Elections with Job-Motivated Bureaucrats

Jean Guillaume Forand*

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Abstract

I introduce a dynamic model of two-party elections in which bureaucratic voters are guided both by their preferences over public goods and by their *job motivation*: they bear costs from policies that downsize the government workforce. Job-motivated voters lead to (i) nonexistence of Condorcet-winning platforms in the short run, as well as (ii) bureaucratic persistence in the long-run: equilibrium outcomes converge to alternations around at most two platforms, in which parties compete over the scale of public investment but keep the size of the bureaucracy fixed. This long-run bureaucratic size is never smaller than the optimal bureaucracy of median private sector workers, and parties typically underinvest, so that public goods are produced inefficiently.

Keywords: Public sector voters; Dynamic elections; Bureaucratic persistence

1 Introduction

"We have the ability, in a sense, to elect our own boss."

Victor Gautbom, municipal union leader in New York City, 1975.[1]

Bureaucrats form a sizeable share of modern electorates. For example, an average of 18% of workers are employed in the public sector in OECD countries, ranging from about

*Department of Economics, University of Waterloo, jgforand@uwaterloo.ca. I thank Odilon Câmara, Rohan Dutta, Michael Ting, Gergely Ujhelyi, as well as participants at the Stanford Political Economy Theory Lunch, CEA 2022, the Formal Theory Virtual Seminar, CPEG 2022 and GLPETC 2023.

[1]Quoted in "Out of control: Why public employee unions don't serve the public's best interests", *USA Today*, Feb. 23, 2021.

5% in Japan to over 30% in Norway (OECD, 2021). For these workers, their stakes, as citizens, in public policy decisions are augmented by concerns about possible impacts on their livelihoods. As highlighted in the quote above, elections provide bureaucrats with the opportunity, not typically afforded to other workers, to participate in choosing their future employers. In turn, politicians competing for office need to cater to their future employees' interests if they want to attract their votes. Alternatively, a politician championing policies that alienate bureaucratic voters must clear a higher hurdle to get elected, garnering, for example, additional support from private sector workers.

In this paper, I study the impact of bureaucrats, as voters, on electoral competition and government policy. To do this, I abstract from the details of bureaucratic policy-making; instead, I extend a standard model of two-party competition through public goods provision to include a meaningful role for bureaucratic votes. My main innovation is to assume that bureaucrats are *job motivated*: this captures the idea that, all else equal, a public sector worker will be less favourable than her private sector counterparts to proposals that reduce the government workforce, given that her own job may be on the line. A job-motivated public sector worker may agree that government should be scaled back, but she prefers doing this by sparing employment and concentrating cuts on other categories of government spending. Hence, as a voting block, job-motivated bureaucratic voters can promote demands both for increased government size and for labour-biased inefficiencies in production. My main results show that job-motivated bureaucrats (*i*) are a source of electoral instability in the short run, whereas (*ii*) they drive bureaucratic persistence in the long run.

The 2014 election in the Canadian province of Ontario offers a striking example of the electoral clout of public sector voters. The opposition Progressive Conservative Party, favoured to replace an unpopular Liberal government at the outset of the campaign, proposed cutting 100 000 positions in the public sector, a substantial fraction of approximately 1.1 million such positions.[2] The affected workers formed a large share of 9.5 million registered voters in this election, and an even larger share of likely voters, given a 51% turnout rate. This campaign promise mobilised public sector unions and left-leaning interest groups, who rapidly pointed out how many jobs would be in jeop-

[2] Toronto Star, "Who are the 1.1 million public servants Tim Hudak is talking about?", May 28, 2014.

ardy in each provincial riding,[3] a task made more politically salient by the fact that potentialy affected jobs were widely distributed geographically. A leader of one of the main public sector unions noted that "it will be interesting to watch candidates for the Progressive Conservatives on the campaign trail going door-to-door to deliver the news that there's a good chance that a family member, neighbour or friend could become [...] unemployed".[4] The Conservatives went on to lose the election, with many observers pointing to their proposed job cuts as a key reason.[5] A right-leaning newspaper column later lamented how bureaucratic votes constrain changes to public employment: "Ontario is so overloaded with public sector workers that today they and their families form a huge voting block large enough to influence multi-party elections. [...] Government is no longer our servant. We are working just to keep its workers employed."[6]

In the model, a one-shot election involves two policy-motivated parties, one on the right and one on the left, that compete for the votes of a citizenry divided into an incumbent bureaucracy (a minority) and the private sector (a majority). Policies specify the levels of the two complementary inputs for producing public goods: a new bureaucratic size and a level of public investment. All citizens are differentiated by their willingness to pay for public goods, whether they work in the public or the private sector. On top of that, I model bureaucratic job motivation by assuming that incumbent bureaucrats bear costs from policies that downsize the bureaucracy. These costs are increasing in the scale of cutbacks to public employment, reflecting an individual bureaucrat's increased risk of job loss. Job motivation is meant to capture the narrow employment concerns of low-level bureaucrats. In particular, when it comes to evaluating platforms that call for a bigger bureaucracy, or that fix public sector employment but vary the level of investment, I assume that job-motivated bureaucrats are indistinguishable from their private sector counterparts. This is distinct from modelling high-level bureaucrats or public sector union leaders as empire builders who value government growth per se, as in Niskanen (1971).[7] A typical bureaucrat has a negligible impact on both government policy and on electoral outcomes, whereas layoff risks are likely to loom large in her eval-

[3] Ontario Federation of Labour news release, May 22, 2014.

[4] Warren (Smokey) Thomas, Ontario Public Sector Employees Union press release, May 9, 2014.

[5] Steve Paikin, "Untangling the complicated legacy of Tim Hudak", August 10, 2016, https://www.tvo.org/article/untangling-the-complicated-legacy-of-tim-hudak.

[6] Toronto Sun, "Hudak lost because he told the truth", August 16, 2016.

[7] I study a version of the model with *growth-motivated* bureaucrats in an extension in Section 6.1.

uation of politicians' platforms. For example, in the Ontario election described above, the Conservatives' platform called for the loss of public sector jobs to be offset by the creation of a million new private sector jobs over their term in office. However, it seems unlikely that, for low-level public sector workers, the prospect of easy access to future employment in the private sector outweighed the prospect of losing their current job.

My first main result shows that job-motivated bureaucrats destabilise electoral competition in one-shot elections. This result is notable because my model is standard in all other respects. In particular, if bureaucrats only vote according to their preferences for public goods (i.e., if they have no job motivation), then the optimal platform of median private sector workers is a Condorcet winner. In this setting, the fact that policies are multi-dimensional does not impact the decisiveness of median citizens, because citizens are ordered by their willingness to pay for public goods. In contrast, Condorcet-winning platforms can fail to exist when incumbent bureaucrats' votes are swayed by their job motivation. More precisely, there are two cases. If median private sector workers' optimal bureaucracy is larger than the current bureaucracy, then median citizens' preferences are independent of their occupation and their optimal platform is a Condorcet winner. However, when median private sector workers want to downsize the current bureaucracy, then they disagree with their counterparts in the bureaucracy, and no Condorcet-winning platform exists.

The electoral instability introduced by job-motivated votes stems from a conflict between parties' incentives to cater to private sector workers, who are a majority of voters and care about the efficiency of public goods provision, and their incentives to distort government production to attract less numerous public sector workers, who are biased against bureaucratic downsizing. The first incentive pushes parties to propose the optimal policies of median private sector workers, as in the absence of job motivation. Here, however, these policies cannot be a Condorcet winner when they entail downsizing: at the margin, median private sector workers' votes are much less sensitive to reductions in planned employment cutbacks than those of median bureaucrats.

The empirical literature on the voting behaviour of public sector workers, which I review below, details various quantitative impacts of bureaucratic votes on elections and public policy. My theoretical result on the absence of Condorcet-winning policies suggests that these votes can have important qualitative effects on the nature of electoral competition. To the best of my knowledge, this is a novel suggestion.

A distinguishing feature of elections with large public sector workforces is that current bureaucrats get to vote on parties' plans for future public goods provision, and on future bureaucracies in particular. Correspondingly, I also focus on the impact of job motivation on the dynamics of government activity. To this end, I specify a simple model of dynamic two-party elections, introduced by Kramer (1977) and Wittman (1977), and studied further by Forand (2014) and Nunnari and Zápal (2017): in each period, the incumbent party is committed to the platform that brought it to power in a previous election; the opposition party commits to a new platform; and a majority vote determines the next period's incumbent and corresponding platform. I assume that voters and parties are myopic,[8] and I characterise long-run equilibrium outcomes: those platforms that parties implement in the limit of some equilibrium of the game.

My second main result shows that job motivation generates bureaucratic persistence in the long run. Specifically, in the long run of any equilibrium, policy paths either (i) converge to the optimal platform of median private sector workers, or (ii) they converge to an alternation around two platforms with the same bureaucratic size. In the latter case, the persistent bureaucracy is larger than the optimal bureaucracy of median private sector workers, and parties' platforms are only differentiated by their levels of public investment: the rightwing party invests less than median private sector workers would want (given the long-run equilibrium bureaucracy), and the leftwing party invests more.[9] Because downsizing the bureaucracy incurs resistance from bureaucratic voters but growing it does not, any inefficiency in government policy must take the form of underinvestment. In particular, I show that the rightwing party always underinvests, but that the leftwing party's long-run platform can be efficient.

Long-run equilibrium policy outcomes are starkly different in the benchmark in which bureaucratic voters are not job motivated. As above, whenever equilibrium dynamics do not converge to the optimal platform of median private sector workers, they converge to an alternation around two platforms. Here, however, the long-run bureaucracy is not persistent: the rightwing party downsizes it when it comes to power,

[8] I study the robustness of my results to forward-looking parties in an extension in Section 6.2.

[9] This result is a necessary condition for long-run equilibrium outcomes. In Section 5.2, I provide sufficient conditions, i.e., I characterise those platform alternations that can be supported as long-run alternations in some equilibrium.

after which the leftwing party expands it again. Furthermore, because the bureaucracy is not fixed in the long run, both parties can adjust public investment to their choice of bureaucracy so that both limiting platforms are efficient.

It is intuitive that job-motivated bureaucratic voters should act as a force that limits downsizing both in the short run and in the long run. It is less obvious that they should also constrain long-run change in public employment. This is a dynamic effect that hinges on how job motivation impacts future electoral competition. More specifically, a rightwing party that successfully downsizes the government workforce will sap future support for further growth. This party must overcome opposition among public sector workers with the support of a supermajority of private sector workers. However, a leftwing party that tries to grow the bureaucracy back to its pre-downsizing level does not activate bureaucrats' job motivation, who now vote like their private sector counterparts. Therefore, a supermajority of all voters would now refuse a return to the previous government size. Past downsizing acts as a moderating force on future growth.

The result above ties bureaucratic bloat to public sector votes, which is in line with some results in the empirical literature reviewed below. The link between public sector votes and bureaucratic persistence is more surprising, and is novel. In particular, my result suggests that public sector employment should be less variable over time than other inputs into government production which don't get to vote on their own utilisation. To my knowledge, this and other related questions tied to the political dynamics of bureaucratic size have not been studied empirically.

1.1 Related Literature

The idea that the electoral power of public sector workers can increase the demand for government services dates back at least to Tullock (1972),[10] who proposed this theory as an alternative to that of Niskanen (1971), who argued that budget-maximising bureaucrats would increase the supply of these services. A number of empirical studies describe the impact of bureaucratic votes on both electoral outcomes and government policy. Public sector employees turn out to vote at higher rates than their private sector counterparts (Frey and Pommerehne, 1982; Corey and Garand, 2002; Bhatti and

[10]See also Tullock (1974). Courant et al. (1979) flesh out a related, and more detailed, model.

Hansen, 2013; Geys and Sørensen, 2022), and this effect can be even higher for bureaucrats employed by local governments, who tend to form a bigger share of the electorate in lower-turnout elections (Johnson and Libecap, 1991). Survey evidence supports the existence of a "public-private sector cleavage" in political attitudes and voting behaviour (Blais et al., 1990; Tepe, 2012). Interestingly, Rattsø and Sørensen (2016) show that this cleavage diminishes after bureaucrats retire, suggesting that occupational concerns matter for bureaucrats' political preferences. Bureaucrats have been found to influence the identity of winning candidates in local elections, through endorsements by their unions (Moe, 2006; Sieg and Wang, 2013). Bureaucratic voting power has also been found to have an effect on policy, through increased wages (Anzia, 2011) and higher public spending (Hyytinen et al., 2018). In a closely related literature focusing on the effect of public sector unions, Hoxby (1996) has linked teachers' collective bargaining with inefficiencies in input use.

There is a growing theoretical literature in which bureaucrats influence elections through their actions in government production (Fox and Jordan, 2011; Barseghyan and Coate, 2014; Ujhelyi, 2014; Vlaicu and Whalley, 2016; Forand, 2019; Forand and Ujhelyi, 2021; Li et al., 2020; Forand et al., 2023; Martin and Raffler, 2021; Sasso and Morelli, 2021). However, there is much less theoretical work that connects bureaucrats' occupational interests to their voting behaviour, with the aim to study the corresponding feedback into government policy. A notable exception is the literature on using public sector jobs to buy votes (Robinson and Verdier, 2013; Alesina et al., 2000; Robinson et al., 2006; Shchukin and Arbatli, 2021), although these papers typically focus on less developed civil service systems that leave politicians with numerous opportunities for patronage.[11] For example, Robinson and Verdier (2013) show that politicians underinvest in public goods to decrease private sector productivity and increase the clientelistic value of public sector jobs. In my model, underinvestment results from incentives to spare the jobs of current public sector workers, not to attract the votes of future workers. Maybe the most closely related model is by Babcock et al. (1997), who consider a bargaining problem between a median voter and a public sector union. The point of contact with my paper is that they model the political power of

[11] Relatedly, Huber and Ting (2021) present a model in which patronage appointments are more productive for generating votes, but less productive for generating public goods, than civil service appointments.

current bureaucrats as endogenous to the proposed public policy. They show that if increasing public employment (and hence public goods provision) changes the identity of the median voter to a wealthier citizen, then the union can extract higher wages for its members by growing, as opposed to shrinking, employment. This exploits the fact that wealthier citizens have a higher demand for public goods.[12]

Finally, my paper is related to recent work on elections with loss-averse voters (Alesina and Passarelli, 2019; Lockwood and Rockey, 2020). For job-motivated bureaucrats, the incumbent bureaucracy serves as a reference point, and they bear costs when the current bureaucracy moves below this. However, notable differences are that this reference point is a policy variable and not a utility level, and that it is irrelevant for bureaucrats' payoffs if the bureaucracy grows. Furthermore, only bureaucrats have such reference-dependent preferences, and also the number of bureaucrats is endogenous and evolves over time. That being said, there are connections between how loss aversion and bureaucratic job motivation affect electoral outcomes. For example, both Alesina and Passarelli (2019) and Lockwood and Rockey (2020) show that policy outcomes under loss aversion display persistence (termed *status quo bias* by the former and *platform rigidity* by the latter). Also, the *political endowment effect* in Alesina and Passarelli (2019), which says that a policy transition that overcomes loss aversion through a simple majority requires a supermajority to be reversed, is related to the logic underpinning bureaucratic persistence in my model. More broadly, both loss aversion and job motivation are forms of costly policy change, as studied by Gersbach et al. (2020), whose results also feature policy persistence, but in the form of incumbency advantage.

2 Model

In each period $t = 1, 2, ...$, one of two parties, R and L, forms the government and oversees the production of public goods. There is a mass 1 of citizens. Of these, a mass $B_{t-1} \leq \overline{B} < 1/2$ currently staffs the bureaucracy, with the complementary mass of citizens, $1 - B_{t-1}$, working in the private sector. Bureaucrats receive the wage w^b, whereas private sector workers receive wage w^p. $\overline{B}$ is an upper bound on the mass of

[12]Relatedly, Glaeser and Shleifer (2005) show that politicians can have incentives to redistribute wealth inefficiently in order to shape their electorate.

bureaucrats, and the assumption that $\overline{B} < 1/2$ ensures that, in any period, government employees are not a majority of citizens. This is an empirically sensible assumption, and, in my results, it limits the electoral power of bureaucrats by ensuring that private sector workers are decisive whenever they agree on platform choices. However, as I detail below, citizens disagree on the ideal size of government irrespective of their employment status, so that most elections will oppose two coalitions of voters containing both private and public sector workers with similar tastes for public goods.

Citizens pay taxes to finance the production of public goods, which requires both labour (bureaucrats) and other supporting resources, like capital goods or program funding. Specifically, the government in period t sets a level of public investment $K_t \geq 0$ along with a size $B_t \geq 0$ for the bureaucracy, which produces a level $G(K_t, B_t) \geq 0$ of public goods. I assume that the production function G is strictly concave, has decreasing returns to both inputs (i.e., $G_K, G_B < 0$) and has both inputs be complements (i.e., $G_{KB} > 0$). The government's budget is balanced by a lump sum tax $T_t \geq 0$ imposed on all citizens, so that, using the normalisation of the mass of citizens, the government's spending decisions (K_t, B_t) satisfy the budget constraint $K_t + w^b B_t = T_t$, where I also normalise the price of public investment to 1.

