Two-Party Competition with Persistent Policies

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Abstract

When campaigning for reelection, incumbent parties' promises to voters are constrained by their records in office, while opposition parties can use their time away from power to develop new platforms. In this paper, I introduce *incumbent policy persistence* in a dynamic game of electoral competition between two policy-motivated parties and characterise the long-run outcomes of its Markov perfect equilibria. Incumbents' reduced policy flexibility leads to alternation in power and policy dynamics that converge to alternations at policies equally preferred by the median voter. Convergence of policies *towards* the median is a robust equilibrium outcome while convergence *to* the median is not; while alternations far from the median are never limits of equilibrium dynamics, alternations close to the median can be reached only if policy dynamics start exactly there. I show that these results are robust to voters being forward-looking and to limited policy commitment by parties.

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1 Introduction

> *Ideological immobility is characteristic of every responsible party, because it cannot repudiate its past actions unless some radical change in conditions justifies this.* (Downs (1957), p. 110)

Politicians face constraints on their ability to champion policies that differ from those with which they are associated through past terms in office or party affiliation. On the one hand, voter behaviour generates costs to policy flexibility. There is evidence from both party-centered and candidate-centered elections that voters punish politicians for 'flip-flops', that is,

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for reversing their policy positions.[1] There is also substantial evidence that voters engage in retrospective voting, that is, they disregard campaign promises and focus on parties' records.[2] On the other hand, internal party politics can inhibit substantial platform changes. Party activists may resist shifts away from a party's traditional objectives, either by threatening to reduce their involvement in electoral activities, or by sustaining challenges to the party's current leaders.[3] The latter also have a stake in the success of the party's current program, in which they have invested political capital and which typically reflects their preferences.

Whether parties can distance themselves from their past policies depends on their performance in past elections. For example, using electoral manifesto data, Janda et al. (1995) and Somer-Topcu (2009) show that the programs of parties that lost votes in previous elections exhibit more change than those of the parties that gained votes. In this paper, I propose a dynamic model of two-party elections whose key feature is that policy persistence disproportionately affects incumbent parties while opposition parties can more freely take up new policies. Parties that have lost elections can more credibly propose new policies to voters, as they often have new leaders who owe their position to rejecting the previous leadership. Also, not having observed opposition parties' actions in office, voters rely on their campaign promises to evaluate their aptitude for office. For example, in U.S. presidential elections, Miller and Wattenberg (1985) show that voters evaluate incumbents retrospectively and challengers prospectively. My main results highlight how the electoral constraints incumbents inherit from their time in office shape the policy dynamics of partisan competition.

In the policy-competition game, two policy-motivated parties have ideal policies on each side of that of the median voter (see Figure 1). Under *incumbent policy persistence*, parties commit to enact a policy for their entire tenure in office. In each election, incumbents defend the policies that they implemented in their previous term, while opposition parties, released from their past commitments by electoral defeat, choose a new platform. The party-level disadvantage stemming from participation in government is well-documented in parliamentary elections, in which governing parties typically lose votes.[4] In such party-centered systems, this aggregate disadvantage swamps any candidate-level incumbency advantage, as identified for the U.S. Congress, which may or may not be present.[5] Even in more candidate-centered systems, parties wielding executive power can be hamstrung by their tenure in office. In the U.S., for example, a sitting president's party typically fares poorly in mid-term elections.[6]

[1]See Adams et al. (2006), DeBacker (2010), Tavits (2007) and Tomz and Van Houweling (2009).

[2]See Fiorina (1981).

[3]See Miller and Schofield (2003).

[4]See Adams et al. (2006), Clark (2009) and Muller and Strom (2000)

[5]See Gelman and King (1990). Evidence for the existence of incumbency advantage outside the U.S. is mixed: see Krashinsky and Milne (1985), Gaines (1998) and Uppal (2009).

[6]See Tufte (1975).

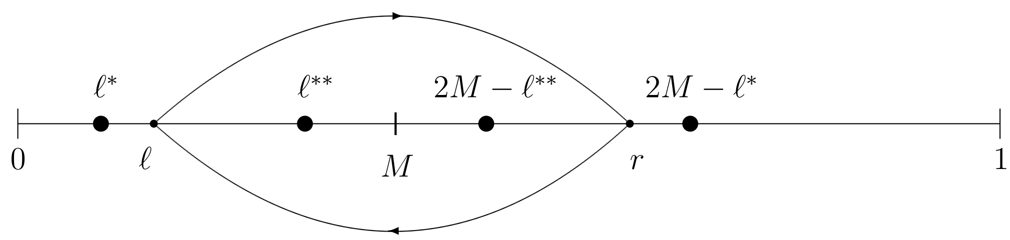

Figure 1: *The policy space is* $[0, 1]$*, the ideal policies of party L, party R and the median voter are* 0*,* 1 *and* M*, respectively. Symmetric alternations at policies in* $[\ell^*, 2M - \ell^*]$ *(e.g., at* (ℓ, r)*) describe all long-run policy outcomes, while symmetric alternations at policies in* $(\ell^{**}, 2M - \ell^{**})$ *describe those long-run policy outcomes that are not robust.*

Opposition parties' ability to outmanoeuvre incumbents drives alternation in power. Anticipating their reduced flexibility as incumbents, opposition parties trade off winning current elections with policies they prefer against setting a more moderate tone to future campaigns. In all Markov perfect equilibria, successive opposition parties win elections by committing to increasingly moderate policies, and policy dynamics converge to some alternation at policies equally preferred by the median voter (a limiting alternations in Figure 1 is (ℓ, r)). While the model admits a complex set of Markov perfect equilibria, I show that its set of *long-run policy outcomes*, those alternations that are limit points of equilibrium paths given some initial incumbent, consists of all alternations at policies that are sufficiently moderate (more moderate than ℓ^* in Figure 1). As all sufficiently extreme policies are transient in any equilibrium, the model generates a dynamic variant of policy convergence.

Periods in which party competition features alternation in government and gradual policy convergence are historically common. In the U.K., following WWII and the adoption by both Labour and Conservative parties of the basic tenets of Keynesian economic policies and the welfare state, the term 'Consensus Politics' was coined to label the period of policy convergence which ended with the election of Margaret Thatcher in 1979.[7] Following a decade of Conservative party dominance and increased polarisation, policy convergence resumed as of the late 1980's.[8] Also, decreasing polarisation has been identified in other European democracies such as Germany and the Netherlands since the end of the Cold War.[9] In these cases, gradual policy convergence appears to be part of an adjustment of party systems to shocks in parties' preferences or to the space of salient issues (e.g., first WWII and then stagflation in the U.K., and the end of the Soviet threat in Germany and the Netherlands).

[7]See Dutton (1997) and Kavanagh and Morris (1994).
[8]See Adams et al. (2012a,b).
[9]See Munzert and Bauer (2012) and Adams et al. (2012), respectively.

Similar dynamics have also been highlighted in an important literature in political science which argues that periods separating what Key (1955) has termed 'critical elections' in the U.S. reflect a process of stabilisation in which 'polarization gives way to conciliation. As it does, the parties move from the poles toward the centre and the distance between them narrows.'[10] Recently, much attention has been given to the observed divergence of the voting records of members of the U.S. Congress from the two major parties since the 1970's. An important note is that this recent trend obscures the fact that even in the US, periods of gradual convergence have occured, as evidenced by the voting records of members of Congress from the 1930's to the 1970's.[11] Furthermore, many accepted contributors to this increase in polarisation, such as party primaries, are peculiar to the U.S. political system.[12]

Is full median convergence a limiting outcome of equilibrium policy dynamics under incumbent policy persistence? On the one hand, the indefinite repetition of the median policy is supported in any equilibrium, since all policy dynamics that start at the median stay there (in Figure 1, M is in the set of long-run policy outcomes). However, median convergence is not a *robust* outcome of the model, where policies in robust long-run outcomes are those that can be reached by policy dynamics that start at more extreme policies and converge over time. In other words, median convergence occurs in the long-run of some equilibrium only if policy dynamics start there. I show that the set of robust long-run policy outcomes under equilibria in *consistent* strategies consists only of those long-run policy outcomes that are sufficiently extreme (more extreme than ℓ^{**} in Figure 1). Gradual convergence to policies near the median fails since parties' benefits from influencing their opponents' future platforms vanish when elections are contested at moderate policies. Hence, while partisan competition leads to convergence towards centrist policies, policy divergence persists even in the long-run and political parties that are initially differentiated remain so.

The tight bounds on the extremism and moderation of long-run policy outcomes reflect the parties' time preferences, which can be interpreted as measuring the strength of party elites or the duration of terms in office. A party with a higher discount factor will better internalise the costs of the opportunities allowed to future challengers by its current policy choices and behave more moderately. Since any party can unilaterally ensure that future electoral campaigns are fought with moderate policies, the bound on long-run extremism is determined by the preferences of the party that most values moderation. However, the bound on moderation for robust long-run policy outcomes reflects the scope for sustained compromise, so that it is determined by the preferences of both parties. Median convergence

[10]Sundquist (1983), p.319. Key (1955) defines critical elections as 'a type of election in which there occurs a sharp and durable electoral realignment between parties' (p.16), such as the presidential election of 1932 that brought F.D. Roosevelt and the New Deal to power.

[11]See Poole and Rosenthal (2007) for both convergence and divergence findings.

[12]See Fiorina (1999).

is restored only in the limit when parties are arbitrarily patient.

To maintain the tractability of my model and focus on how incumbent inflexibility affects the dynamics of electoral competition, I adopt the reduced-form approach of assuming that incumbents who are reelected find it too costly to implement policies that differ from those of their first term. In particular, I do not generate incumbent policy persistence as an equilibrium outcome, as is the case in dynamic models of competition between candidates with privately known policy preferences.[13] These models assume that politicians cannot commit to policies and focus on office-holders' incentives to choose moderate policies to maintain a reputation for moderate preferences. In equilibrium, voters require policy persistence by incumbents as any policy variability is attributed to the most extreme ideological types and leads to the election of an opposition representative. This outcome could also be sustained in dynamic variants of the models of Callander (2008) and Kartik and McAfee (2007), in which privately informed politicians signal valence characteristics as opposed to policy preferences. Alternatively, in Alesina (1988) and Van Weelden (2013), policy persistence is sustained without either private information or policy commitments by implicit equilibrium agreements in a repeated game between voters and politicians.

In their simplest variant, models of dynamic elections with asymmetric information do not generate alternation in the long-run and successive extreme incumbents are replaced after a single term until a sufficiently moderate candidate is elected and survives all challenges.[14] Hence, incumbent policy persistence alone does not generate the alternating dynamics of my model: policy commitment by opposition parties allows them to credibly respond to incumbents' actions in office, ensuring that moderation does not guarantee reelection. In Section 5.2, I also show that equilibrium policy dynamics feature alternation when incumbent parties can pay a cost to adjust their policies.

My dynamic model extends a number of classic models of partisan competition. The key mechanism of my model is a dynamic variant of the trade-off between preferred policies and the probability of winning in static models with policy-motivated candidates and median uncertainty based on Wittman (1983) and Calvert (1985). While these models produce equilibrium outcomes that are bounded away from both the median and the extremes, my model's corresponding bounds reflect equilibrium predictions of future opponents' policy choices and depend critically on parties' degree of farsightedness. Kramer (1977) and Wittman (1977) introduce dynamic models of asynchronous policy competition between myopic parties. Kramer (1977) assumes that parties are office-motivated and maximise votes, while Wittman (1977) assumes that they are policy-motivated. Myopic policy choices gen-

[13] See Banks and Duggan (2008), Bernhardt et al. (2004), Bernhardt et al. (2009), Bernhardt et al. (2011), Duggan (2000) and Kalandrakis (2009a). Closely related is Van Weelden (2013).

