents. Note that $R^{2,i}(\gamma) \leq R^{1,i}(\gamma) = \frac{\bar{S}^{2,i}}{2}$ implies that $2R^{2,i}(\gamma) \leq \bar{S}^{2,i}$ and hence that profits in the two-buyer state are non-negative. The sufficiency argument is direct: fixing some profile that satisfies the assumptions of the proposition, I first show that with $\pi^* = 1$ and mechanism efficiency, no deviation consisting of either individual or joint shifts (not necessarily local) in $R^{1,i}(\gamma)$ and $R^{2,i}(\gamma)$ can achieve higher profits. Since the candidate profile has full information and an efficient mechanism, considering changes in rents where surplus in both states is maximized gives an upper bound on the profitability of deviations that involve the same changes in rents but that include a decrease in information provision and/or allocative efficiency.

Consider a symmetric posting profile (π^*, γ^*) with $\pi^* = 1$ and associated rents $R^{1,i}(\gamma^*) \geq R^{2,i}(\gamma^*)$. Consider a deviation by seller A to a mechanism $\hat{\gamma}$ in which

$$\begin{aligned} R^{1,i}(\hat{\gamma}) &= R^{1,i}(\gamma^*) + \Delta^1 \\ R^{2,i}(\hat{\gamma}) &= R^{2,i}(\gamma^*) + \Delta^2, \end{aligned}$$

where Δ^η for $\eta \in \{1,2\}$ need not be positive. Let $\hat{q} = q((\pi^*, \hat{\gamma}), (\pi^*, \gamma^*))$. Clearly, seller A cannot profitably deviate to any mechanism for which $\hat{q} = 0$. Also, the most profitable deviation to some mechanism such that $\hat{q} = 1$ is such that any less generous mechanism leads to $\hat{q} < 1$. Hence we can restrict attention to pairs (Δ^1, Δ^2) such that the level of traffic $\hat{q} \in (0,1]$ is given by (2). Hence $\hat{q}$ is given by

$$\begin{aligned} \hat{q} &= \frac{(R^{1,i}(\gamma^*) - R^{2,i}(\gamma^*)) + \Delta^1}{2((R^{1,i}(\gamma^*) - R^{2,i}(\gamma^*))) + \Delta^1 - \Delta^2} \\ &= \frac{1}{2} + z \\ &\quad \text{with } z = \left(\frac{1}{2}\right) \frac{\Delta^1 + \Delta^2}{2((R^{1,i}(\gamma^*) - R^{2,i}(\gamma^*))) + \Delta^1 - \Delta^2}. \end{aligned}$$

The difference in profits $\mathcal{P}_A((1,\hat{\gamma}),(1,\gamma^*),q^*) - \mathcal{P}_A((1,\gamma^*),(1,\gamma^*),q^*)$ can be written as

$$\begin{aligned} &\left[\bar{S}^{2,i} - 2R^{2,i}(\gamma^*)\right](z(z+1)) - 2\left[\bar{\theta} - R^{1,i}(\gamma^*)\right] z^2 \\ &\quad - 2\Delta^2 \left(\frac{1}{2} + z\right)^2 - 2\Delta^1 \left(\frac{1}{2} + z\right)\left(\frac{1}{2} - z\right) \\ &= C\Bigg[\left[\bar{S}^{2,i} - 2R^{2,i}(\gamma^*)\right]\left(4((R^{1,i}(\gamma^*) - R^{2,i}(\gamma^*))) + 3\Delta^1 - \Delta^2\right)\left(\Delta^1 + \Delta^2\right) \\ &\quad - 2\left[\bar{\theta} - R^{1,i}(\gamma^*)\right]\left(\Delta^1 + \Delta^2\right)^2 \\ &\quad - 8\left((R^{1,i}(\gamma^*) - R^{2,i}(\gamma^*))\right)\left((R^{1,i}(\gamma^*) - R^{2,i}(\gamma^*)) + \Delta^1\right)\left(\Delta^1 + \Delta^2\right) \Bigg], \end{aligned}$$

where $C = \left(\frac{1}{4}\right)\left[\frac{1}{2(R^{1,i}(\gamma^*)-R^{2,i}(\gamma^*))+\Delta^1-\Delta^2}\right]^2 > 0$. Set the original candidate profile to be such that

$$R^{1,i}(\gamma^*) = \frac{\bar{S}^{2,i}}{2}$$
$$R^{2,i}(\gamma^*) = \frac{\bar{S}^{2,i}}{2} - \epsilon, \text{ for } \epsilon \geq 0.$$

simplifying the profit difference yields

$$\mathcal{P}_A((1,\hat{\gamma}),(1,\gamma^*),q^*) - \mathcal{P}_A((1,\gamma^*),(1,\gamma^*),q^*) = C\left[(\Delta^1+\Delta^2)^2(-2\epsilon - (2\bar{\theta} - \bar{S}^{2,i}))\right]$$
$$< 0 \text{ for any } (\Delta^1,\Delta^2), \text{ since } \epsilon > 0 \text{ and } 2\bar{\theta} > \bar{S}^{2,i}.$$

Thus no deviations from (π^*, γ^*) are profitable. □

Proposition 3. *Suppose that Assumption 1 is satisfied. Then the set of symmetric equilibrium outcomes of the model with commitment to mechanisms but no commitment to information provision includes the set of symmetric equilibrium outcomes of the model with full commitment.*

Proof. Fix any symmetric equilibrium posting strategy (π^*, γ^*) of the model with full commitment. Consider a mechanism γ^{**}, identical to γ^* except that $y^{1,u} = y^{2,u} = 0$, and let π^{**} be the ex post optimal level of information provision when both sellers post mechanism γ^{**}. In the proof of Proposition 2, I show that $\mathcal{P}_A((\pi^*,\gamma^*),(\pi^*,\gamma^*),q^*) > 0$ and since $\pi^* = 1$, it must be that $\pi^{**} = 1$. Finally, it must be that (π^{**},γ^{**}) is a symmetric equilibrium posting strategy of the game with commitment to mechanisms only. To see this, note that any deviation from (π^{**},γ^{**}) for seller A can be mimicked by that seller in the model with full commitment. Hence, since the profile $((\pi^{**},\gamma^{**}),(\pi^{**},\gamma^{**}))$ attains the same outcomes as $((\pi^*,\gamma^*),(\pi^*,\gamma^*))$, the existence of a profitable deviation from (π^{**},γ^{**}) in the model with commitment only to mechanisms would contradict the optimality of (π^*,γ^*) in the model with full commitment. □

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