Public goods provision in period t is determined by an election held at the beginning of that period. I model dynamic electoral competition which pits a flexible opposition party against an incumbent party that is committed to proposing the policy that initially gave it access to power (Kramer, 1977; Wittman, 1977; Forand, 2014; Nunnari and Zápal, 2017). In the election at period t, the incumbent $I_t = R, L$ is the party that held office in period $t-1$, and the opposition party $J \neq I_t$ commits to a platform $(K_{J,t}, B_{J,t})$. Incumbents, for their part, are bound by the policy promises that brought them to government: the incumbent party's platform in the election at time t is $(K_{I_t,t-1}, B_{I_t,t-1})$. A majority vote determines the winning party at t. This model of asynchronous platform choices, in which the incumbent's past policies are an analogue to the "endogenous status quo" of the literature on dynamic legislative bargaining,[13] yields a tractable setting in which to study dynamic two-party competition. As shown by Anesi (2010) and Forand (2014), this modelling approach is well suited to focus on long-run policy outcomes. It is also a useful model to study electoral competition

[13]See Eraslan et al. (2022) for a recent survey.

in policy spaces without Condorcet-winning platforms, which I show to be the case with bureaucratic voters. In fact, this feature was an important motivation for the introduction of this model by Kramer (1977) and Wittman (1977).

Notice that bureaucrats inherited from period $t-1$ vote in the election at t that determines the current size of the bureaucracy. This is particularly important given my assumption that bureaucrats' voting choices are driven by their job motivation. Specifically, given government spending decisions (K_t, B_t), I assume that a private sector worker with type $0 \leq \theta \leq \overline{\theta}$ receives stage payoff $\theta G(K_t, B_t) + w^p - T_t$, where the parameter $\overline{\theta} > 0$ bounds the type space. On the other hand, a bureaucrat with type θ receives stage payoff $\theta G(K_t, B_t) + w^b - T_t - \Psi(B_t, B_{t-1})$, where $\Psi(B_t, B_{t-1}) \geq 0$ captures bureaucrats' *job-loss penalty*. I assume that $\Psi(B_t, B_{t-1}) = 0$ if $B_t \geq B_{t-1}$, whereas if $B_t < B_{t-1}$ then $\Psi(B_t, B_{t-1})$ is strictly positive and both strictly decreasing and strictly convex in B_t. This captures the fact that bureaucrats suffer professionally only if the government plans to downsize the bureaucracy, and that these career costs increase in the extent of this downsizing effort. For example, any individual bureaucrat is more likely to be laid off if more jobs are cut. But other than her job security, because private and public sector wages are exogenous, a bureaucrat with type θ has the same preferences over government policy as a private sector worker with the same type. In particular, citizens with a higher value of θ are more willing to trade taxes against public goods.

In what follows, I let $\Psi_B(B_t, B_{t-1})$ denote the partial derivative of Ψ with respect to the new bureaucracy B_t. When $B_t = B_{t-1}$, this derivative need not be defined: in this case, Ψ_B is the lefthand derivative of Ψ, i.e., the marginal increase in job loss costs when the bureaucracy is downsized from B_{t-1} (the righthand derivative at this point is always 0). Therefore, I allow the case of $\Psi_B(B_{t-1}, B_{t-1}) < 0$. The existence (or not) of positive marginal costs from marginal downsizings will play a role in determining the extent of long-run bureaucratic bloat, as I detail in Section 5.2 below.

I assume that the distribution of types in the citizenry is time independent and given by F, with density f. Let θ_M denote the median citizen type. In principle, the conditional type distributions of both private and public sector workers can differ from F. Because the masses of bureaucrats and private sector workers evolve over time, these conditional distributions cannot both be time-independent *and* be consistent with F at each time t. Correspondingly, let $F^b_{B_{t-1}}$ and $F^p_{B_{t-1}}$ denote the conditional type distributions of both private and public sector workers when given incumbent bureaucracy B_{t-1}, with densities $f^b_{B_{t-1}}$ and $f^p_{B_{t-1}}$, respectively. Therefore, for all types θ and any mass of bureaucratic voters B_{t-1} in period t, we have $F(\theta) = B_{t-1}F^b_{B_{t-1}}(\theta) + (1-B_{t-1})F^p_{B_{t-1}}(\theta)$. I assume that $F^b_{B_{t-1}}$ and $F^p_{B_{t-1}}$ are continuous in B_{t-1}. Even though I allow for ideological skew in the preferences for public spending between bureaucrats and other citizens on top of differences in professional interests, all my results hold when the bureaucracy is ideologically representative, i.e., when $F^b_{B_{t-1}} = F^p_{B_{t-1}} = F$ for all B_{t-1}. In what follows, I will refer to median private sector workers and median bureaucrats as the citizens in these occupations that have the population median type θ_M. Notice that the medians of the conditional distributions $F^b_{B_{t-1}}$ and $F^p_{B_{t-1}}$ need not be θ_M, but I ignore these conditional medians because they do not play a role in my results. I also impose the technical assumption that there exist bounds $\underline{f}$ and $\overline{f}$ such that $0 < \underline{f} < f^b_{B_{t-1}}(\theta), f^p_{B_{t-1}}(\theta) \leq \overline{f}$ for all $0 \leq \theta \leq \overline{\theta}$ and all B_{t-1}. In my results below, electoral incentives will be driven by the choices of marginal citizen types, in both the private and public sectors, who are indifferent between competing platforms. These bounds ensure that, at the margin, no types have either negligible or overwhelming influence on parties' vote shares.

Parties are policy-motivated, and have types $\theta_R < \theta_M < \theta_L$. Therefore, parties' preferences are identical to those of a private sector worker who has the same type. I assume that parties and voters are myopic, so that they only consider time t payoffs when making choices at time t. In Section 6.2, I show how my main result on long-run bureaucratic persistence can be extended to a model in which parties are forward-looking. However, notice that the game is dynamic even when all players only focus on current choices, because the size of the bureaucracy, and hence the current electoral clout of bureaucrats, evolves over time according to past electoral outcomes.

Discussion of job motivation. Job motivation is the only component of my model that is not standard. For simplicity, I model it in reduced form, which has the advantage that its impact on equilibrium outcomes is transparent. The assumption posits that bureaucrats' career interests in electoral outcomes are borne primarily by the threat of job loss. Job loss is costly to workers in any sector, notably because of lost job-specific human capital or foregone learning-by-doing during transitions between jobs (Couch and Placzek, 2010; Burdett et al., 2020). Many public sector occupations have limited substitutes in the private sector (e.g., teachers or social workers), which can exacerbate the costs of public-private sector transitions. Relatedly, the public administration literature shows that bureaucrats select into the public sector according to their "public service motivation": a preference for working in government to serve community or country, or to participate in public policy (Perry and Vandenabeele, 2015). Finally, Ψ can also capture the expected utility costs of layoff risks. The relevance of these costs for public sector workers is magnified by the fact that more risk averse workers tend to sort into government employment (Dohmen and Falk, 2010; Buurman et al., 2012).

There is plenty of anecdotal evidence of public sector workers being politically activated by threats of job loss, as I related above for the 2014 Ontario election. More empirical support is provided by available evidence on privatisation and contracting out. These represent two important sources of job loss for bureaucrats, and we would expect them to use their political power to limit them. This is reflected in empirical work that finds that local governments in the U.S. privatise fewer services when public sector unions are strong, or when there are rules restricting political activities by bureaucrats (Lopez-de Silanes et al., 1997; Jerch et al., 2017).[14]

2.1 Equilibrium and Long-Run Bureaucracies

Given any platform (K_I, B_I) for incumbent $I = R, L$ in any period, a policy strategy $\sigma_J(K_I, B_I)$ specifies a platform for the opposition party $J \neq I$. Furthermore, let $V(K_J, B_J; K_I, B_I)$ denote the mass of votes for the opposition party if it commits to platform (K_J, B_J), supposing that a citizen of type θ votes sincerely and supports the party proposing the platform she prefers. Whenever parties obtain the same mass of votes, I assume that the opposition party wins.[15] I say that a profile of policy strategies $\sigma^* = (\sigma_R^*, \sigma_L^*)$ forms an *equilibrium* if, given any incumbent $I = R, L$ and any platform (K_I, B_I), the platform choice $\sigma_J(K_I, B_I)$ of opposition party $J \neq I$ is a solution to

$$\max_{K_J \geq 0, 0 \leq B_J \leq \overline{B}} \theta_J G(K_J, B_J) - K_J - w^b B_J \text{ subject to } V(K_J, B_J; K_I, B_I) \geq 1/2. \quad (1)$$

[14]In the public administration literature, Pallesen (2004) finds that local governments in Denmark contract out less when under fiscal stress, and he argues that this reflects bureaucrats' reduced opposition to privatisation when government budgets do not put the level of government employment at risk.

[15]By standard arguments, this is required by equilibrium in most cases.

Notice that the opposition party can always win an election by proposing $(K_J, B_J) = (K_I, B_I)$. Because the party is myopic, it is indifferent between choosing a losing policy and winning with the incumbent's platform. Including the electoral constraint in problem (1) reduces to requiring that in such cases the opposition party chooses to win the election. This assumption would be satisfied if parties valued holding office per se.

Even if parties are myopic, the policy dynamics in this model can be complicated in the short run, because platforms are multidimensional and exact policy paths are history-dependent. Furthermore, there can exist multiple equilibria in general: because the set of voters contains both private and public sector workers, and these respond differently to platforms involving downsizing, the constraint in (1) need not be convex, so that opposition parties' optimal platforms need not be unique. My goal is to focus on long-run policy outcomes, i.e., those policies that are limits of *some* equilibrium dynamics. More precisely, given any strategy profile $\sigma = (\sigma_R, \sigma_L)$, some initial platform (K_0, B_0) and incumbent I_1, let $A(\sigma)$ denote the set of limit points of the sequence $(K_t, B_t)_{t=1}^{\infty}$ of implemented policies generated by σ. Therefore, given an equilibrium σ^*, the set $A(\sigma^*)$ describes the set of policies that are observed in the long run of this equilibrium. My results will describe the qualitative properties that these long-run policies share across all equilibria of the model.

3 Benchmark: No Job Motivation

To highlight the impacts of job-motivated bureaucrats on elections, I first present the benchmark in which bureaucrats have the same policy preferences as the private sector workers who share their type ($\Psi(B_t, B_{t-1}) = 0$ for all B_t and B_{t-1}). In this case, electoral outcomes at t are independent of the incumbent bureaucracy B_{t-1}.

Fix a private sector worker of type θ. This type's optimal platform, denoted $(\hat{K}^p_\theta, \hat{B}^p_\theta)$, is a solution to

$$\max_{K \geq 0, 0 \leq B \leq \overline{B}} \theta G(K, B) - [K + w^b B]. \tag{2}$$

By the concavity of G, the solution to this problem is characterised by the first-order conditions[16]

$$G_K(\hat{K}^p_\theta, \hat{B}^p_\theta) = \frac{1}{\theta}, \text{ and} \tag{3}$$

$$G_B(\hat{K}^p_\theta, \hat{B}^p_\theta) = \frac{w^b}{\theta}. \tag{4}$$

These two conditions imply that

$$\frac{G_K(\hat{K}^p_\theta, \hat{B}^p_\theta)}{G_B(\hat{K}^p_\theta, \hat{B}^p_\theta)} = \frac{1}{w^b}. \tag{5}$$

In what follows, I say that a platform (K, B) is *efficient* if it satisfies condition (5), which says that the technical rate of substitution between investment and bureaucrats in the production of public goods must match their relative costs. Notice that (5) does not depend on θ. Therefore, although private sector worker types disagree on the desirable level of public goods production, all types agree on how investments and bureaucrats should be used to produce some fixed amount of public goods. Below, this will not be true for job-motivated bureaucrats, who agree with their private sector counterparts when a platform calls for increases in the size of the bureaucracy, but are biased against cuts to the government workforce.

The next result says that, when bureaucrats are not job motivated, (*i*) the median private sector worker's ideal platform is a Condorcet winner in all elections, and (*ii*) in the long run, policies converge to an alternation around at most two platforms, which must furthermore be efficient.[17]

Proposition 1. *Suppose that bureaucrats have no job motivation. Then, in any period t, platform $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$ is a Condorcet winner. Furthermore, fix any equilibrium $\sigma^* = (\sigma^*_R, \sigma^*_L)$. Then, in the long-run, either*

1. *Platforms are optimal for median private sector workers:*

$$A(\sigma^*) = \{(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})\}.$$

2. *Or there is alternation around two efficient platforms:*

$$A(\sigma^*) = \{(\hat{K}^p_{\theta'}, \hat{B}^p_{\theta'}), (\hat{K}^p_{\theta''}, \hat{B}^p_{\theta''})\}, \text{ where } \theta_R \leq \theta' < \theta_M < \theta'' \leq \theta_L.$$

[16] I ignore corner solutions, which can be ruled out by standard assumptions.
[17] The proofs of all results are in the Appendix.

The result is illustrated in Figure 1. The ideal platforms of private sector workers are contained on the grey line, which collects all efficient platforms.[18] Because investment and bureaucrats are complements in the production of public goods, private sector workers' optimal platforms from problem (2) are ordered by type: we have that both $\hat{K}^p_{\theta'} > \hat{K}^p_{\theta}$ and $\hat{B}^p_{\theta'} > \hat{B}^p_{\theta}$ whenever $\theta' > \theta$. Therefore, this policy space is effectively single-dimensional, and hence the optimal platform of the median private sector worker, $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$, is a Condorcet winner. In the absence of job motivation, parties have no incentives to propose inefficient platforms: given any such platform, there exists an alternative which produces the same level of public goods at lower costs, and hence is strictly preferred by all citizens and parties. Therefore, an opposition party's equilibrium problem reduces to choosing the efficient platform on its "side" of the median private sector worker's optimal platform that keeps this type indifferent with the incumbent's efficient platform, which is on that party's own "side" of $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$. Equilibrium dynamics settle quickly on an alternation around, at most, two platforms, as illustrated in Figure 1.

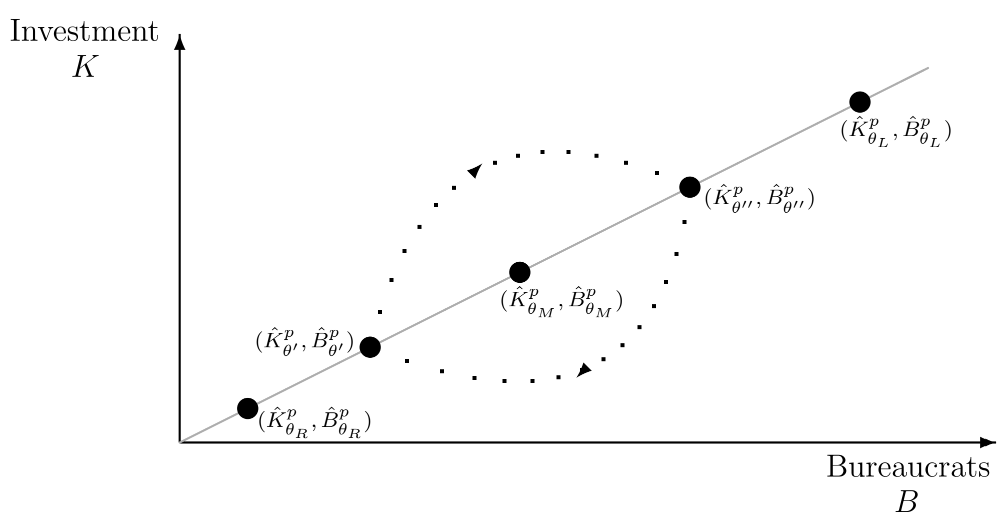

Figure 1: Optimal platforms and equilibrium alternation without job motivation.

The preferences of the electorate are more complex when bureaucrats are job motivated, which explains why electoral competition and policy dynamics are richer. To

[18] If the production function G is Cobb-Douglas, then this is indeed linear, as depicted.

help provide some intuition for my main results in Sections 4 and 5, Figure 2 illustrates the ideal platforms of private and public sector workers when the latter are job motivated. As in Figure 1, the ideal platforms of private sector workers are located on the grey line of efficient platforms. The ideal platforms of bureaucrats, however, evolve over time, because they depend on the current size of the bureaucracy B_{t-1}. Suppose that the incumbent bureaucracy is small, or $B_{t-1} = \hat{B}^p_{\theta'}$ as illustrated. All bureaucrats with types $\theta \geq \theta'$ have the same ideal platforms as their private sector counterparts, which lie on the small-dotted line.[19] Although bureaucrats with types $\theta < \theta'$ would prefer a smaller government, their ideal platforms are inefficient and biased against laying off bureaucrats: in Figure 2, these lie below the grey line. Notice that when the initial bureaucracy is small, the median citizen type's ideal platform is the same whether she works in the private or public sector. This is not the case when the initial bureaucracy is larger than $\hat{B}^p_{\theta_M}$, or $B_{t-1} = \hat{B}^p_{\theta''}$ as illustrated, where bureaucrats' ideal platforms lie on the large-dotted line.