[14] Exceptions are Kalandrakis (2009a) and Bernhardt et al. (2004).

erate trivial policy dynamics very different from those of my model. In a single-dimensional setting, Wittman (1977) predicts complete and persistent policy divergence while Kramer (1977) predicts immediate median convergence.[15]

Dynamic models of elections are less common. Other than the models with adverse selection mentioned above, a recent branch of the literature focuses on the dynamic distortions to public goods provision, government debt and private investment due to electoral incentives. In probabilistic voting models, Battaglini (2013) focuses on office-motivated candidates and variability in voters' preferences and Azzimonti (2011) focuses on policy-motivated candidates and incumbency advantage.[16] My model is also formally related to models of dynamic legislative bargaining with endogenous status quo.[17] Baron (1996) characterises an equilibrium in which, since the median legislator is eventually recognised and proposes his ideal policy, all policy paths converge to the median policy. The parties in my model can be interpreted as the only legislators that are ever recognised to propose policies. The absence of median convergence then follows from my assumption that neither party shares the policy preferences of the median voter.

2 Model

Two parties, L and R, contest an infinite sequence of elections at times $t = 0, 1, \ldots.$ Each period starts with the incumbent party in power and the remaining party in opposition. The *state* (I, x), with $I \in \{L, R\}$ and $x \in X = [0, 1]$, records the identity of the incumbent party along with its policy commitment. The opposition party $-I = \{L, R\} \setminus \{I\}$ commits to implementing a policy $z \in X$, if elected, and for as long as it remains in power: this is the assumption of incumbent policy persistence. In any election, the incumbent's policy commitment is inherited from the election that brought it to power. A party may also choose not to participate in the election, written $z = Out$.

Given policy commitments (x, z) for both parties, an election is held. An odd number of voters have symmetric single-peaked preferences over policies in X, and some policy M corresponds to the ideal policy of the median voter. For now, I assume that voters are myopic. In all elections over policies (x, z), I restrict attention to the equilibrium in weakly undominated strategies in which voters support the party that commits to the policy closest to their ideal policy and in which the median voter is decisive. In Section 5.1, I show that all equilibrium outcomes under myopic voting can be supported by equilibria of the game

[15] See Anesi (2010) for another forward-looking version of these models.

[16] See also Bai and Lagunoff (2011) and Battaglini and Coate (2007, 2008).

[17] See Baron (1996), Baron et al. (2012), Bowen and Zahran (2012), Duggan and Kalandrakis (2012), Fong (2008), Kalandrakis (2004) and Kalandrakis (2009b).

with forward-looking voters, so that my results in the game with myopic voters identify a selection of equilibria in the game with forward-looking voters. This indirect approach to studying the outcomes of the game with forward-looking voters is both convenient, since the equilibria identified under myopic voting have appealing and tractable properties, and suitably general, since the standard approach in comparable models is to focus on a single equilibrium.

As the median voter is decisive in all elections and voter preferences are symmetric, the incumbent party wins whenever $|x - M| < |z - M|$, and the opposition party wins whenever $|x - M| > |z - M|$. If $|x - M| = |z - M|$ but it is not the case that $x = z = M$, in any equilibrium it must be that the median voter supports the opposition party, since any policy more moderate than z wins with probability 1. If $x = z = M$, then no policy other than M can win any future election, so that imposing that the median voter supports the opposition party is without loss of generality for equilibrium policy outcomes and payoffs. Hence, given state (I, x), let $W(I, x) = [\min\{2M - x, x\}, \max\{2M - x, x\}]$ be the *set of winning policies* for the opposition party. Transitions between states are as follows: the current period's winning party and policy become next period's incumbent party and incumbent policy, respectively. Formally, define the *state transition function* $\tau : (\{L, R\} \times X) \times (X \cup \{Out\}) \to \{L, R\} \times X$ as

$$\tau((I, x), z) = \begin{cases} (I, x) & \text{if } z \in W(I, x)^c \cup \{Out\}, \\ (-I, z) & \text{if } x \in W(I, x). \end{cases}$$

I assume that parties are policy-motivated, and that parties L and R have single-peaked preferences over policies around 0 and 1 and represented by u_L and u_R respectively. Suppose, without loss of generality, that $M \leq \frac{1}{2}$, so that party L is (weakly) favoured by the median voter. Assume that $u_L(0) = u_R(1) = 0$, u_L (u_R) is strictly decreasing (increasing), twice continuously differentiable and strictly concave. The key feature of parties' preferences is not that their ideal policies are located at the extremes of the policy space but only that these are on opposite sides of the median policy. Similarly, concavity simplifies the results but can be relaxed. It captures two key features of parties' payoffs: the benefits of policy compromise by a party's opponent always more than offset its loss from its own compromise, and parties are more willing to compromise when facing extreme policies.

I restrict attention to equilibria in Markov strategies. My aim is to characterise parties' long-run interactions, for which it is natural to limit implicit equilibrium coordination and assume that opposition parties' behaviour depends on incumbents' policies only insofar as they affect available winning policies. Parties square off in elections that are years apart and often involve different politicians, so that strategies that with all else equal differentiate between events that occurred even a few elections ago would have problematic interpretations. Formally, a *Markov strategy* for party J is a function $\sigma_J : \{L, R\} \times X \to X \cup \{Out\}$, with the restriction that $\sigma_J(J, x) = x$ for all $x \in X$, which captures incumbent policy persistence. Let Σ_J be the set of Markov strategies for party J. Henceforth, the term strategy refers to a Markov strategy.

The *state path induced by profile* (σ_L, σ_R) *starting from* (I, x) is a sequence $\{(I^t, x^t)\}_{t=1}^{\infty} \in (\{L, R\} \times X)^{\infty}$ defined recursively by

$$(I^1, x^1) = \tau((I, x), \sigma_{-I}(I, x)),$$
$$(I^t, x^t) = \tau((I^{t-1}, x^{t-1}), \sigma_{-I^{t-1}}(I^{t-1}, x^{t-1})).$$

The *policy path* $\{x^t\}_{t=1}^{\infty}$ *induced by* (σ_L, σ_R) *starting from* (I, x) is the policy sequence of the corresponding state path. Discounted payoffs to party $J \in \{I, -I\}$ from policy path $\{x^t\}_{t=1}^{\infty}$ induced by (σ_L, σ_R) starting from (I, x) are given by

$$V_J(\sigma_L, \sigma_R; (I, x)) = \sum_{t=1}^{\infty} \delta_J^{t-1} u_J(x^t),$$

where $\delta_J < 1$ is party J's discount factor. A *Markov perfect equilibrium* is a strategy profile (σ_L, σ_R) such that, for each state (R, r),

$$\sigma_L(R, r) \in \arg\max_{\sigma'_L \in \Sigma_L} V_L(\sigma'_L, \sigma_R; (R, r)),$$

and for each state (L, ℓ)

$$\sigma_R(L, \ell) \in \arg\max_{\sigma'_R \in \Sigma_R} V_R(\sigma_L, \sigma'_R; (L, \ell)).$$

Henceforth, the term equilibrium refers to Markov perfect equilibrium.

3 Outcomes Without Incumbent Policy Persistence

As a benchmark, consider the model without incumbent policy persistence in which incumbent and opposition parties simultaneously commit to policies in all periods. Calvert (1985) shows that the associated stage game has a unique Nash equilibrium in which each party commits to the median policy, and the following result shows that the unique subgame perfect equilibrium of the repeated game has full convergence and trivial policy dynamics.

Proposition 1. *In the unique subgame perfect equilibrium of the model without incumbent policy persistence, parties commit to the median policy after all histories.*[18]

With simultaneous competition, both parties can enforce policy path $(M, M, ...)$ after all histories, while with policy persistence party J can enforce policy path $(M, M, ...)$ only when in opposition. When there is alternation in power, parties can be worse off relative to policy path $(M, M, ...)$ as incumbents, but their gain from accessing office with policies they prefer to the median policy is sufficient to balance this (discounted) loss. Equilibrium policies under policy persistence are dynamically inconsistent since if they could, incumbents would prefer to free themselves of their record and commit to the median policy.

4 Outcomes With Incumbent Policy Persistence

The restriction to Markov strategies does not eliminate equilibrium multiplicity and the model's set of equilibria admits no simple description. I focus instead on characterising equilibrium outcomes, and in particular those that persist in the long-run.

Definition 1. Policy y is a *long-run policy outcome under equilibrium* (σ_L, σ_R) *starting from* (I, x) if y is a limit point of the policy path induced by (σ_L, σ_R) starting from (I, x).[19]

A policy that is a long-run policy outcome under some equilibrium starting from some state is called simply a *long-run policy outcome*. Since I focus on the set of equilibrium outcomes reached from any initial state and Markov strategies are independent of history, I treat the initial state as determined outside the model. Proposition 1 shows that if the initial state were to be determined by a round of simultaneous competition, then the initial incumbent, and all future office-holders, would be located at the median. However, this case is not of much theoretical or substantive interest.

4.1 Equilibrium Policy Dynamics: Alternation

I first characterise the model's moderating equilibrium dynamics.

Proposition 2. *Consider some equilibrium (σ_L, σ_R) and some state (I, x) along with the policy path $\{x^t\}$ induced by (σ_L, σ_R) starting from (I, x). Suppose that $(I, x) = (R, r)$.*

i. If $r \leq M$, then $x^t = r$ for all t.

[18] The proof, in the Supplementary Appendix, is not trivial since parties have different discount factors.

[19] A point y is a *limit point* of a sequence $\{x^t\}$, also called an *accumulation point,* or a *cluster point*, if and only if it is the limit of some subsequence of $\{x^t\}$.

ii. *If $r > M$, then (a) incumbents are always defeated on the equilibrium path, unless $x^t = M$ for some i, (b) $\{x^t\}$ has a pair of limit points $(\hat{\ell}, 2M - \hat{\ell})$ for some $\hat{\ell} \leq M$, and (c) $\sigma_L(R, 2M - \hat{\ell}) = \hat{\ell}$ and $\sigma_R(L, \hat{\ell}) = 2M - \hat{\ell}$.*

The case of $(I, x) = (L, \ell)$ is symmetric.[20]

Party L has no incentive to win the election if incumbent party R champions a policy r to the left of M. The policy path most favourable to L that can be sustained in any equilibrium from such a state is $(r, r, r, ...)$, which L can attain by choosing *Out*. Furthermore, such an outcome occurs in equilibrium only if the initial state is (R, r), since opposition party R strictly prefers setting policy M to r in any state in which policy $r < M$ is winning. Since item *i* of Proposition 2 shows that all policies can be reached by some equilibrium dynamics, I call policy outcome $y \neq M$ *trivial* if it is a long-run policy outcome under (σ_L, σ_R) starting from (I, x) if and only if $y = x$ and the policy path $\{x^t\}$ induced by (σ_L, σ_R) from (I, x) is such that $x^t = y$ for all $t \geq 1$. From now on, the term long-run policy outcome refers to a long-run policy outcome that is not trivial. Item *ii* of Proposition 2 ensures that nontrivial equilibrium dynamics entail alternation in power (unless the median policy is reached, following which office holding dynamics are undetermined) and convergence to symmetric pairs of policies $(\ell, 2M - \ell)$ for some $\ell \leq M$. Hence, given state (R, r) with $r > M$, in any equilibrium opposition parties commit to winning policies on their side of the median and the policy path (weakly) approaches the median and has at most a pair of limit points $(\hat{\ell}, \hat{r})$ such that $\hat{r} = 2M - \hat{\ell}$. Finally, although the limits of alternating equilibrium dynamics need not be reached in finite time, they are absorbing: if the dynamics start at one of the limiting policies, they stay there.

My results depend only on properties of parties' preferences over symmetric policy alternations. Define the functions $U_L^+ : [0, M] \to \mathbb{R}$ and $U_L^- : [0, M] \to \mathbb{R}$ for party L such that

$$U_L^+(\ell) = u_L(\ell) + \delta_L u_L(2M - \ell), \text{ and}$$
$$U_L^-(\ell) = u_L(2M - \ell) + \delta_L u_L(\ell),$$

with the corresponding functions $U_R^+ : [0, M] \to \mathbb{R}$ and $U_R^- : [0, M] \to \mathbb{R}$ defined symmetrically. Hence, $\frac{1}{1-\delta_L^2} U_L^+(\ell)$ is party L's payoff from alternation at policies $(\ell, 2M - \ell)$ *starting from* ℓ, while $\frac{1}{1-\delta_L^2} U_L^-(\ell)$ is its payoff to the same alternation when *starting from* $2M - \ell$. Strict concavity of parties' utility functions yields a natural preference order over alternations.