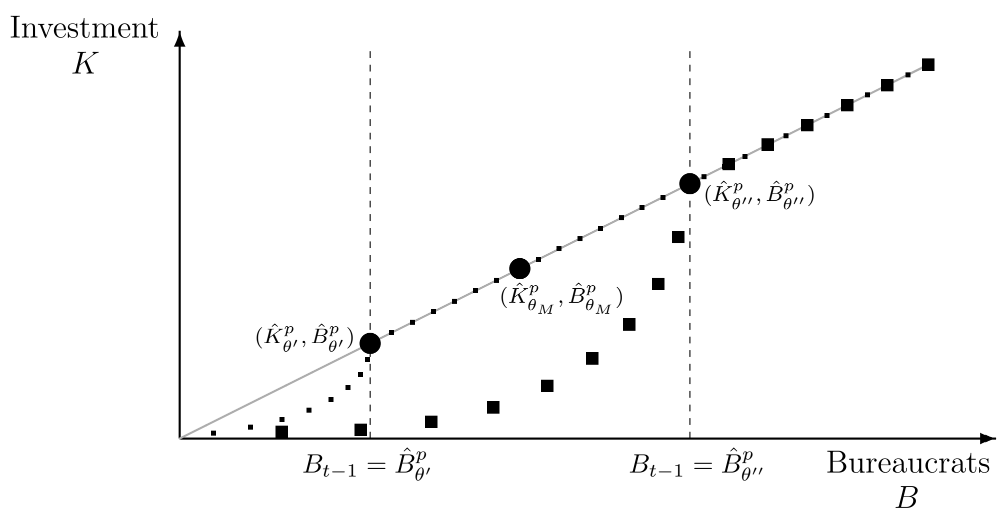

Figure 2: Optimal platforms when bureaucrats are job motivated.

[19]The exact shape of this line in the figure relies on a Cobb-Douglas production function and a quadratic job-loss penalty.

4 Job Motivation and Electoral Instability

In this section, I focus on my model's stage game and show that job-motivated bureaucratic voters can be a source of political instability in one-shot elections. More precisely, I show that, given any election at t in which the incumbent is committed to platform (K_{t-1}, B_{t-1}), no Condorcet-winning platform exists if the median private sector citizen has an ideal bureaucracy that is smaller than B_{t-1} (as was the case when $B_{t-1} = \hat{B}^p_{\theta''}$ in Figure 2). When the electorate contains bureaucrats, the optimal platform of median private sector workers is only a Condorcet winner if it does not conflict with bureaucrats' job motivation, which only happens if median private sector workers do not want to downsize the bureaucracy (as was the case when $B_{t-1} = \hat{B}^p_{\theta'}$ in Figure 2).

Proposition 2. *Consider an election in period t in which the incumbent implements platform (K_{t-1}, B_{t-1}). If $B_{t-1} \leq \hat{B}^p_{\theta_M}$, then platform $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$ is a Condorcet winner. If instead If $B_{t-1} > \hat{B}^p_{\theta_M}$, then no Condorcet winner exists.*

When the median private sector worker wants to downsize the bureaucracy, the nonexistence of a Condorcet-winning platform is the result of a conflict between (i) the efficient production of public goods, which gives incentives for parties to propose $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$, and (ii) the electoral impact of bureaucrats' job motivation, which gives incentives for parties to propose platforms with a bloated bureaucracy.

To unpack the intuition behind the result, a first remark is that, in any Condorcet-winning platform, the mixture of public investment and bureaucrats would have to be efficient. The reason for this is that private sector workers are a majority, so that any platform that produces a fixed level of public goods inefficiently could be defeated by an alternative platform that produces the same amount of public goods at lower costs. Private sector workers would unanimously support the latter platform over the former, so that bureaucrats' preferences over the two are irrelevant. A second remark is that, given some fixed number of bureaucrats, a Condorcet-winning platform would have to provide the median type's optimal level of investment. Otherwise, a platform that keeps the number of bureaucrats fixed but brings investment closer to the level preferred by the median citizen would gather majority support both among private sector workers and the bureaucracy. Putting the two remarks together: if a Condorcet-winning platform would have to be efficient and provide a level of investment that is an optimal response to the size of the bureaucracy for median private sector workers, then it follows that $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$ is the only possible Condorcet winner.

However, whenever median private sector workers want to downsize the bureaucracy, their ideal platform can always be defeated in a majority vote by an alternative platform with an inefficiently large bureaucracy that limits bureaucrats' costs from job loss. To see this, consider a platform that, relative to $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$, keeps public investment unchanged but increases the size of the bureaucracy slightly. It follows that this platform will lose supporters among private sector workers but gain supporters among the bureaucracy. Here, the critical insight is that this platform must gain a lot more bureaucratic votes than it sheds private sector votes. The reason for this is tightly related to the intuition behind a standard envelope theorem: because this new platform is close to their ideal platform, the marginal payoff loss to near-median private sector workers is second order, or negligible, so that few of them will have their votes swayed. However, the marginal drop in the job loss penalty for bureaucrats has a first-order effect on their payoff, so that a much higher mass of them will vote for the new platform.

Although the multi-dimensional policy space does not hinder the existence of Condorcet-winning platforms in the benchmark without job-motivated bureaucrats, it is critical for why such policies fail to exist with job motivation. In fact, the version of my model in which bureaucrats are the only government input is a special case of the model of De Donder (2013) with single-dimensional policies and heterogenous groups of voters, all with single-crossing preferences. There, he shows that this model always admits a Condorcet-winning policy, which weighs the electoral power of the different voter groups (here, public and private sector workers). With two government inputs, no single ordering of private and public sector voters can be obtained; in particular, my result shows that their multi-dimensional conflict over efficiency and downsizing cannot be overcome.

5 Job Motivation and Persistent Bureaucracies

I now turn to describing the long-run outcomes of electoral competition with job-motivated bureaucratic voters. My key finding it that this leads to bureaucratic persistence: in the long-run, the equilibrium bureaucracy must converge to some fixed level, with parties' platforms differing, if at all, only in their levels of public investment.

Proposition 3. *Fix any equilibrium $\sigma^* = (\sigma_R^*, \sigma_L^*)$. Then, in the long-run, either*

1. *Platforms are optimal for median private sector workers:*

$$A(\sigma^*) = \{(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})\}.$$

2. *Or there is alternation around two platforms with the same bureaucracy:*

$$A(\sigma^*) = \{(K_R^*, B^*), (K_L^*, B^*)\}, \text{ where } B^* > \hat{B}^p_{\theta_M} \text{ and } K_R^* < K_L^*.$$

As would be expected, bureaucratic voting power does indeed lead to larger bureaucracies: in the long run, no party, including party R, ever shrinks the bureaucracy below the preferred level of median private sector workers. Furthermore, the bureaucracy is permanently above this level when platforms alternate. This contrasts with the results from the benchmark in Proposition 1. There, a constant bureaucracy in the long run required median convergence, and alternation always featured party R shrinking the bureaucracy below that level. In Figure 3, which illustrates Part 2 of Proposition 3, the platform of party L converges to the optimal platform of type $\theta_M < \theta'' < \theta_L$, which is efficient (Section 5.1 shows that party L may choose inefficient platforms in the long run). Party R's long-run platform, which has the same bureaucracy ($B^* = \hat{B}^p_{\theta''}$) but strictly less investment, is inefficient (Section 5.1 shows that party R's limiting platform must underinvest). In the long run, median private sector workers are indifferent between the platforms of the two parties.

Large bureaucracies also arose in the benchmark in which bureaucrats had no special electoral power, because party L prefers high levels of public goods, whose production requires more bureaucrats. However, in that case large bureaucracies were always downsized when party R took office. The persistence of bloated bureaucracies with job motivation is driven by how downsizing the public sector workforce in the face of bureaucratic opposition shapes future electoral competition. Intuitively, a government needs a supermajority of private sector workers to reduce the size of the bureaucracy, whereas future proposed increases only require a simple majority. Therefore, a successful contraction in the bureaucracy ensures that there will be no support in the electorate for growing it back to its previous size. It is by introducing this force for intertemporal moderation that job motivation leads to convergence in bureaucratic size.

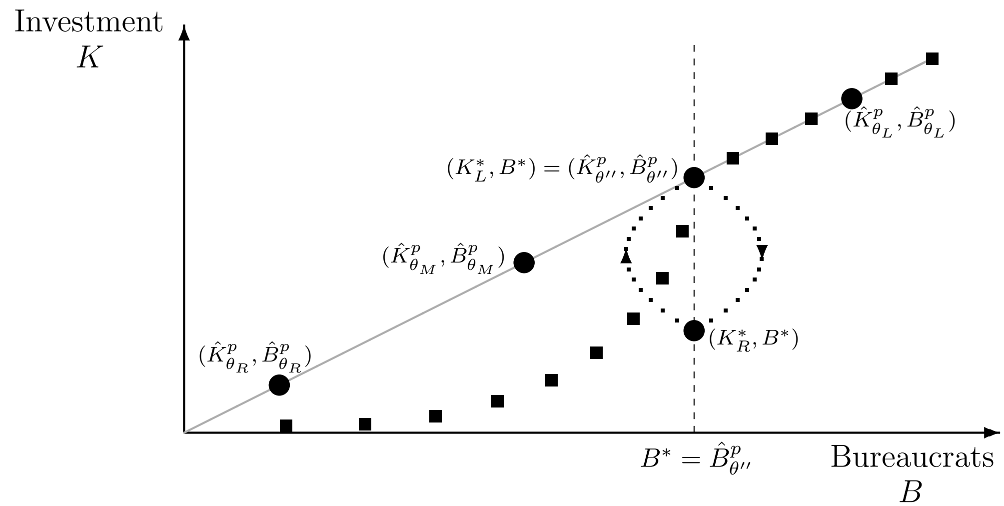

Figure 3: Equilibrium alternation with job motivation.

The proof of Proposition 3 is, maybe surprisingly, a bit involved. The difficulties in the result come from the need to pin down the equilibrium dynamics of arbitrary equilibria. For example, many properties of long-run policy outcomes, and in particular the convergence of equilibrium bureaucracies to a fixed level, must be established before ruling out non-alternating outcomes. That being said, *assuming* that the policy dynamics of some equilibrium converge to an alternation at two platforms (K_L^*, B_L^*) and (K_R^*, B_R^*) with $B_R^* < B_L^*$, which would be inconsistent with Proposition 3, the logic tying job-motivated bureaucrats to persistent bureaucracy is simple to explain. In the limit, when opposition party R proposes (K_R^*, B_R^*) and defeats incumbent party L, which is implementing (K_L^*, B_L^*), it commits to downsizing the bureaucracy, incurring an electoral cost among bureaucratic voters. In particular, the median bureaucrat supports party L in this election. Because party R nevertheless wins the election, it must be the case that median private sector workers strictly prefer its platform. In the next election, party L is supposed to defeat party R by proposing (K_L^*, B_L^*). Here, the party is proposing to grow the bureaucracy, so that median private and public sector workers' preferences coincide. But the previous step already concluded that median private sector workers strictly prefer (K_R^*, B_R^*), so that party L can only attain office by proposing a policy that compromises more than (K_L^*, B_L^*). By facing job-motivated bureaucratic voters when it competes against an incumbent party L, party R is at a short-run disadvantage. However, because bureaucrats' job motivation is inactive when party R is the incumbent, this short-run hurdle generates a long-run advantage for party R, by forcing party L to be more moderate. Clearly, the asymmetry in bureaucratic voters' evaluations of the two parties' policies is critical for this result; I revisit this in Section 6.1, where I allow public sector workers to value bureaucratic growth.

5.1 Efficiency

Contrary to the benchmark without job motivation, long-run alternations are typically inefficient. This is a predictable consequence of bureaucratic persistence: public goods production requires two inputs, one of which is fixed in the long run as a result of electoral competition. Therefore, the two parties' choices on the remaining variable input, public investment, cannot both be efficient. Furthermore, because bureaucratic voters only resist downsizing the bureaucracy, any inefficiency takes the form of investments that are too low relative to the equilibrium number of bureaucrats. More precisely, given platform (K, B) I say that there is *underinvestment* if $G_K(K,B)/G_B(K,B) > 1/w^b$, i.e., if, at the margin, replacing bureaucrats with investment while keeping government production constant reduces government expenses. Finally, because party R prefers lower government spending, it always underinvests in limiting alternations, whereas party L can (but need not) propose efficient platforms in some limiting alternations. These remarks, which follow from Proposition 3, are collected in the following result.

Corollary 1. *Fix a long-run alternation from Proposition 3 at platforms (K_R^*, B^*) and (K_L^*, B^*) with $K_R^* < K_L^*$. Then*

1. *Party R underinvests.*

2. *If furthermore $B^* > \hat{B}^p_{\theta_L}$, then party L also underinvests. If instead $B^* \leq \hat{B}^p_{\theta_L}$, then party L's platform is efficient if K_R^* is low enough, but party L underinvests if K_R^* is high enough.*

What impact does underinvestment have on median private sector workers' payoffs? A first note is that, when their hands are tied by a persistent bureaucracy, median private sector workers do not oppose inefficiency per se. In particular, given a fixed bureaucratic size that is above $\hat{B}^p_{\theta_M}$, the optimal level of public investment for median private sector workers will entail underinvestment. Second, for a given persistent bureaucracy, median private sector workers always prefer to reduce the inefficiency in the platform of party R and increase the inefficiency in the platform of party L. This is because, when faced with a bloated bureaucracy, party R always prefers to starve the government of investment more than the median type, whereas party L always prefers more investment. Persistent bureaucracies can generate voter demands for reduced government efficiency.

To explain the result in Corollary 1 further, notice that there are two reasons why party L can underinvest. First, this can happen if, in equilibrium, bureaucratic voting power is high. Specifically, if the equilibrium bureaucracy is larger than even party L's ideal bureaucracy. In this case, the two parties and median private sector workers agree that a smaller bureaucracy would be best, but bureaucratic voters can block downsizing. But it is also possible for party L to underinvest when the equilibrium bureaucracy is smaller than its ideal bureaucracy. This happens when the competition between the two parties is stiff, i.e., when party R's equilibrium investments are relatively high. In such cases, median citizen's payoffs are high, so that party L only gains office by proposing levels of investment that these citizens prefer, which feature underinvestment.

5.2 Characterising Long-Run Platforms

Proposition 3 says that, absent median convergence, equilibrium dynamics converge to alternating platforms. However, it does not describe describe these limiting platforms, or show how they depend on the model's parameters. I do this in the following result, which presents a condition that is necessary and sufficient for a pair of platforms, one for each party, to be supported as long-run alternations. This condition only applies to pairs of platforms that are consistent with the result of Proposition 3. To this end, I say that platforms (K^*_R, B^*) and (K^*_L, B^*) are *admissible* if (i) $B^* > \hat{B}^p_{\theta_M}$ and $K^*_R < K^*_L$, (ii) (K^*_R, B^*) underinvests and (K^*_L, B^*) is either efficient or underinvests, (iii) no platform (K, B^*) with $K > K^*_R$ is preferred to (K^*_R, B^*) by R and no platform (K, B^*) with $K < K^*_L$ is preferred to (K^*_L, B^*) by L, and (iv) median private sector workers are indifferent between (K^*_R, B^*) and (K^*_L, B^*). Platforms (K^*_R, B^*) and (K^*_L, B^*) are *long-run alternations* if they are admissible and if, furthermore, (K^*_J, B^*) is optimal for all opposition parties $J = L, R$ against the policy (K_I^*, B^*) of the incumbent $I \neq J$. Finally, to obtain a more tractable formulation of the tradeoffs involved, I assume that the conditional type distributions of both private and public sectors are uniform.

Proposition 4. *Assume that both $F^b_{B_{t-1}}$ and $F^p_{B_{t-1}}$ are uniform on $[0, \overline{\theta}]$ for all B_{t-1}. Then admissible platforms (K_R^*, B^*) and (K_L^*, B^*) are long-run alternations if and only if*

$$- B^* \Psi_B(B^*, B^*) \geq w^b \max \left\{ \frac{(\theta_L - \theta_M) G_B(K_L^*, B^*)}{\theta_L G_K(K_L^*, B^*) - 1} \left[\frac{G_K(K_L^*, B^*)}{G_B(K_L^*, B^*)} - \frac{1}{w^b} \right], \right. \\ \left. \frac{(\theta_M - \theta_R) G_B(K_R^*, B^*)}{1 - \theta_R G_K(K_R^*, B^*)} \left[\frac{G_K(K_R^*, B^*)}{G_B(R_L^*, B^*)} - \frac{1}{w^b} \right] \right\}. \quad (6)$$

The condition that platforms (K_R^*, B^*) and (K_L^*, B^*) are admissible ensures that many of the parties' optimality conditions are satisfied. For example, condition (*ii*), which says that both parties are (at least weakly) underinvesting, ensures that neither of them can gain by proposing a platform with $B > B^*$: to limit the efficiency loss, such a deviation would have to also propose a higher level of investment. But such a platform, if proposed by party L, fails to win the election, whereas, if proposed by party R, fails to improve its payoff.

The key condition is inequality (6), which deals with deviations to platforms that downsize the bureaucracy and activate bureaucrats' job motivation. This combines two local optimality conditions: first, that party L cannot, at the margin, benefit from deviating to a platform with $B_L < B^*$, while adjusting K_L so that it still achieves reelection; and second, that party R cannot benefit from an analogous deviation to $B_R < B^*$. Because the production function G is strictly concave and the job loss penalty Ψ is strictly convex in $B_t < B_{t-1} = B^*$, I show in the Appendix that this local necessary condition is also sufficient.

To discuss the intuition behind condition (6), I focus on the optimality condition for party L, which is reflected in the first term of the max operator. In a long-run alternation, the bureaucracy is constant so that each party has the support of median private and public sector workers. By deviating to a platform with $B_L < B^*$, party L loses the support of median bureaucrats, and to win the election it must make up those votes among private sector workers. The righthand side of (6) reflects the electoral benefits to party L from downsizing the bureaucracy. Two factors which affect these benefits are worth mentioning. First, the marginal policy gains for L of compromising with median private sector workers is increasing in the intensity of ideological conflict between them, which is partially captured by $\theta_L - \theta_M$. When this is high, inequality (6) is more difficult to satisfy, so that fewer alternations can be supported as long-run outcomes. More ideologically-driven parties have more incentives to downsize the bureaucracy to achieve policy gains in the short run; but in the long run, this compresses possible equilibrium outcomes. Second, party L attracts private sector votes by making its platform (K_L, B_L) more efficient than its equilibrium platform (K_L^*, B^*), which is valuable both to the party and to citizens. The benefit of increased governmental efficiency is captured by the term ${}^{G_K(K_L^*,B^*)}/_{G_B(K_L^*,B^*)} - {}^1/_{w^b}$: when this is high party L's equilibrium platform entails a lot of underinvestment and inequality (6) is more difficult to satisfy. Notice also that (6) is always satisfied for party L when its equilibrium platform is efficient: in this case, party L cannot offer a more efficient platform that benefits both the party and private sector workers.

The lefthand side of (6) reflects the electoral costs to party L from downsizing the bureaucracy. First, bureaucratic voters have more power in elections when they are more numerous: (6) is easier to satisfy when B^* is higher. Second, bureaucrats have more electoral power when their votes are sensitive to proposals for downsizing: this happens when job loss penalties are steep which, at the margin, is measured by $\Psi_B(B^*, B^*)$. In particular, if job loss penalties from marginal downsizing are low, then long-run outcomes are close to the ideal platform of median private sector workers. This is the case even if job loss penalties are very high when $B_t \ll B_{t-1}$. In this case, equilibrium bureaucracies cannot converge to a level exceeding the optimal bureaucracy of private sector workers because, at the margin, party R can always chip away at such a bureaucracy by proposing small downsizings that gain more private sector votes than they lose public sector votes.

It is interesting that bureaucratic job motivation can actually benefit median private sector workers in the long run. which is not what we would expect from the static model. In the benchmark without job-motivated bureaucrats, the parties face no constraints on how they use the two inputs to compete over time. With job motivation, bureaucratic votes dampen competition through bureaucratic size, which ultimately leads to bureaucratic persistence. But when bureaucratic voting power is not that high, at least at the margin, the dampened competition on bureaucratic size can indirectly intensify competition on the other input, public investment, which benefits the voter.

6 Extensions

6.1 Growth-Motivated Bureaucrats

Job-motivated bureaucrats directly constrain parties that want to downsize the bureaucracy. My main results also show that, through an indirect dynamic effect, job-motivated bureaucrats can also constrain parties that want to grow the bureaucracy: any successful downsizing overcomes bureaucrats' resistance, but reversing that downsizing does not generate job-motivated support among the bureaucracy. However, bureaucratic voters can be motivated by more than the fear of losing their job. In particular, as with the budget-maximising bureaucrats of Niskanen (1975), they may also value growth in the bureaucracy per se. This could be because they anticipate that a larger bureaucracy will grant more power and influence to the programs they value, or that it will provide individual bureaucrats with better advancement opportunities. If bureaucrats are growth-motivated, the dynamic moderating effect from above is reversed: if successful downsizing gains support among private sector workers, successive growth could rely on supermajorities among bureaucrats.

In this section, I highlight the implications of growth motivation on my results in the simplest possible setting: I assume that bureaucrats don't have any job motivation, but that they benefit from bureaucratic growth. Specifically, a bureaucrat with type θ receives stage payoff $\theta G(K_t, B_t) + w^b - T_t + \Psi(B_t, B_{t-1})$, where $\Psi(B_t, B_{t-1}) \geq 0$ now captures bureaucrats' growth benefit. I assume that $\Psi(B_t, B_{t-1}) = 0$ if $B_t \leq B_{t-1}$, whereas if $B_t > B_{t-1}$ then $\Psi(B_{t-1}, B_t)$ is strictly positive and strictly increasing in B_t. A model in which bureaucrats have both growth and job motivations is more general and realistic, but its analysis is less tractable. Here, I focus on pure growth motivation for two reasons. First, this maximises the contrast with the case of pure job motivation, which is the key innovation of my main model. Second, this special setting is sufficient to show how, contrary to job motivation, growth motivation introduces a force for policy extremism. This generates long-run equilibrium outcomes that are quite different from those of Section 5.

Proposition 5. *Suppose that bureaucrats are growth motivated.*

1. *Equilibrium policy dynamics do not converge to alternations with a persistent bureaucracy:*

$$\text{there is no equilibrium } \sigma^* = (\sigma_R^*, \sigma_L^*) \text{ such that } A(\sigma^*) = \{(K_R^*, B^*), (K_L^*, B^*)\} \text{ with } K_R^* < K_L^*.$$

   Furthermore, if $\Psi_B(\hat{B}^p_{\theta_M}, \hat{B}^p_{\theta_M}) > 0$, then equilibrium policy dynamics do not converge to the ideal platform of median private sector workers:

$$\text{there is no equilibrium } \sigma^* \text{ such that } A(\sigma^*) = \{(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})\}.$$

2. *Equilibrium policy dynamics can converge to alternations with different bureaucracies:*

$$\text{there exists } (I_1, K_0, B_0) \text{ along with an equilibrium } \sigma^* = (\sigma_R^*, \sigma_L^*)$$
$$\text{such that } A(\sigma^*) = \{(K_R^*, B_R^*), (K_L^*, B_L^*)\} \text{ and } B_R^* \neq B_L^*.$$

   Furthermore, in any such equilibria, one party implements its optimal platform:

$$\text{either } (K_R^*, B_R^*) = (\hat{K}^p_{\theta_R}, \hat{B}^p_{\theta_R}) \text{ or } (K_L^*, B_L^*) = (\hat{K}^p_{\theta_L}, \hat{B}^p_{\theta_L}).$$

Part 1 says that the long-run outcomes under job motivation are not long-run outcomes under growth motivation. First, bureaucracies are not persistent. The reason for this is that, under growth motivation, parties can eliminate inefficiencies from their platforms in any long-run alternation involving a fixed bureaucracy. Therefore, because a fixed bureaucracy B^* has a unique efficient complementary investment K^*, both parties' platforms in a long-run alternation with a persistent bureaucracy could not both be efficient, ruling out such equilibria. To see why this is true, return to the model with job-motivated bureaucrats. There, parties could always eliminate overinvestment from their platforms: reducing investment while increasing the bureaucracy was supported by private sector workers and could only reduce any electoral penalty imposed by bureaucratic voters. On the other hand, parties could only eliminate underinvestment from their platforms if the benefits to private sector workers outweighed the downsizing costs to public sector workers. With growth motivation, parties can always eliminate underinvestment from their platforms, because decreases in bureaucracy are not penalised.[20] Furthermore, with growth motivation, parties can also eliminate overinvestment from their platforms, because growing the bureaucracy is rewarded by public sector voters.

The second claim in Part 1 says that convergence to the ideal platform of the median private sector worker is ruled out as long as marginal benefits to bureaucratic voters from an increase in the size of the current bureaucracy are positive. This invokes the "envelope theorem" argument used in the proof of Proposition 2 to show that $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$ is not a Condorcet winner with job-motivated bureaucratic voters when $B_{t-1} > \hat{B}^p_{\theta_M}$. With growth-motivated bureaucrats, the claim is stronger: $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$ is not a Condorcet winner even when $B_{t-1} = \hat{B}^p_{\theta_M}$. This is because an alternative platform $(\hat{K}^p_{\theta_M}, B)$ with $B > \hat{B}^p_{\theta_M}$ close to $\hat{B}^p_{\theta_M}$ loses the support of a negligible mass of near-median private sector workers, whereas, because marginal benefits to bureaucratic voters from increasing the size of the bureaucracy are positive, this platform gains the support of a non-negligible mass of public sector workers.

Part 2 says that, when bureaucrats are growth motivated, it is possible to support alternations in the long run between two platforms with different bureaucracies. Furthermore, any such alternation must be relatively extreme, in that one party (or both) implements its ideal platform whenever it is in power. With job-motivated bureaucrats, persistent bureaucracies ensured that parties' long-run platforms catered to different ideological types, but not to different employment types: in the long run, party R obtained support from conservative citizens and party L obtained support from liberals, but no citizen's voting behaviour depended on their occupation. Growth motivation allows for parties to specialise in appealing to different kinds of workers: party L has a permanent advantage, in the long run, with bureaucratic voters, which is balanced (in some equilibria) by a permanent disadvantage with private sector workers.

To understand this result further, consider a long-run alternation at two platforms (K^*_R, B^*_R) and (K^*_L, B^*_L) with $B^*_R < B^*_L$. When party L is in opposition and party R

[20] A remark here is that this argument applies to alternations in which the bureaucracy is persistent. Suppose, for example, that in the long-run party L proposes a platform with a higher bureaucracy than party R. In this case, party L may rely on the additional votes garnered among bureaucrats to attain office, so that increasing its platform's efficiency by proposing a smaller bureaucracy need not be electorally beneficial.

proposes (K_R^*, B_R^*), voters' preferences over platforms are independent of their occupation and, therefore, median private sector workers must weakly prefer (K_R^*, B_R^*) to (K_L^*, B_L^*). If that preference is strict, then party R can offer a platform it prefers unless $(K_R^*, B_R^*) = (\hat{K}^p_{\theta_R}, \hat{B}^p_{\theta_R})$. In this case party R gains office with a supermajority of votes. If instead the median private sector workers are indifferent between the two parties' platforms, then party R attains office with a simple majority of votes. But, in this case, when party L proposes (K_L^*, B_L^*), which has a larger bureaucracy, it obtains a simple majority of private sector votes but a supermajority of public sector votes. By an analogous argument to the one above, this requires that $(K_L^*, B_L^*) = (\hat{K}^p_{\theta_L}, \hat{B}^p_{\theta_L})$.[21]

6.2 Forward-Looking Parties

I assume that parties are myopic. This rules out potentially important dynamic incentives for parties to manipulate bureaucratic size in order to influence their future opponents' choices. For example, a leftwing party could inflate the bureaucracy in order to make it harder for future rightwing opponents to reduce the provision of public goods. Conversely, my results have already illustrated how successful downsizing by a rightwing party saps future support for government growth; this could be actively exploited by a forward-looking rightwing party.

Characterising a single subgame perfect Nash equilibrium in the model with forward-looking parties is difficult, never mind making claims about all equilibrium outcomes, even in restricted classes like stationary Markov equilibria. Here, I take a different tack: I show that my key result about policy dynamics, that job-motivated bureaucratic voters induce bureaucratic persistence, is robust to forward-looking parties. More specifically, I show that, for all subgame perfect Nash equilibria in some broad class, all limit points of equilibrium platforms will have the same bureaucracy. This highlights the fact that my result on persistent bureaucracies is due to the behaviour of (myopic) voters, not parties. In fact, the proof of the result itself is part of the proof of Proposition 3. Studying the incentives for parties to manipulate future bureaucracies to hamstring their opponents is a fruitful topic for further research, but it lies beyond the scope of this paper.

Consider the same game as before, except that now parties care about their future

[21] I show in the Appendix that what determines whether the limiting alternations favour parties R or L is which party's optimal platform is preferred by median private sector workers.

payoffs as well.[22] I focus on subgame perfect Nash equilibria of this game between the two parties that satisfy a natural refinement. Specifically, I say that a subgame perfect Nash equilibrium $\sigma^* = (\sigma_R^*, \sigma_L^*)$ has *nontrivial elections* if the equilibrium policy path $((K_t^*, B_t^*))_{t=1}^{\infty}$ generated by σ^* is such that $G(K_{t+1}^*, B_{t+1}^*) \neq G(K_t^*, B_t^*)$ for all $t \geq 1$. What this means is that, under equilibrium σ^*, elections are determined by voters' preferences for public goods. When two platforms have the same level of public goods, then private sector workers all vote for the most efficient platform, i.e., the one with the lowest costs. In this case public sector workers also vote as a bloc, except that their preferences trade off efficiency against costs from job loss, if applicable.

Proposition 6. *Let $\sigma^* = (\sigma_R^*, \sigma_L^*)$ be a subgame perfect Nash equilibrium with nontrivial elections. Then, in the long run, the equilibrium bureaucracy is persistent:*

$$\text{there exists } B \text{ such that } B^* = B \text{ for all } (K^*, B^*) \in A(\sigma^*).$$

Equilibria with forward-looking parties, even if they can feature incentives and behaviour that are very different than in the myopic model, will nevertheless feature persistent bureaucracies. This result is due to the dynamic properties of voters' payoffs, which play a key role in Proposition 3. The first property is that median private sector workers' payoffs must increase over time. This happens because, in any election, either (i) the winning platform does not downsize the bureaucracy, in which case private sector workers are decisive and must weakly prefer this platform, or (ii) the winning platform downsizes the bureaucracy, in which case some moderate bureaucrats vote for the incumbent and the opposition party only wins the election if median private sector workers strictly prefer this new platform. Therefore, the sequence of median private sector workers' payoffs converges. The second property is that median bureaucrats' payoffs must converge to the same limit. If this was not true, then it must be that there are non-negligible downsizings of the bureaucracy in the limit, which is the only way to explain this gap. But this would mean that the intensity of bureaucrats' payoff losses in these cases would dominate the gains of private sector workers, which is inconsistent with these downsizings being part of winning platforms.[23] Together, these two properties of median citizens' equilibrium payoff dynamics ensure that, in the long run, the

[22] In the interest of space, I forego a formal definition of this game, which is standard.

[23] This is the technical analogue of the explanation I gave in Section 5 for persistent bureaucracies.

bureaucracy must be persistent.

In the myopic model, the proof of Proposition 3 shows that equilibria always have nontrivial elections, whereas in Proposition 6 I need to assume that this is the case for the subgame perfect equilibrium under consideration. With myopic parties, the argument exploits the continuity properties of parties' static optimal platform problems from (1), but these properties are not inherited by the continuation payoffs of the game with forward-looking parties. This additional complexity explains why the result of Proposition 6 is weaker than my characterisation of long-run outcomes from Proposition 3. In the forward-looking model, I can show that the bureaucracy is persistent but cannot pin down the corresponding levels of public investments; in particular, I cannot guarantee that limiting dynamics involve alternation around two platforms.

7 Conclusion

This paper starts from a natural observation, namely, that bureaucrats are inputs into government production that get to vote on their own utilisation. In this paper, I introduced job motivation as a key driver of bureaucratic votes, and my results establish novel connections to the nature of electoral competition, the long-run dynamics of bureaucratic size and the efficiency of government production. My results show that a voting block of job-motivated bureaucrats is qualitatively different from a block of voters that just support larger government: the latter increase the median demand for public goods, whereas the former puts into question the existence of a median voter. I also show that electoral competition over the use of this input with voting rights leaves it fixed in the long run, in effect transferring competition to the remaining, "nonenfranchised", inputs. Therefore, the electoral power of job-motivated bureaucrats is directly tied to government inefficiency.

Given the limited amount of theoretical work on the impact of bureaucrats as voters, this paper is a fist step that leaves many questions open. In my setting, the governing party only controls the amount of downsizing, not which workers it targets and how. In practice, governments can reduce the size of the public workforce through attrition, or they can choose to provide exit packages and retraining programs. My results suggest that these managerial actions also have electoral consequences. In terms of my model, this would imply opening up the black box of job-loss costs, currently captured exogenously by Ψ. A related question is how specific a downsizing party wants to be about its personnel plans during an election. For example, should a party single out specific services or departments for cuts or, as in the case of the 2014 Ontario election, announce a layoff target while remaining fairly vague about the exact workers that will be affected? In the first case, many public sector workers with secure jobs may choose to vote according to their interests as citizens as opposed to as bureaucrats, but those targeted bureaucrats will have very intense preferences for the status quo. In the second case, more bureaucrats' job motivation is activated, but the extent of the uncertainty can mitigate the effect of planned layoffs on any individual bureaucrat's voting choice. These issues would be magnified if the turnout of bureaucrats is related to their preference intensity.