[20] The proofs of all main results are in the Appendix.

Lemma 1. *There exist uniquely defined policies ℓ^* and r^* such that*

$$\ell^* = \underset{\ell \in [0,M]}{\arg\max}\, U_L^+(\ell) \in [0, M), \text{ and}$$
$$r^* = 2M - \underset{\ell \in [0,M]}{\arg\max}\, U_R^+(\ell) \in (M, 2M].$$

U_L^- (U_R^-) is strictly increasing (decreasing), and, given $J \in \{L, R\}$, both U_J^+ and U_J^- are strictly concave.

Given $\ell \in [0, M)$, the concavity of u_L ensures that the cost to L of a moderate move away from ℓ is dominated by the benefit of a moderate move away from $2M - \ell$. However, that U_L^+ is single-peaked around $\ell^* < M$, *L's preferred alternation*, follows from discounting. When the payoff to party L is evaluated starting from L's policy, a shift to a more moderate alternation ensures that L suffers the full loss to moderating its own policy, while the larger benefit of R's moderation is discounted. However, when the payoff to party L is evaluated starting from R's policy, L always prefers more moderate alternations, and L's favoured alternation is at M.

4.2 Long-Run Policy Outcomes: A Bound on Extremism

I show that long-run policy outcomes exhibit bounded extremism: while sufficiently extreme policies can be observed on some equilibrium paths, they are transient.

Proposition 3. *Policy $\ell \leq M$ is a long-run policy outcome if and only if $\ell \in [\max\{\ell^*, 2M - r^*\}, M]$.*

Non-median outcomes are driven by parties' trade-off between myopically leaning towards their preferred policies and farsightedly preempting their opponents' own leanings. If an alternation is too extreme to support a long-run policy outcome, some party has an incentive to rein in future opponents' policies by unilaterally committing to more moderation.

Corollary 1. *The set of long-run policy outcomes has the following properties.*

i. *If v_J is obtained from u_J by a concave transformation, then* $[\max\{\ell^*, 2M - r^*\}, M]|_{v_J} \subseteq [\max\{\ell^*, 2M - r^*\}, M]|_{u_J}$.

ii. *If $\delta'_J > \delta_J$, then* $[\max\{\ell^*, 2M - r^*\}, M]|_{\delta'_J} \subseteq [\max\{\ell^*, 2M - r^*\}, M]|_{\delta_J}$.

iii. $\lim_{\delta_L \to 1}[\max\{\ell^*, 2M - r^*\}, M] = \lim_{\delta_R \to 1}[\max\{\ell^*, 2M - r^*\}, M] = \{M\}$.

iv. $\lim_{\delta_J \to 0}[\max\{\ell^*, 2M - r^*\}, M] = [\ell^* \mathbb{I}_{J=R} + 2M - r^* \mathbb{I}_{J=L}, M]$.

When $\ell^* > 0$, it is uniquely determined by $\frac{u'_L(\ell^*)}{u'_L(2M-\ell^*)} = \delta_L$, is increasing in δ_L and converges to M as δ_L converges to 1. If party L is more farsighted, the cost of R's future policies increases and its preferred alternation is closer to the median. The discount factor δ_L can reflect a host of institutional features that drive the farsightedness of party L, such as (a) the expected tenure of party elites, (b) their level of control over either representatives' actions when in office or the party base at the candidate nomination stage, or (c) the length of terms in office. Similarly, ℓ^* is increasing in L's disutility for policies away from its ideal point, captured by the concavity of u_L, which can reflect the intensity of partisanship or institutional factors within the party that facilitate or inhibit compromise. The policies observed in the long-run are not determined symmetrically by both parties' preferences, but rather by the preferences of the party most willing to compromise. If some party is arbitrarily myopic, then the set of long-run outcomes is determined solely by the preferences of its opponent. If, instead, at least one party is arbitrarily farsighted, policies converge to the median in the long-run.[21]

The bound on extremism hinges on a useful lower bound on party L's equilibrium payoff: any equilibrium path following a commitment to some winning policy ℓ yields a payoff of at least $\frac{1}{1-\delta_L^2}U_L^+(\ell)$. To see this, consider a strategy for opposition party L which sets policy ℓ in the current election and responds with its most extreme winning policy to all of R's subsequent policies. The payoff to L from this strategy is $u_L(\ell)$ in this election, along with a sequence of payoffs $\{U_L^-(r^t)\}$ in the subsequent pairs of elections, for some sequence of policies $\{r^t\}$ such that $\ell \leq 2M - r^t$ for all t. By Lemma 1, each payoff in this sequence is at least $U_L^-(\ell)$ and hence the payoff to selecting winning policy ℓ must be at least $\frac{1}{1-\delta_L^2}U_L^+(\ell)$. Thus, if $\ell < \ell^*$ were a long-run policy outcome, then party L could win the election in state $(R, 2M-\ell)$ by committing to policy ℓ^* and guarantee itself a payoff of at least $\frac{1}{1-\delta_L^2}U_L^+(\ell^*)$, its preferred alternation, yielding a contradiction.

That the bound on extremism is tight and that all more moderate policies are also long-run policy outcomes follows from the construction of a single equilibrium under which all policies $\ell \in [\max\{\ell^*, 2M - r^*\}, M]$ are long-run policy outcomes. Consider the strategy $\sigma_L^{\ell^*}$ such that

$$\sigma_L^{\ell^*}(R, r) = \begin{cases} \ell^* & \text{if } r \in [2M-\ell^*, 1], \\ 2M - r & \text{if } r \in [M, 2M-\ell^*), \\ Out & \text{if } r \in [0, M), \end{cases}$$

[21]Dynamic models of elections with private candidate preferences following Duggan (2000) also lead to more compromise as discount factors increase. In Alesina (1988), as in standard folk theorems for repeated games, the set of equilibrium outcomes is larger for larger discount factors.

as well as the myopically optimal strategy for party R, σ_R^{my}, such that

$$\sigma_R^{my}(L,\ell) = \begin{cases} 2M-\ell & \text{if } \ell \geq M, \\ Out & \text{if } \ell > M. \end{cases}$$

If $\ell^* \geq 2M - r^*$, then $(\sigma_L^{\ell^*}, \sigma_R^{my})$ is an equilibrium.[22] In moderate states (L,ℓ) for some $\ell \geq \ell^*$ and (R,r) for some $r \leq 2M-\ell^*$, both parties behave myopically. In these states their preferences over alternations coincide; both prefer more extreme alternations when evaluated starting from their own policy. Parties' preferences over alternations also coincide in extreme states (L,ℓ) for some $\ell < 2M - r^*$ and (R,r) for some $r > r^*$. In these states, both parties prefer more moderate alternations starting from their own policy. However, having both parties committing to more moderate policies cannot be an equilibrium and some party, in this case L, brings policy dynamics towards more moderate alternations. Since party R knows party L will commit to ℓ^* in the next election against any winning policy $r \in [2M-\ell^*, 2M-\ell]$ it champions in the current election, committing to myopic policy $2M-\ell$ is optimal. For intermediate states (L,ℓ) for some $\ell \in [2M-r^*, \ell^*)$ and (R,r) for some $r \in (2M-\ell^*, r^*]$, parties' preferences over alternations differ and party L, which prefers moderation, ensures that policy paths converge.

Consider the version of the game with finite horizon $T \geq 2$ and suppose that $\ell^* \geq 2M-r^*$. Suppose that T is even and that party R is the initial incumbent, with policy $r > M$. In the unique subgame perfect equilibrium of this game, party R is in opposition in the final period. In the limit, these equilibria for even horizons T select Markov perfect equilibrium $(\sigma_L^{\ell^*}, \sigma_R^{my})$ in the infinite-horizon game. To see this, fix $T \geq 2$ even and first note that whenever party L is in opposition and party R's strategy calls for its myopically optimal policy in all future periods, then party L's optimal policy is described by $\sigma_L^{\ell^*}$. Second, note that whenever party R is in opposition and party L's strategy calls for policies chosen according to $\sigma_L^{\ell^*}$ in all future periods, then party R's myopically optimal policy is optimal. The rest of the claim follows by induction, since party R is in opposition in the final period of the game and its optimal policy is described by σ_R^{my}. A broader point is that in any finite-horizon version of the game no convergence occurs to a policy more moderate than $\max\{\ell^*, 2M-r^*\}$, since in the final period no party is willing choose moderate policies, which leads to an unravelling of the incentives to make earlier moderate moves that are not unilaterally optimal for some party. In the next section, I show that convergence to policies more moderate than $\max\{\ell^*, 2M-r^*\}$ occurs in equilibria of the infinite-horizon game. Hence, these results depend critically on

[22]This involves a straightforward verification argument which is provided in the Supplementary Appendix. If $\ell^* < 2M - r^*$, strategies $\sigma_R^{r^*}$ and σ_L^{my} can be defined by reversing the roles of the two parties and then $(\sigma_L^{my}, \sigma_R^{r^*})$ is an equilibrium.

the possibility of continued compromise.

4.3 Robust Long-run Policy Outcomes: A Bound on Moderation

A long-run policy outcome y is the limit of equilibrium policy dynamics given *some* initial state. In this section, I investigate the properties of the equilibrium policy paths that support y as a long-run policy outcome. In particular, 'steady state' outcome y need not be dynamically stable in the following sense: given an initial state with policy more extreme than y, there need not exist an equilibrium policy path that has y as a limit point.

Definition 2. Policy y is a *robust long-run policy outcome* if it is a long-run policy outcome under some equilibrium (σ_L, σ_R) starting from some state (I, x) such that x is not a long-run policy outcome under (σ_L, σ_R) starting from (I, x).

Long-run policy outcomes that are not robust are poor predictions of equilibrium play since they fail to arise given any different initial state. Robustness is a weak requirement of dynamic stability as it necessitates only that policy y be reached by a single moderating equilibrium path from some more extreme policy x.

Verifying robustness for arbitrary Markov perfect equilibria is difficult as it requires a general characterisation of equilibrium convergence paths. An obstacle to such a characterisation is that the opposition party may have multiple best-responses following a deviation by the incumbent, which it can use to move to a new convergence path to punish the incumbent. The asynchronicity of policy choices suggests a natural refinement of Markov equilibria that rules out such off-path coordination. In each state, the opposition party solves a single-agent decision problem, and I require that its strategy not lead to choice behaviour that is inconsistent with standard criteria in decision theory. In particular, suppose that party L chooses winning policy $\ell > 2M - r$ from set of winning policies $[2M - r, r]$ for some $r > M$. Then the Weak Axiom of Revealed Preference is satisfied if Party L chooses the same policy from a set of winning policies $[2M - r', r']$ for some $r' \in [M, r)$ such that $2M - r' < \ell$. The choice of any other policy from the smaller set of winning policies could be justified by equilibrium considerations, but not by political constraints.

Definition 3. Markov strategy σ_{-I} is *consistent* if for any pairs of states (I, x) and (I, x'), whenever

i. $\tau((I, x), \sigma_{-I}(I, x)) = \tau((I, x'), \sigma_{-I}(I, x))$, and

ii. $\sigma_{-I}(I, x) \neq \sigma_{-I}(I, x')$, then $\tau((I,x),\sigma_{-I}(I,x')) \neq \tau((I,x'),\sigma_{-I}(I,x'))$.

A *consistent Markov perfect equilibrium* is a Markov perfect equilibrium in consistent Markov strategies.

Note that if $\tau((I,x),z) = \tau((I,x'),z)$ for some opposition party policy z that is winning in both states (I,x) and (I,x'), then the sequences of policies induced by z are the same in both states. Hence, Definition 3 states that if $\sigma_{-I}(I,x)$ induces identical outcomes in both states (I,x) and (I,x') and $\sigma_{-I}(I,x)$ is not chosen in state (I,x'), then $\sigma_{-I}(I,x')$ cannot induce identical outcomes in both states. Consistency defines a history to be 'payoff-irrelevant' if it is revealed to be irrelevant by a party's strategy at some other history.