Another limitation of my setting is that, in reality, government inputs other than bureaucrats can also have electoral power. For example, if the inputs into government investment are obtained through procurement, then this gives electoral incentives to the workers involved in their production. My setting can be thought of as applying to the case in which one government input has disproportionate electoral power, in which case my results suggest that productive inefficiencies will be loaded onto that input. However, this leaves open the question of how electoral competition would arbitrate between the competing demands of inputs with more even levels of political power.

A Appendix

Proof of Proposition 1. Given some initial bureaucracy B_0, consider an equilibrium σ^* along with the corresponding sequence $(K_t^*, B_t^*)_{t=1}^{\infty}$ of implemented equilibrium policies.
Step 1. Suppose that $(K_t^*, B_t^*) = (\hat{K}_{\theta_M}^p, \hat{B}_{\theta_M}^p)$ for some t. By the argument in the text preceding Proposition 2, $(\hat{K}_{\theta_M}^p, \hat{B}_{\theta_M}^p)$ is a Condorcet winner when bureaucratic voters have no job-motivation, so that we must have that $(K_{t'}, B_{t'}) = (\hat{K}_{\theta_M}^p, \hat{B}_{\theta_M}^p)$ for all $t' > t$.
Step 2. Suppose that $(K_t^*, B_t^*) \neq (\hat{K}_{\theta_M}^p, \hat{B}_{\theta_M}^p)$ for some t. I show that (K_{t+1}^*, B_{t+1}^*) must be efficient. If instead, towards a contradiction, (K_{t+1}^*, B_{t+1}^*) is not efficient, then, as described in Step 1 of Proposition 2, there exists an alternative platform (K, B) with $G(K, B) = G(K_{t+1}^*, B_{t+1}^*)$, but which imposes strictly lower taxes $T < T_{t+1}^*$. It

follows that (K, B) is strictly preferred to (K^*_{t+1}, B^*_{t+1}), and hence also to (K^*_t, B^*_t), by all citizen types, including type θ_J of the opposition party at $t+1$. But this contradicts the optimality of (K^*_{t+1}, B^*_{t+1}) for party J at $t+1$, as desired.

Step 3. Consider an efficient platform $(K^*_t, B^*_t) \neq (\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$ for some t. Let θ' denote the type for which $(\hat{K}^*_{\theta'}, \hat{B}^*_{\theta'}) = (K^*_t, B^*_t)$, and assume that $\theta' < \theta_M$ (a symmetric argument applies if instead $\theta' > \theta_M$).

First, suppose that the opposition party at $t+1$ is L. By Step 2, we know that party L proposes an efficient platform at $t+1$, and, recalling from the discussion preceding Proposition 2 that optimal platforms are efficient and ordered by type, among the efficient platforms that θ_M prefer to (K^*_t, B^*_t), the optimal platform for party L is $(\hat{K}_{\min\{\theta_L,\theta''\}}, \hat{B}_{\min\{\theta_L,\theta''\}})$, where $\theta'' > \theta_M$ is such that $U^p_{\theta_M}(\hat{K}^*_{\theta''}, \hat{B}^*_{\theta''}) = U^p_{\theta_M}(\hat{K}^*_{\theta'}, \hat{B}^*_{\theta'})$. In period $t+2$, applying the same argument to opposition party R yields an efficient optimal platform $(\hat{K}_{\max\{\theta_L,\theta'''\}}, \hat{B}_{\max\{\theta_L,\theta'''\}})$, where $\theta''' < \theta_M$ is such that $U^p_{\theta_M}(\hat{K}^*_{\theta'''}, \hat{B}^*_{\theta'''}) = U^p_{\theta_M}(\hat{K}_{\min\{\theta_L,\theta''\}}, \hat{B}_{\min\{\theta_L,\theta''\}})$. It follows that, starting in period $t+1$, policies alternate between $(\hat{K}_{\min\{\theta_L,\theta''\}}, \hat{B}_{\min\{\theta_L,\theta''\}})$ when party L is in power and $(\hat{K}_{\max\{\theta_L,\theta'''\}}, \hat{B}_{\max\{\theta_L,\theta'''\}})$ when party R is in power.

Second, suppose that the opposition party at $t+1$ is R. By mimicking the argument from the previous paragraph, the optimal platform for party R at $t+1$ is $(\hat{K}_{\max\{\theta_L,\theta'\}}, \hat{B}_{\max\{\theta_L,\theta'\}})$. Notice that, as of period $t+2$, the argument from the previous paragraph applies, yielding alternation around two efficient platforms.

□

Proof of Proposition 2. First suppose that that $B_0 \leq \hat{B}^p_{\theta_M}$, and fix any $(K, B) \neq (\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$. A private sector worker of type θ prefers $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$ to (K, B) if $\theta[G(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M}) - G(K, B)] \geq w^b[\hat{B}^p_{\theta_M} - B] + [\hat{K}^p_{\theta_M} - K]$, whereas the same is true for a bureaucrat of type θ if $\theta[G(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M}) - G(K, B)] \geq w^b[\hat{B}^p_{\theta_M} - B] + [\hat{K}^p_{\theta_M} - K] - \Psi(B_0, B)$, where I use the fact that $\Psi(B_0, \hat{B}^p_{\theta_M}) = 0$. Suppose, that $G(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M}) \geq G(K, B)$. In this case, given any type $\theta \geq \theta_M$, we have that

$$\begin{aligned}
\theta[G(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M}) - G(K, B)] &\geq \theta_M[G(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M}) - G(K, B)] \\
&> w^b[\hat{B}^p_{\theta_M} - B] + [\hat{K}^p_{\theta_M} - K] \\
&\geq w^b[\hat{B}^p_{\theta_M} - B] + [\hat{K}^p_{\theta_M} - K] - \Psi(B_0, B),
\end{aligned}$$

so that both public and private sector workers with types $\theta \geq \theta_M$, strictly prefer $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$ to (K, B), as desired. A symmetric argument applies if instead $G(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M}) < G(K, B)$.

Now suppose that $B_0 > \hat{B}^p_{\theta_M}$. Suppose, towards a contradiction, that a Condorcet-winning platform (K^{CW}, B^{CW}) exists.

Step 1: I show that (K^{CW}, B^{CW}) must satisfy

$$\frac{G_K(K^{CW}, B^{CW})}{G_B(K^{CW}, B^{CW})} = \frac{1}{w^b}. \tag{7}$$

Recalling (5), this says that, conditional on the level $G(K^{CW}, B^{CW})$ of public goods produced under (K^{CW}, B^{CW}), it must be that investments and bureaucrats are used efficiently, from the point of view of private sector workers. To show this, first suppose, towards a contradiction, that ${G_K(K^{CW},B^{CW})}/{G_B(K^{CW},B^{CW})} < {1}/{w^b}$. In words, this says that platform (K^{CW}, B^{CW}) overinvests. Therefore, there exists a platform (K, B) with $K < K^{CW}$, $B > B^{CW}$, $G(K, B) = G(K^{CW}, B^{CW})$, and with corresponding taxes such that $T < T^{CW}$. It follows that all citizens, whether in the private of the public sector, strictly prefer (K, B) to (K^{CW}, B^{CW}), yielding the desired contradiction. Suppose instead, towards another contradiction, that ${G_K(K^{CW},B^{CW})}/{G_B(K^{CW},B^{CW})} > {1}/{w^b}$. In this case, costs can be reduced by substituting investment for bureaucrats: there exists a platform (K, B) with $K > K^{CW}$, $B < B^{CW}$, $G(K, B) = G(K^{CW}, B^{CW})$, and with corresponding taxes such that $T < T^{CW}$. It follows that all private sector workers strictly prefer (K, B) to (K^{CW}, B^{CW}) which, because $B_0 < 1/2$, means that (K, B) is majority-preferred irrespective of the preferences of bureaucrats, yielding the desired contradiction.

Step 2: I show that, given bureaucracy B^{CW}, investment K^{CW} must be a solution to $\max_{K \geq 0} \theta_M G(K, B^{CW}) - [K + w^b B^{CW}]$. In words, K^{CW} must be optimal for the median citizen against the fixed bureaucracy B^{CW}. To see this, suppose, towards a contradiction, that there exists investment $K \neq K^{CW}$ such that $\theta_M G(K, B^{CW}) - [K + w^b B^{CW}] > \theta_M G(K^{CW}, B^{CW}) - [K^{CW} + w^b B^{CW}]$. But then, because both platforms (K, B^{CW}) and (K^{CW}, B^{CW}) have the same number of bureaucrats, it follows (by arguments analogous to those for the case of $B_0 \leq \hat{B}^p_{\theta_M}$ above) that a supermajority of citizens strictly prefer (K, B^{CW}), yielding the desired contradiction.

Step 3: I show that we must have $(K^{CW}, B^{CW}) = (\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$. By Step 1, a Condorcet-winning platform must satisfy the efficiency condition (7) for private sector workers. By Step 2, this platform must also satisfy the first-order condition

$$G_K(K^{CW}, B^{CW}) = {}^{1}\!/\!{}_{\theta_M}.$$

for type θ_M, which, along with (7), implies that

$$G_B(K^{CW}, B^{CW}) = {}^{w^b}\!/\!{}_{\theta_M}.$$

These last two equations are the first-order conditions for the problem of finding the optimal platform for a private sector worker of type θ, which yields the desired result.

Step 4: I show that we must have that $(K^{CW}, B^{CW}) \neq (\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$. To do this, I construct a platform (K, B) that is majority-preferred to $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$. Fix $0 < \epsilon < B_0 - \hat{B}^p_{\theta_M}$, and consider the platform $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M} + \epsilon)$. Let $\tilde{\theta}^p(\epsilon)$ denote the private sector worker type that is indifferent between $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$ and $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M} + \epsilon)$. By computation, we have that

$$\tilde{\theta}^p(\epsilon) = \frac{w^b \epsilon}{G(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M} + \epsilon) - G(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})},$$

and

$$\tilde{\theta}^{p\prime}(\epsilon) = \frac{w^b - \tilde{\theta}^p(\epsilon) G_B(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M} + \epsilon)}{G(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M} + \epsilon) - G(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})}.$$

Because type θ_M strictly prefers $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$ to $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M} + \epsilon)$ and $G(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M} + \epsilon) - G(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M}) > 0$, we know that $\tilde{\theta}^p(\epsilon) > \theta_M$ and that a private sector worker strictly prefers $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$ if and only if she has type $\theta < \tilde{\theta}^p(\epsilon)$.

Now let $\tilde{\theta}^b(\epsilon)$ denote the bureaucratic type that is indifferent between $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$ and $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M} + \epsilon)$. By computation, we have that

$$\tilde{\theta}^b(\epsilon) = \frac{w^b \epsilon + \Psi(B_0, \hat{B}^p_{\theta_M} + \epsilon) - \Psi(B_0, \hat{B}^p_{\theta_M})}{G(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M} + \epsilon) - G(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})}$$

and

$$\tilde{\theta}^{b\prime}(\epsilon) = \frac{w^b - \tilde{\theta}^b(\epsilon) G_B(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M} + \epsilon) + \Psi_B(B_0, \hat{B}^p_{\theta_M} + \epsilon)}{G(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M} + \epsilon) - G(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})}.$$

Because $\lim_{\epsilon \to 0} \tilde{\theta}^b(\epsilon) = \theta_M$, the first-order condition (4), along with the fact that $\Psi_B(B_0, \hat{B}^p_{\theta_M} + \epsilon) < 0$, ensures that $\tilde{\theta}^{b\prime}(\epsilon) < 0$, and hence that $\tilde{\theta}^b(\epsilon) < \theta_M$, if ϵ is sufficiently small. Furthermore, by the same reasoning as for $\tilde{\theta}^p$ we know that a bureaucrat strictly prefers $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$ if and only if she has type $\theta < \tilde{\theta}^b(\epsilon)$.

Because we have that

$$\frac{\tilde{\theta}^{b\prime}(\epsilon)}{\tilde{\theta}^{p\prime}(\epsilon)} = \frac{w^b - \tilde{\theta}^b(\epsilon) G_B(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M} + \epsilon) + \Psi_B(B_0, \hat{B}^p_{\theta_M} + \epsilon)}{w^b - \tilde{\theta}^p(\epsilon) G_B(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M} + \epsilon)},$$

then again using the first-order condition (4), it follows that $\lim_{\epsilon \to 0} \frac{-\tilde{\theta}^{b\prime}(\epsilon)}{\tilde{\theta}^{p\prime}(\epsilon)} \to \infty$. For what follows, let $\tilde{\epsilon}$ be small enough that $\frac{-\underline{\theta}'(\epsilon)}{\overline{\theta}'(\epsilon)} > \frac{(1-B_0)\overline{f}}{B_0 \underline{f}}$ for all $\epsilon \leq \tilde{\epsilon}$.

Because all private sector workers with types $\theta > \tilde{\theta}^p(\epsilon)$ strictly prefer $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M} + \epsilon)$ to $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$, it follows that the mass of votes among private sector workers for $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M} + \epsilon)$, which we denote by $V^p(\epsilon)$, is

$$V^p(\epsilon) = (1 - B_0)[1 - F^p_{B_0}(\tilde{\theta}^p(\epsilon))].$$

Similarly, because all bureaucrats with types $\theta > \tilde{\theta}^b(\epsilon)$ strictly prefer $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M} + \epsilon)$ to $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$, it follows that the mass of votes among public sector workers for $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M} + \epsilon)$, which we denote by $V^b(\epsilon)$ is

$$V^b(\epsilon) = B_0[1 - F^b_{B_0}(\tilde{\theta}^b(\epsilon))].$$

Therefore, we have that, for all $\epsilon \leq \tilde{\epsilon}$,

$$\begin{aligned} \frac{V^{b\prime}(\epsilon)}{-V^{p\prime}(\epsilon)} &= \frac{B_0 f^b_{B_0}(\tilde{\theta}^b(\epsilon))\tilde{\theta}^{b\prime}(\epsilon)}{-(1 - B_0) f^p_{B_0}(\tilde{\theta}^p(\epsilon))\tilde{\theta}^{p\prime}(\epsilon)} \\ &> \frac{B_0 \underline{f}}{(1 - B_0)\overline{f}} \frac{-\tilde{\theta}^{b\prime}(\epsilon)}{\tilde{\theta}^{p\prime}(\epsilon)} \\ &> 1. \end{aligned}$$

Because $\lim_{\epsilon \to 0} V^b(\epsilon) + V^p(\epsilon) = 1/2$, it follows that $V^b(\tilde{\epsilon}) + V^p(\tilde{\epsilon}) > 1/2$, as desired.

□

Proof of Proposition 3. Because wages in both the private and the public sector are fixed, the voting decisions of all workers are independent of their income. Therefore, it is useful to define their policy utilities net of wages. Given some initial bureaucracy B_{t-1} and some platform (K_t, B_t), let $U^p_\theta(K_t, B_t) = \theta G(K_t, B_t) - [K_t + w^b B_t]$ and $U^b_\theta(K_t, B_t) = U^p_\theta(K_t, B_t) - \Psi(B_{t-1}, B_t)$ denote these policy utilities for private and public sector workers of type θ, which only differ by the layoff costs associated to (K_t, B_t).

Turning to the proof, given some initial bureaucracy B_0, consider an equilibrium σ^* along with the corresponding sequence $(K^*_t, B^*_t)_{t=1}^\infty$ of implemented equilibrium policies.

Step 1. Suppose that $(K^*_t, B^*_t) = (\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$ for some t. Because, by Proposition 2, $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$ is a Condorcet winner when $B_0 \leq \hat{B}^p_{\theta_M}$, we must have that $(K_{t'}, B_{t'}) = (\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$ for all $t' > t$.

Step 2. I determine the limit points of sequences of equilibrium policies for which $(K^*_t, B^*_t) = (\hat{K}^p_{\theta_J}, \hat{B}^p_{\theta_J})$ for some opposition party J and some t. This eases the proof in the steps that follow because, for those remaining sequences of equilibrium policies in which parties never implement their ideal platforms, their electoral constraint from problem (1) must bind.

Suppose first that $V(\hat{K}^p_{\theta_R}, \hat{B}^p_{\theta_R}; \hat{K}^p_{\theta_L}, \hat{B}^p_{\theta_L}) \geq 1/2$. Because $\hat{B}^p_{\theta_R} < \hat{B}^p_{\theta_L}$, it follows that $U^p_{\theta_M}(\hat{K}^p_{\theta_R}, \hat{B}^p_{\theta_R}) > U^p_{\theta_M}(\hat{K}^p_{\theta_L}, \hat{B}^p_{\theta_L})$, so that $V(\hat{K}^p_{\theta_L}, \hat{B}^p_{\theta_L}; \hat{K}^p_{\theta_R}, \hat{B}^p_{\theta_R}) < 1/2$. Assume that $(K^*_t, B^*_t) = (\hat{K}^p_{\theta_L}, \hat{B}^p_{\theta_L})$, and consider period $t+1$ at which party R is in opposition. Because $V(\hat{K}^p_{\theta_R}, \hat{B}^p_{\theta_R}; \hat{K}^p_{\theta_L}, \hat{B}^p_{\theta_L}) \geq 1/2$, the optimal platform for R at $t+1$ is $(\hat{K}^p_{\theta_R}, \hat{B}^p_{\theta_R})$. But then, because $V(\hat{K}^p_{\theta_L}, \hat{B}^p_{\theta_L}; \hat{K}^p_{\theta_R}, \hat{B}^p_{\theta_R}) < 1/2$, the optimal platform for L at $t+2$ is not $(\hat{K}^p_{\theta_L}, \hat{B}^p_{\theta_L})$, and it must be such that $U^p_{\theta_M}(K^*_{L,t+2}, B^*_{L,t+2}) = U^p_{\theta_M}(\hat{K}^p_{\theta_R}, \hat{B}^p_{\theta_R})$, and furthermore it will be such that $B^*_{L,t+2} > \hat{B}^p_{\theta_R}$. But then this implies that, at time $t+3$, $U^b_{\theta_M}(\hat{K}^p_{\theta_R}, \hat{B}^p_{\theta_R}) < U^b_{\theta_M}(K^*_{L,t+2}, B^*_{L,t+2})$, so that $V(\hat{K}^p_{\theta_R}, \hat{B}^p_{\theta_R}; K^*_{L,t+2}, B^*_{L,t+2}) < 1/2$, implying that the optimal platform for R at $t+3$ is not $(\hat{K}^p_{\theta_R}, \hat{B}^p_{\theta_R})$. Therefore, for all $t' \geq t+3$, we have that $(K^*_{t'}, B^*_{t'}) \neq (\hat{K}^p_{\theta_J}, \hat{B}^p_{\theta_J})$ for $J = R, L$. Notice that if at time t we had that $(K^*_t, B^*_t) = (\hat{K}^p_{\theta_R}, \hat{B}^p_{\theta_R})$, then the latter parts of the argument from above establish that, for all $t' \geq t+2$, we have that $(K^*_{t'}, B^*_{t'}) \neq (\hat{K}^p_{\theta_J}, \hat{B}^p_{\theta_J})$ for $J = R, L$.

Now suppose that $V(\hat{K}^p_{\theta_L}, \hat{B}^p_{\theta_L}; \hat{K}^p_{\theta_R}, \hat{B}^p_{\theta_R}) \geq 1/2$. In this case, we must have that $V(\hat{K}^p_{\theta_R}, \hat{B}^p_{\theta_R}; \hat{K}^p_{\theta_L}, \hat{B}^p_{\theta_L}) < 1/2$: otherwise, the first line from the preceding paragraph requires that $V(\hat{K}^p_{\theta_L}, \hat{B}^p_{\theta_L}; \hat{K}^p_{\theta_R}, \hat{B}^p_{\theta_R}) < 1/2$, a contradiction. Assume that $(K^*_t, B^*_t) = (\hat{K}^p_{\theta_R}, \hat{B}^p_{\theta_R})$, and consider period $t+1$ at which party L is in opposition. Because $V(\hat{K}^p_{\theta_L}, \hat{B}^p_{\theta_L}; \hat{K}^p_{\theta_R}, \hat{B}^p_{\theta_R}) \geq 1/2$, the optimal platform for L at $t+1$ is $(\hat{K}^p_{\theta_L}, \hat{B}^p_{\theta_L})$. But then, because $V(\hat{K}^p_{\theta_R}, \hat{B}^p_{\theta_R}; \hat{K}^p_{\theta_L}, \hat{B}^p_{\theta_L}) < 1/2$, the optimal platform for R at $t+2$ is not $(\hat{K}^p_{\theta_R}, \hat{B}^p_{\theta_R})$, and it must be such that $V(K^*_{R,t+2}, B^*_{R,t+2}; \hat{K}^p_{\theta_L}, \hat{B}^p_{\theta_L}) = 1/2$. There are two cases to consider. First, assume that $B^*_{R,t+2} < \hat{B}^p_{\theta_L}$. In this case, we have that $U^p_{\theta_M}(K^*_{R,t+2}, B^*_{R,t+2}) > U^p_{\theta_M}(\hat{K}^p_{\theta_L}, \hat{B}^p_{\theta_L})$, so that, at time $t+3$, $V(\hat{K}^p_{\theta_L}, \hat{B}^p_{\theta_L}; K^*_{R,t+2}, B^*_{R,t+2})) < 1/2$ and the optimal platform for L is not $(\hat{K}^p_{\theta_L}, \hat{B}^p_{\theta_L})$. Therefore, for all $t' \geq t+2$, we have that $(K^*_{t'}, B^*_{t'}) \neq (\hat{K}^p_{\theta_J}, \hat{B}^p_{\theta_J})$ for $J = R, L$. Second, assume that $B^*_{R,t+2} = \hat{B}^p_{\theta_L}$. In this case, we have that $U^p_{\theta_M}(K^*_{R,t+2}, \hat{B}^p_{\theta_L}) = U^p_{\theta_M}(\hat{K}^p_{\theta_L}, \hat{B}^p_{\theta_L})$, so that, at time $t+3$, $V(\hat{K}^p_{\theta_L}, \hat{B}^p_{\theta_L}; K_{R,t+2}, \hat{B}^p_{\theta_L})) = 1/2$ and the optimal platform for L is $(\hat{K}^p_{\theta_L}, \hat{B}^p_{\theta_L})$. In this case, parties' platforms alternate at $(\hat{K}^p_{\theta_L}, \hat{B}^p_{\theta_L})$ and $(K^*_{R,t+2}, \hat{B}^p_{\theta_L})$ from period $t+1$ onward. Notice that if at time t we had that $(K^*_t, B^*_t) = (\hat{K}^p_{\theta_L}, \hat{B}^p_{\theta_L})$, then the two cases from above establish that either (i) for all $t' \geq t+1$, we have that $(K^*_{t'}, B^*_{t'}) \neq (\hat{K}^p_{\theta_J}, \hat{B}^p_{\theta_J})$ for $J = R, L$, or that (ii) parties' platforms alternate at $(\hat{K}^p_{\theta_L}, \hat{B}^p_{\theta_L})$ and $(K^*_{R,t+2}, \hat{B}^p_{\theta_L})$ from period t onward.

Step 3. I show that if $(K^*_t, B^*_t) \neq (\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$ and $(K^*_t, B^*_t) = (K^*_{t-1}, B^*_{t-1})$, then $(K^*_{t+1}, B^*_{t+1}) \neq (K^*_t, B^*_t)$. To see this, suppose that party R is the opposition party at t that implements (K^*_t, B^*_t) (a symmetric argument applies to party L). Let $\hat{K}^p_\theta(B)$ denote the optimal level of investment for a private sector worker of type θ if the level of the bureaucracy is fixed at B. First, we cannot have that $K^*_t > \hat{K}_{\theta_M}(B^*_t)$: in this case, because $U^p_{\theta_R}(\hat{K}_{\theta_M}(B^*_t), B^*_t) > U^p_{\theta_R}(K^*_t, B^*_t)$ and $U^p_{\theta_M}(\hat{K}_{\theta_M}(B^*_t), B^*_t) > U^p_{\theta_M}(K^*_t, B^*_t)$, platform (K^*_t, B^*_t) is not optimal for R at time t. Second, suppose that $K^*_t < \hat{K}_{\theta_M}(B^*_t)$. But then, because $U^p_{\theta_L}(\hat{K}_{\theta_M}(B^*_t), B^*_t) > U^p_{\theta_L}(K^*_t, B^*_t)$ and $U^p_{\theta_M}(\hat{K}_{\theta_M}(B^*_t), B^*_t) > U^p_{\theta_M}(K^*_t, B^*_t)$, it follows that $(K^*_{t+1}, B^*_{t+1}) \neq (K^*_t, B^*_t)$. Third, suppose that $K^*_t = \hat{K}_{\theta_M}(B^*_t)$. But then, because $(K^*_t, B^*_t) \neq (\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$ is not efficient, we know (from the argument in Step 3 of Proposition 2) that there exists a platform (K', B') that is strictly preferred to (K^*_t, B^*_t) by all private sector workers (and hence type θ_L) and furthermore leads to the election of party L in $t+1$. Hence, we must have that $(K^*_{t+1}, B^*_{t+1}) \neq (K^*_t, B^*_t)$ in this case as well.

From this step, we know that any policies $(K^*_t, B^*_t) \neq (\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$ such that $(K^*_t, B^*_t) = (K^*_{t-1}, B^*_{t-1})$ can never be contained in the set of limit points $A(\sigma^*)$. Therefore, for the purposes of characterizing this set it is enough to restrict attention to the subsequence of $((K^*_t, B^*_t))_{t=1}^\infty$ for which $(K^*_t, B^*_t) \neq (K^*_{t-1}, B^*_{t-1})$.

Step 4. I show that if $(K^*_t, B^*_t) \neq (K^*_{t-1}, B^*_{t-1}), (\hat{K}^p_{\theta_J}, \hat{B}^p_{\theta_J})$, then $G(K^*_t, B^*_t) \neq G(K^*_{t-1}, B^*_{t-1})$. Assume, towards a contradiction, that $G(K^*_t, B^*_t) = G(K^*_{t-1}, B^*_{t-1})$. I show that it must be the case that the opposition party J at t strictly prefers (K_t^*, B_t^*) to (K_{t-1}^*, B_{t-1}^*), which holds if and only if $K_t^* + w^b B_t^* < K_{t-1}^* + w^b B_{t-1}^*$, i.e, if the platform at t delivers the same amount of public goods at strictly lower cost. If instead $K_t^* + w^b B_t^* = K_{t-1}^* + w^b B_{t-1}^*$, then given any $0 < \alpha < 1$ consider the platform $(K_t', B_t') = \alpha(K_t^*, B_t^*) + (1-\alpha)(K_{t-1}^*, B_{t-1}^*)$. This platform has the same cost as both (K_t^*, B_t^*) and (K_{t-1}^*, B_{t-1}^*) but, because G is strictly concave, it achieves a strictly higher level of public goods. Hence, every private sector worker, including type θ_J, strictly prefers (K_t', B_t') to (K_{t-1}^*, B_{t-1}^*), so that, because $B_{t-1}^* < 1/2$, platform (K_t', B_t') leads to the election of opposition party J at t. However, type θ_J is indifferent between (K_t^*, B_t^*) and (K_{t-1}^*, B_{t-1}^*), so that opposition party J strictly prefers to propose (K_t', B_t') at t, contradicting the optimality of (K_t^*, B_t^*). Returning to the main claim: if party J strictly prefers (K_t^*, B_t^*) to (K_{t-1}^*, B_{t-1}^*), then so must every private sector worker and, because $B_{t-1}^* < 1/2$, we have $V(K_t^*, B_t^*; K_{t-1}^*, B_{t-1}^*) > 1/2$. But then, because $(K_t^*, B_t^*) \neq (\hat{K}_{\theta_J}^p, \hat{B}_{\theta_J}^p)$ and G is strictly concave, there exists (K_t', B_t') close to (K_t^*, B_t^*) that is both strictly preferred by J to (K_t^*, B_t^*) and also such that $V(K_t', B_t'; K_{t-1}^*, B_{t-1}^*) > 1/2$. But this contradicts the optimality of (K_t^*, B_t^*) for J, as desired.

Step 5. I show that if $(K_t^*, B_t^*) \neq (\hat{K}_{\theta_J}^p, \hat{B}_{\theta_J}^p)$ and $G(K_t^*, B_t^*) \neq G(K_{t-1}^*, B_{t-1}^*)$, then $V(K_t^*, B_t^*; K_{t-1}^*, B_{t-1}^*) = 1/2$. Suppose, towards a contradiction, that $V(K_t^*, B_t^*; K_{t-1}^*, B_{t-1}^*) > 1/2$. Assume further that $G(K_t^*, B_t^*) < G(K_{t-1}^*, B_{t-1}^*)$ (a symmetric argument applies to the case of $G(K_t^*, B_t^*) > G(K_{t-1}^*, B_{t-1}^*)$). It follows that there exist threshold types $\tilde{\theta}^p$ and $\tilde{\theta}^b$ such that private (pubic) sector worker of type θ votes for J whenever $\theta \leq \tilde{\theta}^p$ ($\theta \leq \tilde{\theta}^b$). Furthermore, by assumption $V(K_t^*, B_t^*; K_{t-1}^*, B_{t-1}^*) = B_{t-1}^* F_{B_{t-1}}^b(\tilde{\theta}^b) + (1 - B_{t-1}^*) F_{B_{t-1}}^p(\tilde{\theta}^p) > 1/2$. But then, because $(K_t^*, B_t^*) \neq (\hat{K}_{\theta_J}^p, \hat{B}_{\theta_J}^p)$ and G is strictly concave, there exists (K_t', B_t') close to (K_t^*, B_t^*) that is both strictly preferred by J to (K_t^*, B_t^*) and also such that $V(K_t', B_t'; K_{t-1}^*, B_{t-1}^*) > 1/2$. But this contradicts the optimality of (K_t^*, B_t^*) for J, establishing that we must have that $V(K_t^*, B_t^*; K_{t-1}^*, B_{t-1}^*) = 1/2$ in this case.

Step 6. I show that the sequence $(U_{\theta_M}^p(K_t^*, B_t^*))_{t=1}^{\infty}$ converges to some limit, which I denote $\tilde{U}_{\theta_M}^p$. To do this, I establish that $U_{\theta_M}^p(K_{t-1}^*, B_{t-1}^*) \leq U_{\theta_M}^p(K_t^*, B_t^*) \leq U_{\theta_M}^p(\hat{K}_{\theta_M}^p, \hat{B}_{\theta_M}^p)$, i.e., the sequence $(U_{\theta_M}^p(K_t^*, B_t^*))$ is a monotone and bounded sequence. To see this, fix t and assume that $B_t^* \geq B_{t-1}^*$. It follows that the electoral constraint $V(K_t^*, B_t^*; K_{t-1}^*, B_{t-1}^*) \geq$

$1/2$ reduces to $U^p_{\theta_M}(K^*_{t-1}, B^*_{t-1}) \leq U^p_{\theta_M}(K^*_t, B^*_t)$. Second, assume that $B^*_t < B^*_{t-1}$ and that, towards a contradiction, $U^p_{\theta_M}(K^*_{t-1}, B^*_{t-1}) > U^p_{\theta_M}(K^*_t, B^*_t)$. But then

$$\begin{aligned} U^b_{\theta_M}(K^*_{t-1}, B^*_{t-1}) &= U^p_{\theta_M}(K^*_{t-1}, B^*_{t-1}) \\ &> U^p_{\theta_M}(K^*_t, B^*_t) - \Psi(B^*_t, B^*_{t-1}) \\ &= U^b_{\theta_M}(K^*_t, B^*_t), \end{aligned}$$

so that both private sector and public sector workers of type θ_M vote for the incumbent, yielding $V(K^*_t, B^*_t; K^*_{t-1}, B^*_{t-1}) < 1/2$, a contradiction.

Step 7. I show that the sequence $(U^b_{\theta_M}(K^*_t, B^*_t))^\infty_{t=1}$ also converges to $\tilde{U}^p_{\theta_M}$. To do this, I establish that, given any $\epsilon > 0$, there exists t large enough that $U^b_{\theta_M}(K^*_t, B^*_t) \geq U^p_{\theta_M}(K^*_{t-1}, B^*_{t-1}) - \epsilon$. Recalling that $U^p_{\theta_M}(K^*_t, B^*_t) \geq U^b_{\theta_M}(K^*_t, B^*_t)$, it follows that the two sequences have the same limit. Therefore, suppose, towards a contradiction, that there exists $\epsilon > 0$ such that, for any t, there exists $t' \geq t$ such that

$$U^b_{\theta_M}(K^*_{t'}, B^*_{t'}) < U^p_{\theta_M}(K^*_{t'-1}, B^*_{t'-1}) - \epsilon. \tag{8}$$

By Step 6, we have that $U^p_{\theta_M}(K^*_{t'}, B^*_{t'}) \geq U^p_{\theta_M}(K^*_{t'-1}, B^*_{t'-1})$, so that (8) implies that $U^b_{\theta_M}(K^*_{t'}, B^*_{t'}) < U^p_{\theta_M}(K^*_{t'}, B^*_{t'})$ and hence that $B^*_{t'} < B^*_{t'-1}$ for any such t'. It then follows from Step 4 that we must have $G(K^*_{t'}, B^*_{t'}) \neq G(K^*_{t'-1}, B^*_{t'-1})$. Assume further that $G(K^*_{t'}, B^*_{t'}) < G(K^*_{t'-1}, B^*_{t'-1})$ (a symmetric argument applies to the case of $G(K^*_{t'}, B^*_{t'}) > G(K^*_{t'-1}, B^*_{t'-1})$). Because a median bureaucrat would get payoff $U^p_{\theta_M}(K^*_{t'-1}, B^*_{t'-1})$ by voting for the incumbent at t, it follows from (8) that the threshold bureaucratic type $\tilde{\theta}^b$ that is indifferent between the incumbent and the opposition party at t is such that $\tilde{\theta}^b < \theta_M$. Letting $\tilde{\theta}^p$ denote the private sector type that is indifferent between the incumbent and the opposition party at t, then because $V(K^*_t, B^*_t; K^*_{t-1}, B^*_{t-1}) = B^*_{t-1} F^b_{B_{t-1}}(\tilde{\theta}^b) + (1 - B^*_{t-1}) F^p_{B_{t-1}}(\tilde{\theta}^p) \geq 1/2$, it follows that $\tilde{\theta}^p > \theta_M$.