I show that robust long-run outcomes under consistent equilibria exhibit bounded moderation. This does not contradict the results of Section 4: policy paths tend to converge *toward* the median, but they do not converge *to* the median. Median politics obtains in the long-run only if the initial incumbent party champions the median, otherwise parties remain differentiated and settle into clearly defined party identities.

Proposition 4. *There exists* $\ell^{**} \in (\max\{\ell^*, 2M - r^*\}, M)$ *such that policy* $\ell \leq M$ *is a robust long-run policy outcomes in consistent Markov strategies if and only if* $\ell \in [\max\{\ell^*, 2M - r^*\}, \ell^{**}]$.

Equilibrium convergence paths are supported by a party's commitment to a more moderate policy being reciprocated in future elections by its opponent. When converging to sufficiently moderate policy alternations, parties' value their opponents' (discounted) moderate moves so little that they are unwilling to commit to policies moderate enough to sustain convergence. Policy ℓ^{**} is the most moderate policy that gives parties sufficient incentives to participate in these successive rounds of compromise.

Corollary 2. *The set of robust long-run policy outcomes in consistent Markov strategies has the following properties.*

i. *If* v_J *is obtained from* u_J *by a concave transformation, then* $[\max\{\ell^*, 2M - r^*\}, \ell^{**}]|_{v_J} > [\max\{\ell^*, 2M - r^*\}, \ell^{**}]|_{u_J}$, *where* $\geq$ *is the weak set order.*

ii. *If* $\delta'_J > \delta_J$, *then* $[\max\{\ell^*, 2M - r^*\}, \ell^{**}]|_{\delta'_J} > [\max\{\ell^*, 2M - r^*\}, \ell^{**}]|_{\delta_J}$.

iii. $\lim_{\delta_L \to 1}[\max\{\ell^*, 2M - r^*\}, \ell^{**}] = \lim_{\delta_R \to 1}[\max\{\ell^*, 2M - r^*\}, \ell^{**}] = \{M\}$.

iv. $\lim_{\delta_J \to 0}[\max\{\ell^*, 2M - r^*\}, \ell^{**}] = \{\ell^* \mathbb{I}_{J=R} + 2M - r^* \mathbb{I}_{J=L}\}$.

More farsighted parties have more moderate preferred alternations but are also more willing to compromise to achieve moderate convergence outcomes, reflected by a value of ℓ^{**} closer to the median. Hence, less myopic parties shift the whole set of robust long-run policy outcomes toward the median. As with the set of long-run policy outcomes, the set of robust outcomes treats the preferences of the parties asymmetrically. If a single party is arbitrarily farsighted, the set of robust outcomes collapses to the median policy. If instead a single party is arbitrarily myopic, then the set of long-run policy outcomes is determined by the preferred alternation of the more farsighted party but the set of robust outcomes collapses to this alternation. While the myopic party cannot affect the set of long-run policy outcomes, it refuses to participate in any converging policy paths.

When studying the convergence outcomes of the model, it is convenient to map converging dynamics into a single increasing sequence of policies. The *convergence path* $\{y^t\}$ *to policy* $\hat{\ell} \in (0, M]$ *under equilibrium* (σ_L, σ_R) *starting from* (I, x) is a sequence such that

i. If $(I, x) = (R, r)$ for some $r > 2M - \hat{\ell}$, then $y^t = x^t$ for t odd and $y^t = 2M - x^t$ for t even, where $\{x^t\}$ is the sequence of policies induced by (σ_L, σ_R) starting from (I, x).

ii. If $(I, x) = (L, \ell)$ for some $\ell < \hat{\ell}$, then $y^t = x^t$ for t even and $y^t = 2M - x^t$ for t odd.

iii. $\{y^t\} \to \hat{\ell}$.

Consistency allows simple characterisations of equilibrium convergence paths. Lemma 3 in the Appendix characterises consistent strategies along convergence paths and is illustrated in Figure 2. Given a section of some convergence path $\{y^t\}$ initiated by party R committing to policy $2M - y^t$, to which L responds by moderating to y^{t+1}, consistency implies that $\sigma_L(R, r) = y^{t+1}$ for all $r \in (2M - y^t, 2M - y^{t+1}]$. That is, L moderates to y^{t+1} when facing an incumbent R championing policies more moderate than $2M - y^t$. Furthermore, consistency implies that $\sigma_R(L, \ell) = 2M - \ell$ for all $\ell \in [y^t, y^{t+1})$, that is, R commits to its most extreme winning policy whenever L stops short of moderating to y^{t+1}.

Section 4.2 noted that $\frac{1}{1-\delta_L^2} U_L^+(y^t)$ is a lower bound on L's payoff in state $(R, 2M - y^t)$. Lemma 4 in the Appendix shows that if $2M - y^t$ lies on a consistent equilibrium convergence path then this payoff is also an upper bound. That is, L's payoff at $(R, 2M - y^t)$ is computed 'as though' equilibrium dynamics were absorbed by an alternation at the symmetric pair of policies $(y^t, 2M - y^t)$. This property implies that L's payoff in state $(R, 2M - y^t)$ satisfies the following second-order difference equation

$$U_L^+(y^t) - U_L^+(y^{t+1}) = \delta_L[U_L^-(y^{t+2}) - U_L^-(y^{t+1})]. \quad (1)$$

The left-hand side of (1) is the cost (computed in payoffs to alternations starting from

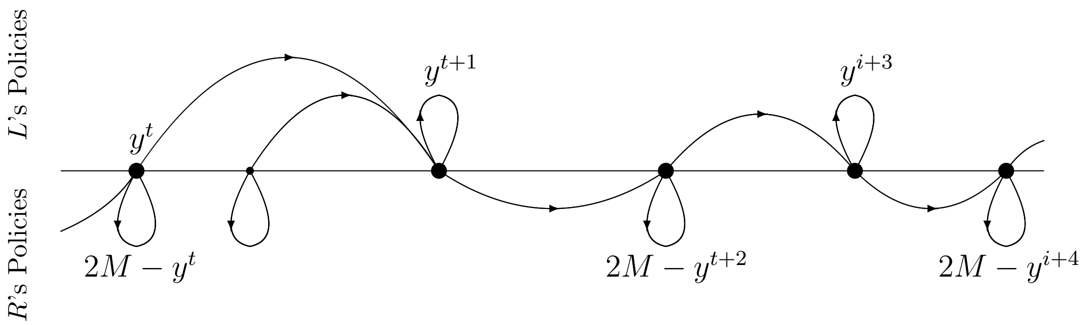

Figure 2: *Convergence paths under consistent equilibria depicted in a 'folded' policy space. The directed curve above (below) the interval from point ℓ represents the equilibrium action of party L (R) in state $(R, 2M - \ell)$ $((L, \ell))$. From state $(R, 2M - y^t)$, party L commits to moderate policy y^{t+1}, to which party R responds with policy $2M - y^{t+2}$, and so on.*

L's policy) of choosing moderate policy y^{t+1} while the right-hand side is the (discounted) benefit (computed in payoffs to alternations starting from R's policy) of party R's subsequent moderate move to $2M - y^{t+2}$. Equation (1) shows that moderation is self-reinforcing: if parties anticipate an end to convergence in the future current incentives to choose moderate policies unravel. That is, if $y^t \geq \ell^*$, then (1) cannot be satisfied for $y^{t+2} = y^{t+1}$ unless $y^{t+1} = y^t$. In fact, this holds for all equilibria, not just those in consistent strategies: only the most extreme long-run policy outcome is ever reached from a more extreme state in a finite number of elections. Equation (1) also explains why party L is willing to sustain convergence paths to alternations more moderate than $(\ell^*, 2M - \ell^*)$, that is, why $\ell^{**} > \ell^*$. Around ℓ^*, the cost of moving to a more moderate alternation is of second-order importance, while the benefit of R's moderation is of first-order importance. That is, around ℓ^*, L is willing to bear almost all of the cost of sustaining convergence.

The recursive relationship in (1), along with the corresponding relationship for party R, generate the bound on moderation $\ell^{**} \in (\ell^*, M)$, which is derived explicitly in the Appendix. Fix one round of moderation from $(R, 2M - y^t)$ as the moves, first by L, then by R, that take the state to $(R, 2M - y^{t+2})$. Then (1) describes the share of the total moderation $y^{t+2} - y^t$ that L is willing to undertake. The most moderate policy for which the parties' 'supply' of moderation is consistent with convergence in the limit as $y^{t+2} \to y^t$ is ℓ^{**}. Convergence to moderate policies fails as the shares of any given round of moderation that parties are willing to undertake become too small. To see this, consider the polar case of convergence to the median. As a convergence path approaches M, moderate moves of similar sizes by parties L or R have similar effects on L's payoffs, yet the gain from R's moderation is discounted. Since the same observation holds for R, both parties require their opponents to make larger moderate moves than they do, which contradicts convergence. That the bound on moderation ℓ^{**} is tight and that all more extreme long-run policy outcomes are robust follows from equilibrium construction: I construct an equilibrium under which policy $\hat{\ell} \in (\max\{\ell^*, 2M - r^*\}, \ell^{**}]$ is a robust long-run policy outcome for each such $\hat{\ell}$.[23]

5 Extensions

5.1 Forward-looking Voters

When voters are myopic, all future office-holders are at least as moderate as the current incumbent. However, forward-looking voters may choose to elect parties with more extreme platforms than their opponents if this generates preferred continuation play. In this section, I allow forward-looking voters and restrict attention to equilibria in which the median voter is decisive.[24] I show that my results in the model with myopic voters identify a selection of equilibria in the model with forward-looking voters. Hence, the equilibrium outcomes of this paper are not due to myopic voting.

Proposition 5. *Consider a consistent equilibrium in the game with myopic voters, a state (I, x) such that either $I = L$ and $x \leq M$ or $I = R$ and $x \geq M$ along with the policy path $\{x^t\}$ induced from (I, x) by that equilibrium. Then there exists an equilibrium with forward-looking voters such that the policy path $\{x'^t\}$ induced from (I, x) by that equilibrium is such that $x'^t = x^t$ for all $t \geq 2$.*[25]

To see both that consistent equilibrium strategies under myopic voting are not equilibria with forward-looking voters and how to construct an equilibrium which supports the same policy dynamics, consider Figure 3. This depicts a consistent equilibrium convergence path $\{y^t\}$, a policy y^t such that $\sigma_L(R, 2M - y^t) = y^{t+1}$ and a state (L, x) for some $x \in (y^t, y^{t+1})$. Under myopic voting, party R responds with its most extreme winning policy in state (L, x). If instead party R deviates to policy $2M - x + \epsilon$ for small ϵ, a forward-looking median voter prefers to deviate from the myopic strategy of voting for party L. If it keeps incumbent party L in power for another term, it delays (a) R's victory at policy $2M - x'$ by one period and (b) the resumption of the original convergence path by two periods. Voting for deviating party R in this election allows party L gain office in the next election with moderate policy y^{t+1}. In the equilibrium constructed under forward-looking voting, the median voter sometimes votes against myopically preferred policies. In particular, a function $z^{t+1}(x) \in [y^t, x)$ can

[23]The details are provided in the Supplementary Appendix.
[24]Banks and Duggan (2006) provide sufficient conditions for median decisiveness in this environment.
[25]The proofs of all extensions are provided in the Supplementary Appendix.

be defined such that $2M - z^{t+1}(x)$ is the most extreme policy by R that the median voter supports against x in state (L, x) in order to 'resume' convergence.