Because type $\tilde{\theta}^b$ is indifferent between $(K^*_{t'}, B^*_{t'})$ and $(K^*_{t'-1}, B^*_{t'-1})$ at t', we have that

$$\tilde{\theta}^b[G(K^*_{t'-1}, B^*_{t'-1}) - G(K^*_{t'}, B^*_{t'})] = w^b[B^*_{t'-1} - B^*_{t'}] + [K^*_{t'-1} - K^*_{t'}] - \Psi(B^*_{t'-1}, B^*_{t'}). \tag{9}$$

Rewriting (8) yields

$$\theta_M[G(K^*_{t'-1}, B^*_{t'-1}) - G(K^*_{t'}, B^*_{t'})] > w^b[B^*_{t'-1} - B^*_{t'}] + [K^*_{t'-1} - K^*_{t'}] - \Psi(B^*_{t'-1}, B^*_{t'}) + \epsilon,$$

which together with (9) yields

$$\theta_M - \tilde{\theta}^b > \frac{\epsilon}{G(K^*_{t'-1}, B^*_{t'-1}) - G(K^*_{t'}, B^*_{t'})}. \tag{10}$$

From the combination of Step 6 and (8), it is possible to choose t' large enough that such that $U^b_{\theta_M}(K^*_{t'}, B^*_{t'}) \leq U^b_{\theta_M}(K^*_{t'-1} B^*_{t'-1}) + \frac{\epsilon \hat{B}^p_{\theta_R} \underline{f}}{(1-\hat{B}^p_{\theta_R})\overline{f}}$, which can be rewritten as

$$\theta_M[G(K^*_{t'-1}, B^*_{t'-1}) - G(K^*_{t'}, B^*_{t'})] \geq w^b[B^*_{t'-1} - B^*_{t'}] + [K^*_{t'-1} - K^*_{t'}] - \frac{\epsilon \hat{B}^p_{\theta_R} \underline{f}}{(1 - \hat{B}^p_{\theta_R})\overline{f}}.$$

Together with the indifference condition for type $\tilde{\theta}^p$, analogous to (9) for type $\tilde{\theta}^b$, this implies that

$$\tilde{\theta}^p - \theta_M \leq \frac{\frac{\epsilon \hat{B}^p_{\theta_R} \underline{f}}{(1-\hat{B}^p_{\theta_R})\overline{f}}}{G(K^*_{t'-1}, B^*_{t'-1}) - G(K^*_{t'}, B^*_{t'})}. \tag{11}$$

Therefore, we have that

$$\begin{aligned} V(K^*_t, B^*_t; K^*_{t-1}, B^*_{t-1}) &= 1/2 - B^*_{t'}[F^b_{B_{t-1}}(\theta_M) - F^b_{B_{t-1}}(\tilde{\theta}^b)] + (1 - B^*_{t'})[F^p_{B_{t-1}}(\tilde{\theta}^p) - F^p_{B_{t-1}}(\theta_M)] \\ &\leq 1/2 - \hat{B}^p_{\theta_R}[F^b_{B_{t-1}}(\theta_M) - F^b_{B_{t-1}}(\tilde{\theta}^b)] + (1 - \hat{B}^p_{\theta_R})[F^p_{B_{t-1}}(\tilde{\theta}^p) - F^p_{B_{t-1}}(\theta_M)] \\ &\leq 1/2 - \hat{B}^p_{\theta_R}\underline{f}[\theta_M - \tilde{\theta}^b] + (1 - \hat{B}^p_{\theta_R})\overline{f}[\tilde{\theta}^p - \theta_M] \\ &< 1/2, \end{aligned}$$

yielding the desired contradiction. The first inequality follows from that fact that, because $\hat{B}^p_{\theta_R} < \hat{B}^p_{\theta_M} < \hat{B}^p_{\theta_L}$, then for any $t > 1$ we must have that $B^*_t \geq \hat{B}^p_{\theta_R}$. The second inequality follows from the upper and lower bounds on the densities of $F^b_{B_{t-1}}$ and $F^p_{B_{t-1}}$, and the final inequality follows from the combination of (10) and (11).

Notice that the result of this step implies that the sequence $(B^*_t)^\infty_{t=1}$ has a limit, which I denote $\tilde{B}$.

Step 8. For each party J, let $(K^*_{J,t})^\infty_{t=1}$ denote the subsequence of $(K^*_t)^\infty_{t=1}$ that collects the investment choices of that party. Then this sequence has a limit $\tilde{K}_J$, which is a solution to $U^p_{\theta_M}(\tilde{K}_J, \tilde{B}) = \tilde{U}^p_{\theta_M}$. Furthermore, recalling that $\hat{K}^p_\theta(B)$ denotes the optimal level of investment for a private sector worker of type θ if the level of the bureaucracy is fixed at B, we have that $\tilde{K}_R \leq \hat{K}^p_{\theta_M}(\tilde{B}) \leq \tilde{K}_L$. To see this, let $\tilde{K}_J$ denote the limit of some converging subsequence of $(K^*_{J,t})$ and note that because $B^*_t - B^*_{t-1} \to 0$ (by Step 7), it follows that the voting constraint $V(K^*_t, B^*_t; K^*_{t-1}, B^*_{t-1}) = 1/2$ from Step 5 converges to

$$U^p_{\theta_M}(\tilde{K}_J, \tilde{B}) = \tilde{U}^p_{\theta_M}, \tag{12}$$

where I use the fact from Claim 6 that the median private sector worker's payoffs converge to $\tilde{U}^p_{\theta_M}$. For fixed B, $U^p_{\theta_M}(K, B)$ is strictly concave in K, and hence (12) has at most two solutions: either $\tilde{K}_R = \hat{K}^p_{\theta_M}(\tilde{B}) = \tilde{K}_L$ or $\tilde{K}_R < \hat{K}^p_{\theta_M}(\tilde{B}) < \tilde{K}_L$. Hence all converging subsequences of of $(K^*_{J,t})$ converge to $\tilde{K}_J$, yielding the claim.

Step 9. If the sequence $((K^*_t, B^*_t))_{t=1}^{\infty}$ converges, then it converges to $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$. This follows from Step 3: the sequence $((K^*_t, B^*_t))$ converging means that, in the limit, $(K^*_t, B^*_t) = (K^*_{t-1}, B^*_{t-1})$. But the arguments from that step show that in this case we must have that $(K^*_{t+1}, B^*_{t+1}) = (K^*_t, B^*_t)$ only if $(K^*_t, B^*_t) = (\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$.

Step 10. If $(K^*_R, B^*), (K^*_L, B^*) \neq (\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$, then $B^* > \hat{B}^p_{\theta_M}$. Suppose, towards a contradiction, that $B^* \leq \hat{B}^p_{\theta_M}$. Because $(K^*_L, B^*) \neq (\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$, we have that $U^p_{\theta_M}(K^*_L, B^*) < U_{\theta_M}(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$. Because $B^* \leq \hat{B}^p_{\theta_M}$, $V(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M}; K^*_R, B^*) > 1/2$, so that we must have that $U^p_{\theta_L}(K^*_L, B^*) \geq U_{\theta_L}(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$. These two facts imply that

$$G(K^*_L, B^*) > G(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M}). \tag{13}$$

Now consider an efficient platform (K', B') such that $G(K', B') = G(K^*_L, B^*)$. By (13) and the fact that $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$ is efficient, we must have that $K' > \hat{K}^p_{\theta_M}$ and $B' > \hat{B}^p_{\theta_M}$. Therefore, $V(K', B'; K^*_R, B^*) > 1/2$ follows from the fact that $U^p_{\theta_M}(K', B') > U^p_{\theta_M}(K^*_L, B^*) \geq U^p_{\theta_M}(K^*_R, B^*)$. But we also have that $U_{\theta_L}(K', B') > U^p_{\theta_L}(K^*_L, B^*)$, contradicting the optimality of (K^*_L, B^*) for party L, as desired.

□

Proof of Corollary 1. To show that that (K^*_R, B^*) involves underinvestment, note that, because $B^* > \hat{B}^p_{\theta M}$, the type θ for which $\hat{B}^p_\theta = B^*$ must be such that $\theta > \theta_M$. Therefore, $\hat{K}^p_{\theta_M}(B^*) < \hat{K}^p_\theta$ and, because $(\hat{K}^p_\theta, \hat{B}^p_\theta)$ is efficient, it follows that $(\hat{K}^p_{\theta_M}(B^*), B^*)$ must be inefficient and involve underinvestment. But then, because $K^*_R < \hat{K}^p_{\theta_M}(B^*)$, (K^*_R, B^*) must also involve underinvestment.

For the claims about party L's platform, let $\theta > \theta_M$ be the type for which $\hat{B}^p_\theta = B^*$. Suppose that $U^p_{\theta_M}(K^*_R, B^*) > U^p_{\theta_M}(\hat{K}_\theta, B^*)$. Notice that, because $K^*_R < \hat{K}^p_{\theta_M}(B^*)$,

$U^p_{\theta_M}(K^*_R, B^*)$ is increasing in K^*_R and this inequality holds if K^*_R is high enough. Because $U^p_{\theta_M}(K^*_R, B^*) = U^p_{\theta_M}(K^*_L, B^*)$ and $\hat{K}_\theta > \hat{K}^p_{\theta_M}(B^*)$, it follows that $K^*_L < \hat{K}_\theta$ and, because $(\hat{K}^p_\theta, \hat{B}^p_\theta)$ is efficient, (K^*_L, B^*) must involve underinvestment.

Now suppose, towards a contradiction, that $U^p_{\theta_M}(K^*_R, B^*) < U^p_{\theta_M}(\hat{K}_\theta, B^*)$, which implies that platform (K^*_L, B^*) has $K^*_L > \hat{K}^p_\theta$ and is inefficient because of overinvestment. But then there exists a platform (K', B') with $K' < K^*_L$ and $B' > B^*$ which has the same level of government production as (K^*_L, B^*) and that is strictly preferred by all citizen types, contradicting the optimality of (K^*_L, B^*) for party L.

Therefore, the remaining case is that $U^p_{\theta_M}(K^*_R, B^*) = U^p_{\theta_M}(\hat{K}_\theta, B^*)$, in which case $K^*_L = \hat{K}^p_\theta$. Notice that this can only happen if $\theta \leq \theta_L$: otherwise, there is a platform (K', B^*) with $K' < \hat{K}^p_\theta$ which is strictly preferred to (K^*_R, B^*) by both party L and all median citizens, a contradiction. In turn, the condition that θ is low requires that the median private sector worker's payoff from party R's platform is high, which requires K^*_R to be high enough.

□

Proof of Proposition 4. Fix platforms (K_{t-1}, B_{t-1}) and (K_t, B_t), and let $\tilde{\theta}^p$ and $\tilde{\theta}^b$ denote the types of private and public sector workers that are indifferent between these two platforms. Assuming that $B_t < B_{t-1}$, the cutoff types are given by

$$\tilde{\theta}^p = \frac{w^b[B_t - B_{t-1}] + [K_t - K_{t-1}]}{G(K_t, B_t) - G(K_{t-1}, B_{t-1})}, \text{ and}$$
$$\tilde{\theta}^b = \tilde{\theta}^p + \frac{\Psi(B_t, B_{t-1})}{G(K_t, B_t) - G(K_{t-1}, B_{t-1})}. \tag{14}$$

Calculating the partial derivatives of these thresholds with respect to B_t and K_t yields

$$\tilde{\theta}^p_B = \frac{w^b - \tilde{\theta}^p G_B(K_t, B_t)}{G(K_t, B_t) - G(K_{t-1}, B_{t-1})},$$
$$\tilde{\theta}^b_B = \frac{w^b - \tilde{\theta}^b G_B(K_t, B_t) + \Psi_B(B_t, B_{t-1})}{G(K_t, B_t) - G(K_{t-1}, B_{t-1})}, \text{ and}$$
$$\tilde{\theta}^i_K = \frac{1 - \tilde{\theta}^i G_K(K_t, B_t)}{G(K_t, B_t) - G(K_{t-1}, B_{t-1})}, \text{ for } i = p, b, \tag{15}$$

where Ψ_B denote the partial derivative of the job loss penalty with respect to B_t.

Consider admissible platforms (K_L^*, B^*) and (K_R^*, B^*). I will focus on proving that part of the result that applies to party L, and explain how to apply these arguments to party R at the end. Because party L does not overinvest, we have that

$$\frac{G_K(K_L^*, B^*)}{G_B(K_L^*, B^*)} \geq \frac{1}{w^b}. \tag{16}$$

Step 1. I show that no deviation to platform (K_L, B_L) with $B_L > B^*$ can benefit party L. First, suppose that $K_L \leq K_L^*$. It follows by (16) that ${G_K(K_L,B_L)}/{G_B(K_L,B_L)} > {1}/{w^b}$, so that platform (K_L, B_L) underinvests. But then there must exist a platform (K_L', B_L') with $B_L' > B^*$, $K_L' > K_L$ and $G(K_L', B_L') = G(K_L, B_L)$ that is strictly preferred to (K_L, B_L) by all citizen types, including type θ_L. Therefore, it is sufficient to consider deviations with $K_L > K_L^*$. Now, because $U_{\theta_M}^p(K_R^*, B^*) = U_{\theta_M}^p(K_L^*, B^*)$ and $K_R^* < K_L^*$, we have that $\frac{\partial}{\partial K} U_{\theta_M}^p(K_L^*, B^*) < 0$, which is equivalent to

$$G_K(K_L^*, B^*) < {1}/{\theta_M}. \tag{17}$$

By combining (16) and (17), we have that $G_B(K_L^*, B^*) < {w^b}/{\theta_M}$, which is equivalent to $\frac{\partial}{\partial B} U_{\theta_M}^p(K_L^*, B^*) < 0$. But then because G, and hence $U_{\theta_M}^p$, is strictly concave in (K, B), it follows that no deviation with $K_L > K_L^*$ and $B_L > B^*$ can yield a higher payoff to all citizens of type θ_M than $U_{\theta_M}^p(K_L^*, B^*) = U_{\theta_M}^p(K_R^*, B^*)$. Therefore, because $V(K_L^*, B^*, K_R^*, B^*) = {1}/{2}$, it follows that (K_L', B_L') does not allow party L to gain office, which is worse for party L than implementing platform (K_L^*, B^*).

Step 2. I show that no deviation to platform (K_L, B^*) can benefit party L. This is by construction of (K_L^*, B^*): any platform with $K_L > K_L^*$ is worse for all citizens with type θ_M than (K_R^*, B^*), and hence does not allow party L to gain office; whereas because $\arg\max_K U_{\theta_L}^p(K, B^*) \geq K_L^*$, any platform with $K < K_L^*$ is worse for party L whether it allows it to gain office or not.