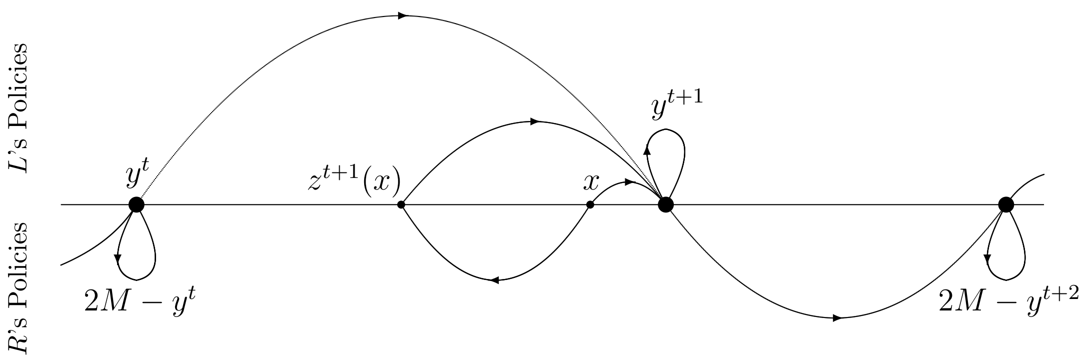

Figure 3: *Policy dynamics of equilibria with forward-looking voters. In state (L, x), the median voter supports party R when it commits to extreme policy $2M - z^{t+1}(x)$.*

5.2 Limited Policy Persistence

To relax the assumption of full commitment while maintaining the relative inflexibility of incumbent parties, suppose that incumbents can, at some cost, free themselves of their platforms. Specifically, given state (I, x), party $-I$ commits to a policy y as before. Second, given y, party I can bear an adjustment cost c to change its policy to some alternative policy x', after which the election is held. The model studied so far has $c = \infty$, but the next result shows that, as long as $c > 0$, alternation is a robust property of equilibrium.

Proposition 6. *Consider the model with cost $c > 0$ to policy adjustments for incumbents. There exists policies $\ell^c \in [0, M)$ and $r^c \in (M, 1]$ such that policy $\ell \leq M$ is a long-run policy outcome if and only if $\ell \in [\max\{\ell^c, 2M - r^c, \ell^*, 2M - r^*\}, M]$.*

Corollary 3. *The set of long-run policy outcomes with adjustment cost $c > 0$ has the following properties.*

i. If $c' > c$, then $[\max\{\ell^{c'}, 2M - r^{c'}, \ell^*, 2M - r^*\}, M] \supseteq [\max\{\ell^c, 2M - r^c, \ell^*, 2M - r^*\}, M]$.

ii. $\lim_{c \to 0}[\max\{\ell^c, 2M - r^c, \ell^*, 2M - r^*\}, M] = \{M\}$

iii. There exists $\bar{c}$ such that $[\max\{\ell^c, 2M - r^c, \ell^*, 2M - r^*\}, M] = [\max\{\ell^*, 2M - r^*\}, M]$ for all $c \geq \bar{c}$.

Incumbent parties are more willing to bear smaller costs to adjust their policies and, in the long-run, equilibrium alternations are closer to the median. In the limit as $c \to 0$, incumbent parties are unconstrained and only the median is observed in the long-run. However, the case of full incumbent policy persistence is not knife-edge. For high enough costs, the set of long-run policy with costly adjustments coincides with that of Proposition 3.

My results are also robust to alternative forms of limited policy persistence. In the Supplementary Appendix, I consider (a) a model in which parties can commit to policies for only $T \geq 2$ periods (i.e., incumbent parties face a T-term limit), and (b) a model in which parties select candidates with observable policy preferences but no ability to commit to polices, where the relative inflexibility of incumbents is obtained by imposing that only opposition parties can replace their representatives. I show that the set of equilibria of both these models are isomorphic to the set of equilibria of my model. For the model with term limits, although Proposition 1 states that in any equilibrium, the median policy obtains when a term-limited incumbent steps down and parties compete simultaneously, first-term incumbents are defeated in all equilibria. Politicians can reach their term limits only by implementing sufficiently moderate policies, but, in equilibrium, they gain by implementing policies they prefer even if these lead to defeat. For the model without commitment, forward-looking candidates implement their ideal policy when in office in all equilibria. Hence, for party I to select a candidate with ideal policy $\hat{x}$ is equivalent to a commitment by party I to implement policy $\hat{x}$, and this commitment persists as long as the party is represented by this politician, that is, as long as this candidate is reelected.

5.3 Office-Motivated Parties

Most static and dynamic models of elections find that parties that care more about holding office per se are more willing to compromise.[26] A party that values office will be willing to compromise *if* compromise leads to a longer tenure, but the link between compromise and tenure is determined in equilibrium. In the Supplementary Appendix, I consider a model in which the payoff to party J from policy y is the sum of its policy payoff $u_J(y)$ and an office benefit $b > 0$. I show that, as $b \to 0$, the set of long-run policy outcomes with office benefits converges to that of Proposition 3. More surprisingly, as $b \to \infty$, the set of long-run policy outcomes with office benefits also converges to that of Proposition 3. That is, if parties rank office and policies lexicographically, the set of long-run policy outcomes is exactly the same as when they are purely office-motivated. To enjoy a longer tenure, an opposition party must commit to a moderate policy that (a) it is willing to champion in exchange for office and

[26]See Calvert (1985) and Duggan (2000).

that (b) the opposition party consents to receiving in exchange for non-participation. For intermediate values of the office benefit b, there are equilibria in which one party retains office by implementing a policy on its opponent's side of the median. However, parties that rank office and policies lexicographically can never offer a compromise policy that induces their opponents not to compete for power. Hence, in equilibrium, both parties are resigned to one-term tenures, and their platforms are guided solely by their policy preferences.

5.4 Median Uncertainty and Incumbency Advantage

Uncertainty about the location of the median policy generates a trade-off between parties' preferred policies and their probability of winning and, in principle, it is not clear whether this would lead to more or less convergence. However, if uncertainty about the median policy is small and favours the incumbent, the benefit to opposition parties in winning probability from small moderate moves should dominate the cost of increased moderation. In the Supplementary Appendix, I consider a model with a probabilistic incumbency bias: in any election the median policy is either M or it is pulled in the direction of the policy championed by the incumbent by a distance of ϵ. I show that, as $\epsilon \to 0$, the only long-run policy outcome of this model is M. The intuition is straightforward: in any (probabilistic) alternation at some $(\ell, 2M - \ell)$, the opposition party L wins the election with a probability that is strictly less than 1 in state $(R, 2M - \ell)$, while policy $\ell + \epsilon$ wins with probability 1. While this points to a discontinuity in the set of equilibrium outcomes of my model when a small amount of median uncertainty is introduced, it admits a substantial qualification. Since the moderate moves ruling out alternations in the long-run when ϵ is small are themselves small, I show that there exist equilibria such that, as $\epsilon \to 0$, parties are 'almost' alternating and the time it takes for policy dynamics to reach M grows without bound. Specifically, given a long-run outcome $\ell > \max\{\ell^*, 2M - r^*\}$ of the model with a known median, there exists an equilibrium of the model with median uncertainty such that, as $\epsilon \to 0$, the time it takes for policy paths to become more moderate than ℓ converges to infinity.

A model with both policy persistence and median uncertainty has a disadvantage: policy paths are considerably less tractable and my approach of characterising all equilibrium outcomes has to be abandoned in favour of studying particular equilibria.[27] However, such a model would present two directions in which to extend my results. First, uncertainty about the median does not affect the mechanism leading to bounded moderation, which relies on the fact that when elections are contested near the median policy, parties' opportunity costs to implementing moderate policies overwhelm the benefits they derive from these in future

[27]Furthermore, Markov perfect equilibria need not exist in general (Duggan and Kalandrakis (2012)).

elections. For this reason, a model with median uncertainty may accommodate dynamic policy divergence. With perfect information, policy dynamics involve alternation at moderate states from which further compromise cannot be supported, as more extreme policies lose with probability 1. With imperfect information, it could be that parties are willing to bear a decrease in winning probability to implement divergent policy paths from such states. Second, with median uncertainty equilibria need not feature deterministic alternation. While my results stress the importance of incumbent policy persistence for generating alternation in government, the patterns of alternation it generates are very simple. Electoral uncertainty may provide a way to focus more closely on the determinants of alternation.

6 Conclusion

This paper studies the policy dynamics of a game of electoral competition between two policy-motivated parties. I highlight the importance of incumbent policy persistence, under which incumbents and challengers are subjected to different standards by voters. Incumbent politicians typically have little choice but to 'run on their record', since their performance in office is fresh in the minds of voters, who have had years to derive information about incumbents' aptitudes and preferences from their decisions. Compounding this effect, opposition candidates or parties often position their platforms relative to the policies enacted by incumbents. Office-holding politicians are acutely aware of this and act accordingly. I show that although opposition parties can outmaneuver inflexible incumbents and gain access to office with extreme policies, they have an incentive to commit to more moderate policies that set the agenda for future electoral campaigns. Hence, convergence *toward* the median is a dynamically robust phenomenon. However, since the incentives to participate in successive rounds of compromise vanish as campaigns are decided by increasingly moderate policies, convergence *to* the median is not.

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Appendix

Proof of Proposition 2. Consider state (R, r) and policy path $\{x^t\}$ induced by (σ_L, σ_R) from (R, r). First note that the policy path following state (R, M) can only be $(M, M, ...)$. To prove the rest of point i and part of point ii, consider the following claim: *In any MPE (σ_L, σ_R), $\sigma_L(R, r) \in X \setminus W(R, r) \cup \{Out\}$ for all $r < M$ and $\sigma_L(R, r) \leq M$ for all $r > M$. The corresponding claims for party R are symmetric.* To show this, consider some MPE (σ_L, σ_R) with $\sigma_L(R, r) \in [r, 2M - r]$ for some $r < M$. Consider a one-shot deviation by L at state (R, r) to *Out*. The payoff to this deviation is

$$u_L(r) + \delta_L V_L(\sigma_L, \sigma_R; (R, r)),$$

while the payoff to $\sigma_L(R, r)$ is $V_L(\sigma_L, \sigma_R; (R, r))$. Hence the deviation is unprofitable if and only if

$$V_L(\sigma_L, \sigma_R; (R, r)) \geq \frac{1}{1 - \delta_L} u_L(r). \tag{2}$$

Since $r < M$, the policy path following (R, r) most favourable to L is $(r, r, ...)$. Hence we have that

$$V_L(\sigma_L, \sigma_R; (R, r)) \leq \frac{1}{1 - \delta_L} u_L(r). \tag{3}$$

(2) and (3) imply that $V_L(\sigma_L, \sigma_R; (R, r)) = \frac{1}{1-\delta_L} u_L(r)$, which holds if and only if $\sigma_L(R, r) = r$ and $\sigma_R(L, r) = r$. Now consider a deviation for R in state (L, r) to $r^d \in (r, 2M - r]$. Any policy path $\{x^t\}$ induced by (σ_L, σ_R) from (R, r^d) must be such that $x^t > r$ for all t. Hence the payoff to r^d is

$$u_R(r^d) + \sum_{t=1}^{\infty} \delta_R^{2t-1} [u_R(x^t) + \delta_R u_R(x^{t+1})] > \frac{1}{1 - \delta_R^2} u_R(r),$$

a contradiction. For the second part of the claim, take (R, r) for some $r > M$ such that $\sigma_L(R, r) > M$. Consider a deviation to some $\ell^d \in (M, \sigma_L(R, r))$. By the first part of the claim, the payoff to ℓ^d is given by

$$\begin{aligned} \frac{1}{1 - \delta_L} u_L(\ell^d) &> \frac{1}{1 - \delta_L} u_L(\sigma_L(R, r)) \\ &= V_L(\sigma_L, \sigma_R; (R, r)), \end{aligned}$$

a contradiction. In a similar manner, if (R, r) for some $r > M$ is such that $\sigma_L(R, r) = Out$, considering a deviation to some $\ell^d \in (M, r)$ yields the desired contradiction.

For point ii of Proposition 2, note that by the previous claim, the sequence $\{x^t\}_{t \text{ odd}}$ is weakly increasing and bounded by x^1 and M, and hence converges to some limit $\hat{\ell}$. The sequence $\{x^t\}_{t \text{ even}}$ is weakly decreasing and bounded by M and x^2, and hence converges to some limit $\hat{r}$. Furthermore, it must be that $\hat{\ell} = 2M - \hat{r}$. Suppose instead that $\hat{\ell} - (2M - \hat{r}) = \epsilon > 0$. Consider $n \in \mathbb{N}$ such that $\hat{\ell} - x^t < \epsilon$ for all $t \geq n$ odd. Then for $j \geq n$ odd

$$
\begin{aligned}
2M - \ell^j &< 2M - \hat{\ell} + \epsilon \\
&= \hat{r} \\
&\leq x^{j+1}
\end{aligned}
$$

and hence $x^{j+1} \notin W(L, x^j)$ and there can be no $\sigma_R(L, x^j)$ such that $\tau((L, x^j), \sigma_R(L, x^j)) = x^{j+1}$, a contradiction. A similar argument shows that it cannot be that $\hat{\ell} < 2M - \hat{r}$. Hence $\hat{r} = 2M - \hat{\ell}$.

To complete the proof of Proposition 2, it remains to be shown that $\sigma_L(R, \hat{r}) = \hat{\ell}$ and $\sigma_R(L, \hat{\ell}) = \hat{r}$. Suppose first that $x^t = \hat{\ell}$ for some t odd. Then $x^j = \hat{\ell}$ for all $j > i$ odd and it must be that $\sigma_L(R, \hat{r}) = \hat{\ell}$ and $\sigma_R(L, \hat{\ell}) = \hat{r}$. Suppose now that $x^t \neq \hat{\ell}$ for all t, and that $\sigma_R(L, \hat{\ell}) = r < \hat{r}$. Consider $\Delta > 0$ such that

$$
u_L(\hat{\ell}) + \frac{\delta_L}{1 - \delta_L^2} U_L^-(2M - r) > \frac{1}{1 - \delta_L^2} U_L^+(\hat{\ell}) + \Delta. \tag{4}
$$

Such a Δ exists by Lemma 1 since $r < \hat{r}$. Since u_L is continuous and $\{x^t\}_{t \text{ odd}} \to \hat{\ell}$, there exists $n \in \mathbb{N}$ and $\epsilon > 0$ such that for all $t \geq n$ odd, $\hat{\ell} - x^t < \epsilon$ and $u_L(x^t) - u_L(\hat{\ell}) < \Delta$. Now, for any $j \geq n$ odd

$$
\begin{aligned}
V_L(\sigma_L, \sigma_R; (R, x^{j-1})) &= u_L(x^j) + \sum_{t=1}^{\infty} \delta_L^{2t-1} [u_L(x^{j+2t-1}) + \delta_L u_L(x^{j+2t})] \\
&\leq u_L(x^j) + \sum_{t=1}^{\infty} \delta_L^{2t-1} U_L^-(x^{j+2t-1}) \\
&\leq u_L(x^j) + \frac{\delta_L}{1 - \delta_L^2} U_L^-(\hat{\ell}) \\
&< \frac{1}{1 - \delta_L^2} U_L^+(\hat{\ell}) + \Delta.
\end{aligned} \tag{5}
$$

The first inequality follows from the fact that $x^{j+2t+1} \geq 2M - x^{j+2t}$ for all t. The second inequality follows by Lemma 1 from the fact that $x^{j+2t} \geq \hat{r}$ for all t. In state (R, x^{j-1}), consider a deviating strategy by L, σ_L^d, such that $\sigma_L^d(R, x^{j-1}) = \hat{\ell}$ and $\sigma_L^d(R, r') = 2M - r'$ for all $r' \leq \hat{r}$. Consider the policy path $\{x'^t\}$ induced by (σ_L^d, σ_R) from (R, x^{j-1}). The payoff to σ_L^d is

$$\begin{aligned} u_L(\hat{\ell}) + \sum_{t=1}^{\infty} \delta_L^{2t-1} U_L^-(2M - x'^{2t}) &\geq u_L(\hat{\ell}) + \frac{\delta_L}{1-\delta_L^2} U_L^-(2M - \hat{r}) \\ &> \frac{1}{1-\delta_L^2} U_L^+(\hat{\ell}) + \Delta \\ &> V_L(\sigma_L, \sigma_R; (R, x^{j-1})), \end{aligned}$$

a contradiction. The first inequality follows from Lemma 1 and the fact that $x^{2t} \leq \hat{r}$ for all t, the second from (4) and the third from (5). The same proof applies to show that $\sigma_L(R, \hat{r}) = \hat{\ell}$. $\square$

Proof of Proposition 3. The following lemma provides a lower bound on equilibrium payoffs.

Lemma 2. *If (σ_L, σ_R) is an equilibrium, then in state (R, r) with $r > M$, the payoff to party L from policy $\ell \in W(R, r)$ for some $\ell \leq M$ is at least $\frac{1}{1-\delta_L^2} U_L^+(\ell)$. The statement for party R is symmetric.*

Proof of Lemma 2. Given state (R, r) with $r > M$, consider the strategy σ'_L for L such that $\sigma'_L(R, r) = \ell \in W(R, r)$ and $\sigma'_L(R, r') = 2M - r'$ for all $r' \leq 2M - \ell$, as well as the policy path $\{x^t\}$ induced by (σ'_L, σ_R) from (R, r). The payoff to σ'_L is

$$u_L(\ell) + \sum_{t=1}^{\infty} \delta_L^{2t-1} U_L^-(x^{2t}) \geq \frac{1}{1-\delta_L^2} U_L^+(\ell),$$

where the inequality follows by Lemma 1 since $x^{2t} \leq 2M - \ell$ for all t. $\square$

The following claim establishes the bound on the extremism of long-run policy outcomes: *If policy ℓ is a long-run policy outcome, then $\ell \geq \max\{\ell^*, 2M - r^*\}$.* To show this, suppose that $\ell^* \geq 2M - r^*$ and that $\ell < \ell^*$ is a long-run policy outcome under (σ_L, σ_R) starting from some state. By Lemma 2, party L can obtain a payoff of at least $\frac{1}{1-\delta_L^2} U_L^+(\ell^*)$ by committing to ℓ^* in state (R, r). However, $V_L(\sigma_L, \sigma_R; (R, r)) = \frac{1}{1-\delta^2} U_L^+(\ell) < \frac{1}{1-\delta_L^2} U_L^+(\ell^*)$ by Lemma 1 since $\ell < \ell^*$, a contradiction. For the equilibrium construction that shows the sufficiency of the bound on extremism, consult the Supplementary Appendix. $\square$

Proof of Corollary 1. Consider v_L obtained from u_L by applying some increasing concave transformation. Then for any $\ell \in (0, M)$, $\frac{v'_L(\ell)}{v'_L(2M-\ell)} < \frac{u'_L(\ell)}{u'_L(2M-\ell)}$. The rest of the claim follows from the discussion in the text. $\square$

The following Lemma characterises convergence paths under consistent strategies.

Lemma 3. *Consider consistent Markov strategies σ_L and σ_R.*

i. If $\sigma_L(R, r) = \ell \in (\max\{2M - r, 0\}, M]$ for some $r > M$, then $\sigma_L(R, r') = \ell$ for all $r' \in [2M - \ell, r)$.

ii. Suppose (σ_L, σ_R) form a consistent equilibrium. If $\sigma_L(R, r) = \ell \in (\max\{2M - r, 0\}, M]$ for some $r > M$, then $\sigma_R(L, \ell') = 2M - \ell'$ for all $\ell' \in [\max\{2M - r, 0\}, \ell)$.

Both statements for R are symmetric.

Proof of Lemma 3. Part i is immediate from the definition of consistent Markov strategies. For part ii, consider consistent equilibrium (σ_L, σ_R), $r > M$ and $\sigma_L(R, r) = \ell > \max\{2M - r, 0\}$. Suppose for some $\ell' \in [\max\{2M - r, 0\}, \ell)$, $\sigma_R(L, \ell') = r' < 2M - \ell'$. There are two cases. First, suppose that $r' \geq 2M - \ell$. Consider the one-shot deviation by R to $2M - \ell'$ in state (L, ℓ'). The payoff to this deviation is

$$
\begin{aligned}
&u_R(2M - \ell') + \delta_R V_R(\sigma_L, \sigma_R; (R, 2M - \ell')) \\
&\qquad > u_R(r') + \delta_R V_R(\sigma_L, \sigma_R; (R, r')) \\
&\qquad = V_R(\sigma_L, \sigma_R; (L, \ell')).
\end{aligned}
$$

a contradiction. The inequality follows since $\sigma_L(R, r') = \ell$ for all $r' \in [2M - \ell, r]$ and $r' < 2M - \ell'$.

Second, suppose $r' < 2M - \ell$. Then by the part i of the lemma it must be that $\sigma_R(L, \ell'') = r'$ for all $\ell'' \in [\ell', 2M - r']$. By reversing the roles in the proof of the first case above, it can be seen that L can profitably deviate to $2M - r'$ at (R, r'). □

The following lemma characterises payoffs on consistent equilibrium convergence paths.

Lemma 4. *Consider long-run policy outcome $\hat{\ell} > \max\{\ell^*, 2M - r^*\}$, associated consistent equilibrium (σ_L, σ_R) and convergence path $\{y^t\} \to \hat{\ell}$ starting from some state. Take state $(R, 2M - y^t)$ such that $\sigma_L(R, 2M - y^t) = y^{t+1}$ with $t > 1$. Then*

$$
V_L(\sigma_L, \sigma_R; (R, 2M - y^t)) = \frac{1}{1 - \delta_L^2} U_L^+(y^t). \tag{6}
$$

Furthermore,

$$
\frac{1}{1 - \delta_L^2} U_L^+(y^t) = u_L(y^{t+1}) + \frac{\delta_L}{1 - \delta_L^2} U_L^-(y^{t+2}). \tag{7}
$$

The case of state (L, y^t) such that $\sigma_R(L, y^t) = 2M - y^{t+1}$ with $t > 1$ is symmetric.

Proof of Lemma 4. Consider state $(R, 2M - y^t)$ such that $\sigma_L(R, 2M - y^t) = y^{t+1}$ with $t > 1$. Since $\hat{\ell} > \max\{\ell^*, 2M - r^*\}$, we have that $y^t < y^{t+1}$ for all t. Since $t > 1$, by Lemma 3 there exists $\epsilon > 0$ such that for all $\ell \in (y^t - \epsilon, y^t]$, $\sigma_R(L, \ell) = 2M - y^t$. For any $\bar{\epsilon} \in (0, \epsilon)$, consider one-shot deviation by L at $(R, 2M - y^t + \bar{\epsilon})$ to $y^{t+1} = \sigma_L(R, 2M - y^t)$. The value to this deviation is given by

$$\begin{aligned} V_L(\sigma_L, \sigma_R; (R, 2M - y^t)) &\leq V_L(\sigma_L, \sigma_R; (R, 2M - y^t + \bar{\epsilon})) \\ &= u_L(y^t - \epsilon) + \delta_L u_L(2M - y^t) + \delta_L^2 V_L(\sigma_L, \sigma_R; (R, 2M - y^t)), \end{aligned}$$

where the inequality follows from equilibrium. This yields

$$V_L(\sigma_L, \sigma_R; (R, 2M - y^t)) \leq \frac{1}{1 - \delta_L^2}[u_L(y^t - \bar{\epsilon}) + \delta_L u_L(2M - y^t)]$$

for any $\bar{\epsilon} \in (0, \epsilon)$, and hence by the continuity of u_L

$$V_L(\sigma_L, \sigma_R; (R, 2M - y^t)) \leq \frac{1}{1 - \delta_L^2} U_L^+(y^t).$$

Lemma 2 yields the opposite inequality and hence

$$V_L(\sigma_L, \sigma_R; (R, 2M - y^t)) = \frac{1}{1 - \delta_L^2} U_L^+(y^t).$$

The final claim of the lemma follow since

$$V_L(\sigma_L, \sigma_R; (R, 2M - y^t)) = u_L(y^{t+1}) + \delta_L u_L(2M - y^{t+2}) \delta_L^2 V_L(\sigma_L, \sigma_R; (R, 2M - y^{t+2})).$$

□

To construct the bound on long-run moderation, define mappings $\alpha_L : [\max\{\ell^*, 2M - r^*\}, M] \to (0, 1]$ and $\alpha_R : [\max\{\ell^*, 2M - r^*\}, M] \to (0, 1]$ such that

$$\begin{aligned} \frac{u_L'(\ell)}{u_L'(2M - \ell)} &= \frac{\delta_L}{\delta_L^2 + \alpha_L(\ell)(1 - \delta_L^2)} \text{ and} \\ \frac{u_R'(2M - \ell)}{u_R'(\ell)} &= \frac{\delta_R}{\delta_R^2 + \alpha_R(\ell)(1 - \delta_R^2)}. \end{aligned} \tag{8}$$

Define ℓ^{**} such that $\alpha_L(\ell^{**}) + \alpha_R(\ell^{**}) = 1$. First show that α_L, α_R and $\ell^{**} \in (\max\{\ell^*, 2M - r^*\}, M)$ are well-defined. To see this, note that since u_L is concave $\frac{u_L'(\ell)}{u_L'(2M-\ell)}$ is strictly increasing in $\ell \in [\ell^*, M]$, with a minimum of δ_L and a maximum of 1. Now $\frac{\delta_L}{\delta_L^2 + \alpha_L(1-\delta_L^2)}$ is strictly decreasing in $\alpha_L \in [0, 1]$, with a minimum of δ_L and a maximum of $\frac{1}{\delta_L}$. $\alpha_L(\ell)$ is well-defined for $\ell \in [\max\{\ell^*, 2M - r^*\}, M]$ since $\frac{u_L'(\max\{\ell^*, 2M-r^*\})}{u_L'(2M-\max\{\ell^*, 2M-r^*\})} \geq \delta_L$. Also, $\alpha_L(\ell) \in (0,1]$ for all ℓ since $\alpha_L(M) = \frac{\delta_L}{1+\delta_L}$ and $\alpha_L(\ell^*) = 1$. Similarly, $\alpha_R(l)$ is well-defined. Furthermore, $\alpha_L(\ell) + \alpha_R(\ell)$ is strictly decreasing in $\ell \in [\max\{\ell^*, 2M - r^*\}, M]$, with $\alpha_L(M) + \alpha_R(M) < 1$ and $\alpha_L(\max\{\ell^*, 2M - r^*\}) + \alpha_R(\max\{\ell^*, 2M - r^*\}) > 1$. Thus $\ell^{**} \in (\max\{\ell^*, 2M - r^*\}, M)$.

To understand the derivation of α_L and α_R, consider y^t, $y^{t+2} = y^t + \Delta$ for some $\Delta > 0$ and the unique $\alpha_L \in (0,1)$ such that

$$\frac{1}{1-\delta_L^2} U_L^+(y^t) = u_L(y^t + \alpha_L \Delta) + \frac{\delta_L}{1-\delta_L^2} U_L^-(y^t + \Delta). \tag{9}$$

The limit of (9) as $\Delta \to 0$ yields that α_L is determined by (8) evaluated at y^t.

Proof of Proposition 4. The following claim establishes the bound on the moderation of robust long-run policy outcomes: *If policy $\hat{\ell} \leq M$ is a robust long-run policy outcome under some consistent equilibrium, then $\hat{\ell} \leq \ell^{**}$.* To show this, the following lemma establishes the properties of the recursive equation (1) that determine consistent equilibrium convergence path policies that allow us to determine possible convergence points.

Lemma 5. *Consider robust long-run policy outcome $\hat{\ell}$ under consistent equilibrium (σ_L, σ_R) and associated convergence path $\{y^t\}$ starting from some state.*

i. Suppose that

$$\frac{u_L'(\hat{\ell})}{u_L'(2M - \hat{\ell})} < \frac{\delta_L}{\delta_L^2 + \alpha_L(1-\delta_L^2)} \tag{10}$$

for some $\alpha_L \in [0,1]$ and that $\sigma_L(R, 2M - y^{t-1}) = y^t$ for some t. Then $y^t - y^{t-1} > \frac{\alpha_L}{1-\alpha_L}(y^{t+1} - y^t)$.

ii. Conversely, suppose that

$$\frac{u_L'(y^j)}{u_L'(2M - y^j)} > \frac{\delta_L}{\delta_L^2 + \alpha_L(1-\delta_L^2)} \tag{11}$$

for some $\alpha_L \in [0,1]$ and that $\sigma_L(R, 2M - y^{j-1}) = y^j$. Then $y^t - y^{t-1} < \frac{\alpha_L}{1-\alpha_L}(y^{t+1} - y^t)$ for all $t \geq j$.

The case for party R is symmetric.

Proof of Lemma 5. To prove part i of the lemma, first prove the following claim: *Suppose that for some $\alpha_L \in [0,1]$ and y, Δ such that $y - \Delta \in [\ell^*, M]$*

$$U_L^+(y - \Delta) - U_L^+(y - (1-\alpha_L)\Delta) \leq \delta_L[U_L^-(y) - U_L^-(y - (1-\alpha_L)\Delta)], \tag{12}$$

then for any $y' \leq y$ and $n \in \mathbb{N}$ such that $y' - 2^n\Delta \in [\ell^*, M]$

$$U_L^+(y' - 2^n\Delta) - U_L^+(y' - 2^n(1-\alpha_L)\Delta) \leq \delta_L[U_L^-(y') - U_L^-(y' - 2^n(1-\alpha_L)\Delta)] \tag{13}$$

with the inequality strict if $y' \neq y$ or $n > 0$. Note that (12) implies that on an infinite convergence path for some consistent equilibrium for which $\sigma_R(L,\ell) = 2M - (y - \Delta)$, $\sigma_L(R, 2M - (y-\Delta)) - y \geq \alpha_L\Delta$. The claim states that if party R's successive policy choices on some consistent equilibrium convergence path are $2M - (y - \Delta)$ and $2M - y$ and party L is (weakly) willing to moderate to $y - (1-\alpha_L)\Delta$ when in state $(R, 2M - (y-\Delta))$,[28] then in another consistent equilibrium convergence path in which party R's successive policies are $2M - (y' - \Delta')$ and $2M - y'$ with $y' \leq y$, then party L is strictly willing to moderate to $y' - (1-\alpha_L)\Delta'$ in state $(R, 2M - (y' - \Delta'))$, where $\Delta' = 2^n\Delta$ for some $n \in \mathbb{N}$.

To prove the claim, note first that, for $y' \leq y$

$$\begin{aligned} U_L^+(y' - \Delta) - U_L^+(y' - (1-\alpha_L)\Delta) &\leq U_L^+(y - \Delta) - U_L^+(y - (1-\alpha_L)\Delta) \\ &\leq \delta_L[U_L^-(y) - U_L^-(y - (1-\alpha_L)\Delta)] \\ &\leq \delta_L[U_L^-(y') - U_L^-(y' - (1-\alpha_L)\Delta)], \end{aligned}$$

with the first and third inequalities strict if $y' \neq y$. The first inequality follows from the strict concavity of U_L^+, the second from (12), and the third from the strict concavity of U_L^-. Given (12), the above shows that

$$\begin{aligned} U_L^+(y - 2\Delta) - U_L^+(y - (2-\alpha_L)\Delta) &< \delta_L[U_L^-(y - \Delta) - U_L^-(y - (2-\alpha_L)\Delta)] \text{ and,} \\ U_L^+(y - (2-\alpha_L)\Delta) - U_L^+(y - 2(1-\alpha_L)\Delta) &< \delta_L[U_L^-(y - (1-\alpha_L)\Delta) - U_L^-(y - 2(1-\alpha_L)\Delta)]. \end{aligned} \tag{14}$$

Hence we have that

$$\begin{aligned} \delta_L[U_L^-(y) - U_L^-(y - 2(1-\alpha_L)\Delta)] &= \delta_L[U_L^-(y) - U_L^-(y - (1-\alpha_L)\Delta)] \\ &\quad + \delta_L[U_L^-(y - (1-\alpha_L)\Delta) - U_L^-(y - 2(1-\alpha_L)\Delta)] \\ &> U_L^+(y - \Delta) - U_L^+(y - (1-\alpha_L)\Delta) \\ &\quad + U_L^+(y - (2-\alpha_L)\Delta) - U_L^+(y - 2(1-\alpha_L)\Delta) \\ &> U_L^+(y - 2\Delta) - U_L^+(y - \Delta(2-\alpha_L)) \\ &\quad + U_L^+(y - (2-\alpha_L)\Delta) - U_L^+(y - 2(1-\alpha_L)\Delta) \\ &= U_L^+(y - 2\Delta) - U_L^+(y - 2(1-\alpha_L)\Delta). \end{aligned}$$

[28] That is, moderate by $\alpha_L\Delta$.

The first inequality follows from (12) and (14), and the second inequality follows from Lemma 1 since $y-(1-\alpha_L)\Delta = y-\Delta(2-\alpha_L)-(y-2\Delta) = \alpha_L\Delta$. The claim follows by applying the above argument recursively.

To complete the proof of part i of Lemma 5, consider (10). This condition guarantees that for arbitrarily small Δ, party L is willing to take up share $\alpha_L\Delta$ of moderation Δ from $y-\Delta$ to y. Hence, there exists some $\tilde{\Delta}$ such that for all $\Delta < \tilde{\Delta}$,

$$U_L^+(\hat{\ell}-\Delta) - U_L^+(\hat{\ell}-(1-\alpha_L)\Delta) < \delta_L[U_L^-(\hat{\ell}) - U_L^-(\hat{\ell}-(1-\alpha_L)\Delta)].$$

Thus, by the earlier claim, for all $y < \hat{\ell}$ and Δ such that $y-\Delta > \ell^*$,

$$U_L^+(y-\Delta) - U_L^+(y-(1-\alpha_L)\Delta) < \delta_L[U_L^-(y) - U_L^-(y-(1-\alpha_L)\Delta)].$$

This implies that for y^t such that $\sigma_L(R, 2M-y^{t-1}) = y^t$, $y^t - y^{t-1} > \frac{\alpha_L}{1-\alpha_L}(y^{t+1}-y^t)$.

The proof of part ii of Lemma 5 follows along the lines of part i. The difference is that while part i is backward-looking, part ii is forward-looking. □

Now to show that moderation is bounded by ℓ^{**}, consider a robust long-run policy outcome $(\hat{\ell}, 2M-\hat{l})$ with $\hat{\ell} > \ell^{**}$ and associated consistent equilibrium (σ_L, σ_R). Consider state (R, r) with $2M-r < \hat{\ell}$ and convergence path $\{y^t\} \to \hat{\ell}$ given (R, r) with $\sigma_L(R, 2M-y^0) = y^1$. Fix n such that $y^n > \ell^{**}$ and $\sigma_L(R, 2M-y^n) = y^{n+1}$. Hence

$$\begin{aligned}\frac{u_L'(y^n)}{u_L'(2M-y^n)} &> \frac{u_L'(\ell^{**})}{u_L'(2M-\ell^{**})} \\ &= \frac{\delta_L}{\delta_L^2 + \alpha_L(\ell^{**})(1-\delta_L^2)},\end{aligned}$$

and hence by part i of Lemma 5, for all $j \geq n$,

$$y^{j+1} - y^j < \frac{\alpha_L(\ell^{**})}{1-\alpha_L(\ell^{**})}(y^{j+2} - y^{j+1}).$$

Similarly, if $j \geq n+1$ and $\sigma_R(L, y^j) = 2M - y^{j+1}$ then

$$y^{j+1} - y^j < \frac{\alpha_R(\ell^{**})}{1-\alpha_R(\ell^{**})}(y^{j+2} - y^{j+1}).$$

This yields that for all $j \geq n+1$,

$$
\begin{aligned}
y^{j+1} - y^j &< \frac{\alpha_L(\ell^{**})}{1-\alpha_L(\ell^{**})}\frac{\alpha_R(\ell^{**})}{1-\alpha_R(\ell^{**})}(y^{j+3} - y^{j+2}) \\
&< (y^{j+3} - y^{j+2}).
\end{aligned}
$$

Hence the convergence path $\{y^t\} \to \hat{\ell}$ contains a nonconverging subsequence, a contradiction.

To show the sufficiency of the bound on moderation, given a strictly increasing sequence $\{y^t\} \to \hat{\ell}$ with $y^0 = \ell^*$ and y^t, y^{t+1} and y^{t+2} satisfying the conditions of Lemma 4 for all $t \geq 1$, an equilibrium construction is provided in the Supplementary Appendix. To complete the proof of Proposition 4, let Y be the set of increasing extended real-valued sequences. Define mapping $B : (\ell^*, M] \to Y$ such that $B(y)^0 = \ell^*$, $B(y)^1 = y$, for each $i \geq 2$ with t even $B(y)^t$ solves

$$
U_L^+(B(y)^{t-2}) - U_L^+(B(y)^{t-1}) = \delta_L[U_L^-(B(y)^t) - U_L^-(B(y)^{t-1})], \tag{15}
$$

and for each $t \geq 3$ with t odd, $B(y)^t$ solves

$$
U_R^+(B(y)^{t-2}) - U_R^+(B(y)^{t-1}) = \delta_R[U_R^-(B(y)^t) - U_R^-(B(y)^{t-1})], \tag{16}
$$

if solutions $B(y)^t \leq M$ exist to (15) and/or (16). If not, set $B(y)^j = \infty$ for all $j \geq t$. Define mapping $\Gamma : (\ell^*, M] \to \mathbb{R} \cup \{\infty\}$ such that $\Gamma(y) = \lim_{t\to\infty} B^t(y)$.

Equations (15) and (16) restate the payoff conditions of Lemma 4. Suppose that $\ell^* \geq 2M - r^*$ and that there exists a consistent equilibrium under which $\hat{\ell} \in (\ell^*, \ell^{**}]$ is a robust long-run policy outcome. In that case, there exists a convergence path $\{y^t\} \to \hat{\ell}$ from state (L, ℓ^*). Suppose that in state (L, ℓ^*) party R selects policy $2M - y$ for $y \in (\ell^*, M]$. The mapping B recovers the full sequence of equilibrium convergence path policies. When no such path exists, we have $B(y)^t = \infty$ for some t. Iteration on B yields a candidate for the sequence posited in the claim for equilibrium $(\sigma_{L^*}^{\hat{\ell}}, \sigma_R^{\hat{\ell}})$, which is acceptable if the limit of of $B(y)$, that is $\Gamma(y)$, is contained in $(\ell^*, \ell^{**}]$. The following claim makes this precise: *Mapping B is such that*

i. The mapping Γ is well-defined, increasing, strictly increasing on $\{y : \Gamma(y) < \infty\}$, right-continuous on $\{y : \Gamma(y) < \ell^{**}\}$ and left-continuous on $\{y : \Gamma(y) < \infty\}$.

ii. For any $\hat{\ell} \in (\ell^*, \ell^{**}]$, there exists y such that $\Gamma(y) = \hat{\ell}$.

iii. A strictly increasing sequence $\{y^t\} \to \hat{\ell}$ with $y^0 = \ell^*$ and y^t,y^{t+1} and y^{t+2} satisfying the conditions of Lemma 4 for all $t \geq 1$.

To show this, not that for $y^1 \in (\ell^*, M]$, $\Gamma(y^1)$ is the limit an increasing extended real-valued sequence and hence is well-defined. For the monotonicity of Γ, consider $y^1, \tilde{y}^1 \in (\ell^*, M]$ such that $y^1 < \tilde{y}^1$, along with induced sequences $\{B(y^1)^t\} = \{y^t\}$ and $\{B(\tilde{y}^1)^t\} = \{\tilde{y}^t\}$. First show that for $t \geq 1$, whenever $\infty > \tilde{y}^{t-1} \geq y^{t-1}$, $\infty > \tilde{y}^t > y^t$, $\tilde{y}^t - \tilde{y}^{t-1} > y^t - y^{t-1}$, and $y^{t+1}, \tilde{y}^{t+1} < \infty$, it is the case that $\tilde{y}^{t+1} - \tilde{y}^t > y^{t+1} - y^t$ and $\tilde{y}^{t+1} > y^{t+1}$. Suppose $\tilde{y}^{t-1} - \epsilon = y^{t-1}$, where $\epsilon \geq 0$. Hence

$$\begin{aligned} U_L^+(\tilde{y}^{t-1} - \epsilon) - U_L^+(\tilde{y}^t - \epsilon) &- \delta_L[U_L^-(y^{t+1}) - U_L^-(\tilde{y}^t - \epsilon)] \\ &> U_L^+(y^{t-1}) - U_L^+(y^t) - \delta_L[U_L^-(y^{t+1}) - U_L^-(y^t)] \\ &= 0, \end{aligned}$$

where the inequality follows by Lemma 1 since $\tilde{y}^t - y^t > \epsilon$. Define $\bar{y}^{t+1}$ such that

$$U_L^+(\tilde{y}^{t-1} - \epsilon) - U_L^+(\tilde{y}^t - \epsilon) - \delta_L[U_L^-(\bar{y}^{t+1}) - U_L^-(\tilde{y}^t - \epsilon)] = 0.$$

It must be that $\bar{y}^{t+1} > y^{t+1}$. By Lemma 1, it is also the case that

$$\begin{aligned} U_L^+(\tilde{y}^{t-1}) - U_L^+(\tilde{y}^t) &- \delta_L[U_L^-(\bar{y}^{t+1} + \epsilon) - U_L^-(\tilde{y}^t)] \\ &> U_L^+(\tilde{y}^{t-1} - \epsilon) - U_L^+(\tilde{y}^t - \epsilon) - \delta_L[U_L^-(\bar{y}^{t+1}) - U_L^-(\tilde{y}^t - \epsilon)] \\ &= 0, \end{aligned}$$

and hence $\tilde{y}^{t+1} > \bar{y}^{t+1} + \epsilon > y^{t+1}$ and $\tilde{y}^{t+1} - \tilde{y}^i > \bar{y}^{t+1} - \tilde{y}^t - \epsilon > y^{t+1} - y^t$. By induction, if $y^1, \tilde{y}^1 \in \{y : \Gamma(y) < \infty\}$, this implies that for each $t \geq 1$, $\tilde{y}^t > y^t$, and

$$\begin{aligned} \Gamma(\tilde{y}^1) &= \lim_{i \to \infty} \tilde{y}^t \\ &> \lim_{i \to \infty} y^t \\ &= \Gamma(y^1). \end{aligned}$$

The above argument also shows that if $y^1 < \tilde{y}^1$, then $y^t < \tilde{y}^t$ for all t such that $\tilde{y}^t < \infty$, and hence that $\Gamma(y^1) \leq \Gamma(\tilde{y}^1)$.

Suppose Γ is not right-continuous at y^1, and that $\Gamma(y^1) < \ell^{**}$. Then there exists $\epsilon > 0$ such that for any $\delta > 0$, $\Gamma(y^1 + \delta) - \Gamma(y^1) > \epsilon$. Take $\bar{\epsilon} \in (0, \min\{\epsilon, \ell^{**} - \Gamma(y^1)\})$. Hence $\Gamma(y^1) + \tilde{\epsilon} < \ell^{**}$ Consider $\tilde{y}^1 \in (y^1, y^1 + \delta)$ and associated sequence $\{\tilde{y}^i\}$. Since $\Gamma(y^1) + \bar{\epsilon} < \ell^{**}$, by part ii of Lemma 5 there exist α_L and α_R with $\alpha_L + \alpha_R > 1$ such that for any $\{\bar{y}^t\} \to \Gamma(\bar{y}^1)$

with $\Gamma(\bar{y}^1) \leq \Gamma(y^1) + \bar{\epsilon}$, $\bar{y}^{t+1} - \bar{y}^i < \frac{\alpha_L}{1-\alpha_L}(\bar{y}^t - \bar{y}^{t-1})$, $\bar{y}^t - \bar{y}^{t-1} < \frac{\alpha_R}{1-\alpha_R}(\bar{y}^{t-1} - \bar{y}^{t-2})$ and

$$\lim_{i\to\infty} \bar{y}^t < \bar{y}^0 + (\bar{y}^1 - \bar{y}^0)\frac{\frac{\alpha_L}{1-\alpha_L}(1 + \frac{\alpha_R}{1-\alpha_R})}{1 - \frac{\alpha_L}{1-\alpha_L}\frac{\alpha_R}{1-\alpha_R}}.$$

Conversely, if $\bar{y}^0 + (\bar{y}^1 - \bar{y}^0)\frac{\frac{\alpha_L}{1-\alpha_L}(1+\frac{\alpha_R}{1-\alpha_R})}{1-\frac{\alpha_L}{1-\alpha_L}\frac{\alpha_R}{1-\alpha_R}} \leq \Gamma(y^1) + \bar{\epsilon}$, then it must be that $\Gamma(\tilde{y}^1) < \Gamma(y^1) + \bar{\epsilon}$. Since $\{y^t\} \to \Gamma(y^1)$, there exists $n \in \mathbb{N}$ such that

$$y^t + (y^{t+1} - y^t)\frac{\frac{\alpha_L}{1-\alpha_L}(1 + \frac{\alpha_R}{1-\alpha_R})}{1 - \frac{\alpha_L}{1-\alpha_L}\frac{\alpha_R}{1-\alpha_R}} < \Gamma(y^1) + \frac{\bar{\epsilon}}{2}$$

for all $t \geq n$. Fix $j \geq n$. Since for all $t \geq 1$, $\tilde{y}^{t+1}$ is a continuous function of $\tilde{y}^t$ and $\tilde{y}^{t-1}$, $\tilde{y}^1$ can be found such that $\tilde{y}^j - y^j < \frac{\bar{\epsilon}}{4}$ and $(\tilde{y}^{j+1} - \tilde{y}^j) - (y^{j+1} - y^j) < \frac{\bar{\epsilon}}{4}\frac{1-\frac{\alpha_L}{1-\alpha_L}\frac{\alpha_R}{1-\alpha_R}}{\frac{\alpha_L}{1-\alpha_L}(1+\frac{\alpha_R}{1-\alpha_R})}$. Then it follows that

$$\begin{aligned}\tilde{y}^j + (\tilde{y}^{j+1} - \tilde{y}^j)\frac{\frac{\alpha_L}{1-\alpha_L}(1 + \frac{\alpha_R}{1-\alpha_R})}{1 - \frac{\alpha_L}{1-\alpha_L}\frac{\alpha_R}{1-\alpha_R}} &< y^j + \frac{\bar{\epsilon}}{4} + (y^{j+1} - y^j)\frac{\frac{\alpha_L}{1-\alpha_L}(1 + \frac{\alpha_R}{1-\alpha_R})}{1 - \frac{\alpha_L}{1-\alpha_L}\frac{\alpha_R}{1-\alpha_R}} + \frac{\bar{\epsilon}}{4} \\ &< \Gamma(y^1) + \bar{\epsilon}.\end{aligned}$$

Hence $\Gamma(\tilde{y}^1)$ is such that $\Gamma(\tilde{y}^1) < \Gamma(y^1) + \bar{\epsilon}$, a contradiction.

Suppose Γ is not left-continuous at y^1, and that $\Gamma(y^1) < \infty$. Then there exists $\epsilon > 0$ such that for any $\delta > 0$, $\Gamma(y^1) - \Gamma(y^1 - \delta) > \epsilon$. Take $j \in \mathbb{N}$ such that $y^j > \Gamma(y^1) - \epsilon + \eta$ for $\eta \in (0, \epsilon)$. Fix $\tilde{y}^1$ such that $y^j - \tilde{y}^j < \eta$. Hence $\tilde{y}^j > y^j - \eta > \Gamma(y^1) - \epsilon$, and hence $\Gamma(\tilde{y}^1) > \Gamma(y^1) - \epsilon$, since $\{\tilde{y}^t\}$ is increasing, a contradiction. The set $\{y : \Gamma(y) < \ell^{**}\}$ is nonempty since $\lim_{y^1\to\ell^*} \Gamma(y^1) = \ell^*$, and hence by continuity of Γ on $\{y : \Gamma(y) < \ell^{**}\}$, for each ℓ with $\ell < \ell^{**}$, there exists y such that $\Gamma(y) = \ell$. Finally, since Γ is left-continuous on $\{y : \Gamma(y) < \infty\}$, there exist a y such that $\Gamma(y) = \ell^{**}$. □

Proof of Corollary 2. The claim follows from 1 and the properties of ℓ^{**} established above. □

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