Step 3. I provide a condition that is necessary for no deviation to platform (K_L, B_L) with $B_L < B^*$ benefiting party L. Any such deviation must be a solution to the problem

$$\max_{K \geq 0, 0 \leq B \leq B^*} \theta_L G(K, B) - [K + w^b B] \text{ subject to } B^* \frac{\tilde{\theta}^b}{\overline{\theta}} + (1 - B^*) \frac{\tilde{\theta}^p}{\overline{\theta}} = {1}/{2}, \tag{18}$$

where I use the fact that F^b and F^p are uniform on $[0, \overline{\theta}]$. The first-order necessary conditions for this problem are

$$\theta_L G_K(K_L, B_L) - 1 + \frac{\lambda}{\bar{\bar{\theta}}}\left[B^*\tilde{\theta}^b_K + (1-B^*)\tilde{\theta}^p_K\right] = 0, \text{ and}$$
$$\theta_L G_B(K_L, B_L) - w^b + \frac{\lambda}{\bar{\bar{\theta}}}\left[B^*\tilde{\theta}^b_B + (1-B^*)\tilde{\theta}^p_B\right] \geq 0,$$

where λ is the multiplier attached to the electoral constraint and the second condition holds with equality whenever $B_L < B^*$. If (K^*_L, B^*) is optimal for party L, then because $B_t = B_{t-1} = B^*$, it follows that $\tilde{\theta}^p = \tilde{\theta}^b = \theta_M$, so that by substituting our expressions for the derivatives of the threshold types from (15), these first-order conditions become

$$\theta_L G_K(K^*_L, B^*) - 1 = -\frac{\lambda}{\bar{\bar{\theta}}}\left[\frac{1-\theta_M G_K(K^*_L, B^*)}{G(K^*_L, B^*) - G(K^*_R, B^*)}\right], \text{ and} \tag{19}$$
$$\theta_L G_B(K^*_L, B^*) - w^b \geq -\frac{\lambda}{\bar{\bar{\theta}}}\left[\frac{w^b - \theta_M G_B(K^*_L, B^*) + B^*\Psi_B(B^*, B^*)}{G(K^*_L, B^*) - G(K^*_R, B^*)}\right], \tag{20}$$

Substituting (19) into (20), we obtain $\theta_L G_B(K^*_L, B^*) \geq \frac{\theta_L G_K(K^*_L, B^*)-1}{1-\theta_M G_K(K^*_L, B^*)}[w^b - \theta_M G_B(K^*_L, B^*) + B^*\Psi_B(B^*, B^*)]$. Recalling that, by construction of (K^*_L, B^*), we have that $1-\theta_M G_K(K^*_L, B^*) > 0$, this last expression can be reduced to

$$-\Psi_B(B^*, B^*) \geq \frac{w^b}{B^*}\frac{(\theta_L - \theta_M)G_B(K^*_L, B^*)}{\theta_L G_K(K^*_L, B^*) - 1}\left[\frac{G_K(K^*_L, B^*)}{G_B(K^*_L, B^*)} - \frac{1}{w^b}\right]. \tag{21}$$

Step 4. I show that the necessary condition (21) from Step 3 is also sufficient. Suppose that (21) is satisfied but that (K_L, B_L) with $B_L < B^*$ is a solution to (18), so that $U^p_{\theta_L}(K_L, B_L) \geq U^p_{\theta_L}(K^*_L, B^*)$. I will show that, for any $0 < \alpha < 1$, the platform $(K_\alpha, B_\alpha) = \alpha(K^*_L, B^*) + (1-\alpha)(K_L, B_L)$ is such that $V(K_\alpha, B_\alpha; K^*_R.B^*) \geq 1/2$. This, along with the fact that U_{θ_L} is strictly concave, yields the desired contradiction.

First suppose that $G(K_\alpha, B_\alpha) = G(K^*_R, B^*)$. Notice that the median private sector worker supports party L against party R when in proposes either platform (K^*_L, B^*) or (K_L, B_L). Therefore,

$$\begin{aligned}0 &\leq \alpha[U^p_{\theta_M}(K^*_L, B^*) - U^p_{\theta_M}(K^*_R, B^*)] + (1-\alpha)[U^p_{\theta_M}(K^*_L, B^*) - U^p_{\theta_M}(K^*_R, B^*)]\\ &< U^p_{\theta_M}(K_\alpha, B_\alpha) - U^p_{\theta_M}(K^*_R, B^*)\\ &= K^*_R + w^b B^* - [\alpha K^*_L + (1-\alpha)K_L + w^b[\alpha B^* + (1-\alpha)B_L]],\end{aligned}$$

where the second inequality follows from the strict concavity of G. Therefore, platform (K_α, B_α) has the same amount of public goods but lower expenditures than platform

(K_R^*, B^*), so that all private sector workers support party L and, because $B^* \leq 1/2$, $V(K_\alpha, B_\alpha; K_R^*, B^*) \geq 1/2$ as desired.

Now suppose that $G(K_\alpha, B_\alpha) > G(K_R^*, B^*)$ (a similar argument applies to the case in which $G(K_\alpha, B_\alpha) < G(K_R^*, B^*)$), and let $\tilde{\theta}_\alpha^p$ and $\tilde{\theta}_\alpha^b$ denote the threshold types for platform (K_α, B_α). We have that

$$\begin{aligned}
&B^* F_{B^*}^b(\tilde{\theta}_\alpha^b) + (1 - B^*) F_{B^*}^p(\tilde{\theta}_\alpha^p) \\
&\quad = B^* \frac{\tilde{\theta}_\alpha^b}{\overline{\theta}} + (1 - B^*) \frac{\tilde{\theta}_\alpha^p}{\overline{\theta}} \\
&\quad = \frac{w^b[\alpha B^* + (1-\alpha) B_L - B^*] + [\alpha K_L^* + (1-\alpha) K_L - K_R^*] + B^* \Psi(\alpha B^* + (1-\alpha) B_L, B^*)}{\overline{\theta}[G(K_\alpha, B_\alpha) - G(K_R^*, B^*)]} \\
&\quad < \frac{\alpha\, [K_L^* - K_R^*] + (1-\alpha) \left[w^b [B_L - B^*] + [K_L - K_R^*] + B^* \Psi(B_L, B^*) \right]}{\overline{\theta}[G(K_\alpha, B_\alpha) - G(K_R^*, B^*)]} \\
&\quad = \frac{\alpha \theta_M \,[G(K_L^*, B^*) - G(K_R^*, B^*)] + (1-\alpha) \theta_M \,[G(K_L, B_L) - G(K_R^*, B^*)]}{\overline{\theta}[G(K_\alpha, B_\alpha) - G(K_R^*, B^*)]} \\
&\quad < \frac{\theta_M}{\overline{\theta}} \\
&\quad = 1/2.
\end{aligned}$$

The first inequality follows from the strict convexity of Ψ and the second inequality from the strict concavity of G. The third equality follows from the facts that $U_{\theta_M}^p(K_L^*, B^*) = U_{\theta_M}^p(K_R^*, B^*)$ and $V(K_L, B_L; K_R^*.B^*) = 1/2$. This last expression, after substituting the appropriate expressions for threshold types from (14), reduces to

$$\frac{w^b[B_L - B^*] + [K_L - K_R^*] + B^* \Psi(B_L, B^*)}{G(K_L, B_L) - G(K_R^*, B^*)} = \frac{\overline{\theta}}{2} = \theta_M.$$

Step 5. The results of Steps 1-4 can be reproduced for party R's choice of (K_R^*, B^*). Because party R underinvests, we have that

$$\frac{G_K(K_R^*, B^*)}{G_B(K_R^*, B^*)} > \frac{1}{w^b}. \tag{22}$$

The same argument as in the beginning of Step 1 ensures that, if we consider deviations to platforms (K_R, B_R) with $B_R > B^*$, it is enough to restrict attention to the case of $K_R > K_R^*$. The second part of Step 1 can then be adapted by noting that because $\arg\max_K U_{\theta_R}^p(K, B^*) \leq K_R^*$, it follows that $G_K(K_R^*, B^*) \leq 1/\theta_R$. When combined with

(22), we also obtain that $G_B(K_R^*, B^*) < {}^{w^b}/_{\theta_R}$. Therefore, by the strict concavity of $U^p_{\theta_R}$, no such deviation can benefit party R. Step 2 can straightforwardly be modified to show that no deviation to some platform (K_R, B^*) can benefit party R. The first-order conditions for party R as in Step 3 can be computed to reduce to

$$-\Psi_B(B^*, B^*) \geq \frac{w^b}{B^*} \frac{(\theta_M - \theta_R) G_B(K_R^*, B^*)}{1 - \theta_R G_K(K_R^*, B^*)} \left[\frac{G_K(K_R^*, B^*)}{G_B(K_R^*, B^*)} - \frac{1}{w^b} \right]. \tag{23}$$

Finally, the argument from Step 4 can be adapted to show that (23) is also sufficient for the optimality of platform (K_R^*, B^*).

□

Proof of Proposition 5.

Part 1. To prove the first claim, suppose that, towards a contradiction, there exists an equilibrium σ^* such that $A(\sigma^*) = \{(K_R^*, B^*), (K_L^*, B^*)\}$ with $K_R^* < K_L^*$. I claim that, for each $J = R, L$, (K_J^*, B^*) must be efficient. Because $K_R^* \neq K_J^*$, this yields the desired contradiction. To prove the claim, first suppose that (K_J^*, B^*) underinvests. Then there exists platform (K, B) with $K > K_J^*$, $B < B^*$ and $G(K, B) = G(K_J^*, B^*)$. Also, because $\Psi(B, B^*) = 0$, then any citizen, whether bureaucrat or a private sector worker, that prefers (K_J^*, B^*) to (K_I^*, B^*), where $I \neq J$, must strictly prefer (K, B) to (K_I^*, B^*). But this contradicts the optimality of (K_J^*, B^*) for party J against (K_I^*, B^*), as desired. Second, suppose that (K_J^*, B^*) overinvests. Then there exists platform (K, B) with $K < K_J^*$, $B > B^*$ and $G(K, B) = G(K_J^*, B^*)$ that is strictly preferred to (K_J^*, B^*) by all citizens, both in the private and the public sector, the latter's preference being reinforced because $\Psi(B, B^*) > 0$. But, by the same argument as for the case above, this yields a contradiction.

To prove the second claim of Part 1, suppose, towards a contradiction, that there exists an equilibrium σ^* such that $A(\sigma^*) = \{(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})\}$. I claim that, because $\Psi_{B_t}(\hat{B}^p_{\theta_M}, \hat{B}^p_{\theta_M}) > 0$, there exists a platform with $K = \hat{K}^p_{\theta_M}$ and $B > \hat{B}^p_{\theta_M}$ that defeats platform $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$ in a majority vote. Because party L strictly prefers this platform to $(\hat{K}^p_{\theta_M}, \hat{B}^p_{\theta_M})$ given that B is close to $\hat{B}^p_{\theta_M}$, this yields the desired contradiction. This can be proved by mimicking Step 4 from the proof of Proposition 2, which provides an analogous construction for the case of job-motivated bureaucrats.

Part 2, I first characterise sets of long-run platforms for those equilibria for which these consist of two platforms with distinct bureaucracies, assuming that such equilibria exist. Fix an equilibrium $\sigma^* = (\sigma_R^*, \sigma_L^*)$ such that $A(\sigma^*) = \{(K_R^*, B_R^*), (K_L^*, B_L^*)\}$ and $B_R^* \neq B_L^*$. Given $J = R, L$ and $I \neq J$, I show that if

$$U_{\theta_M}^p(\hat{K}_{\theta_J}^p, \hat{B}_{\theta_J}^p) \geq U_{\theta_M}^p(\hat{K}_{\theta_I}^p, \hat{B}_{\theta_I}^p), \tag{24}$$

then $(K_J^*, B_J^*) = (\hat{K}_{\theta_J}^p, \hat{B}_{\theta_J}^p)$. To do this suppose, towards a contradiction, that $(K_J^*, B_J^*) \neq (\hat{K}_{\theta_J}^p, \hat{B}_{\theta_J}^p)$. By arguments akin to those of Steps 4 and 5 of the proof of Proposition 3, we must then have that

$$V(K_J^*, B_J^*; K_I^*, B_I^*) = 1/2. \tag{25}$$

There are two cases to consider. First, assume that $B_I^* < B_J^*$. Here, because $\Psi(B_J^*, B_I^*) > 0$, (25) can only hold if $U_{\theta_M}^p(K_J^*, B_J^*) < U_{\theta_M}^p(K_I^*, B_I^*)$, which contradicts (24). The second case is when $B_I^* > B_J^*$. Here, because $\Psi(B_J^*, B_I^*) = 0$, (25) requires that

$$U_{\theta_M}^p(K_J^*, B_J^*) = U_{\theta_M}^p(K_I^*, B_I^*). \tag{26}$$

It must also be the case that

$$V(K_I^*, B_I^*; K_J^*, B_J^*) = 1/2. \tag{27}$$

Otherwise, we have that $(K_I^*, B_I^*) = (\hat{K}_{\theta_I}^p, \hat{B}_{\theta_I}^p)$, which, by (24), implies that $(\hat{K}_{\theta_J}^p, \hat{B}_{\theta_J}^p)$ receives majority support against $(\hat{K}_{\theta_I}^p, \hat{B}_{\theta_I}^p)$, so that $(K_J^*, B_J^*) = (\hat{K}_{\theta_J}^p, \hat{B}_{\theta_J}^p)$, a contradiction. Therefore, because $\Psi(B_I^*, B_J^*) > 0$, (27) requires that $U_{\theta_M}^p(K_I^*, B_I^*) < U_{\theta_M}^p(K_J^*, B_J^*)$, contradicting (26).

The existence of equilibria with two long-run platforms follows by construction. First, suppose that $U_{\theta_M}^p(\hat{K}_{\theta_R}^p, \hat{B}_{\theta_R}^p) \geq U_{\theta_M}^p(\hat{K}_{\theta_L}^p, \hat{B}_{\theta_L}^p)$. Let (K_L^*, B_L^*) be a solution to (1) for party L when $(K_R, B_R) = (\hat{K}_{\theta_R}^p, \hat{B}_{\theta_R}^p)$, which must have $B_L^* > \hat{B}_{\theta_R}^p$, and hence $\Psi(B_L^*, \hat{B}_{\theta_R}^p) > 0$. From this, it follows that $U_{\theta_M}^p(K_L^*, B_L^*) \leq U_{\theta_M}^p(\hat{K}_{\theta_R}^p, \hat{B}_{\theta_R}^p)$. Therefore, when party R is in the opposition and the incumbent party L implements (K_L^*, B_L^*), then because $\Psi(B_L^*, \hat{B}_{\theta_R}^p) = 0$, platform $(\hat{K}_{\theta_R}^p, \hat{B}_{\theta_R}^p)$ gains majority support, and hence is optimal for party R. To complete the argument, we set $(I_0, K_0, B_0) = (L, K_L^*, B_L^*)$.

Second, suppose that $U_{\theta_M}^p(\hat{K}_{\theta_R}^p, \hat{B}_{\theta_R}^p) < U_{\theta_M}^p(\hat{K}_{\theta_L}^p, \hat{B}_{\theta_L}^p)$. Let (K_R^*, B_R^*) be a solution to (1) for party R when $(K_L, B_L) = (\hat{K}_{\theta_L}^p, \hat{B}_{\theta_L}^p)$, which must have $B_R^* < \hat{B}_{\theta_L}^p$, and hence $\Psi(B_R^*, \hat{B}_{\theta_L}^p) = 0$. Therefore, it must also be the case that $U_{\theta_M}^p(K_R^*, B_R^*) \geq$

$U^p_{\theta_M}(\hat{K}^p_{\theta_L}, \hat{B}^p_{\theta_L}) > U^p_{\theta_M}(\hat{K}^p_{\theta_R}, \hat{B}^p_{\theta_R})$, which implies that $(K^*_R, B^*_R) \neq (\hat{K}^p_{\theta_R}, \hat{B}^p_{\theta_R})$ and $U^p_{\theta_M}(K^*_R, B^*_R) = U^p_{\theta_M}(\hat{K}^p_{\theta_L}, \hat{B}^p_{\theta_L})$. Therefore, when party L is in the opposition and the incumbent party R implements (K^*_R, B^*_R), then because $\Psi(B^*_R, \hat{B}^p_{\theta_L}) > 0$, platform $(\hat{K}^p_{\theta_L}, \hat{B}^p_{\theta_L})$ gains a supermajority of votes, and hence is optimal for party L. To complete the argument, we set $(I_0, K_0, B_0) = (R, K^*_R, B^*_R)$.

□

Proof of Proposition 6. A first remark is that, given any subgame perfect Nash equilibrium σ^* of the game with forward-looking parties, the corresponding set $A(\sigma^*)$ of limit points is nonempty. To see this, recall, from Step 6 of Proposition 3, that the sequence $(U^p_{\theta_M}(K^*_t, B^*_t))^\infty_{t=1}$ is monotone. Therefore, the sequence of equilibrium path policies, $((K^*_t, B^*_t))^\infty_{t=1}$, must lie in the set $\{(K, B) \in \mathbb{R}_+ \times [0, \overline{B}] : U_{\theta_M}(K, B) \geq U^p_{\theta_M}(K^*_1, B^*_1)\}$, which is closed. This set is also bounded, given the assumption that $\lim_{K\to\infty} G_K(K, B) = 0$ for all B: this implies that there exists $\overline{K}$ such that, for all $B \in [0, \overline{B}]$, $U_{\theta_M}(\overline{K}, B) \leq U^p_{\theta_M}(K^*_1, B^*_1)$. To see this, let $B^*(K) = \arg\max_{0\leq B\leq \overline{B}} U^p_{\theta_M}(K, B)$, and note that $\lim_{K\to\infty} U^p_{\theta_M}(K, B^*(K)) = -\infty$. Finally, define $\overline{K}$ such that $U^p_{\theta_M}(\overline{K}, B^*(\overline{K})) \leq U^p_{\theta_M}(K^*_1, B^*_1)$. Therefore, $((K^*_t, B^*_t))$ has at least one converging subsequence, which is contained in $A(\sigma^*)$.

The claim in the proposition is proved by Steps 6 and 7 of Proposition 3. The only difference is that, in the first paragraph of Step 7, we cannot guarantee that $G(K^*_{t'}, B^*_{t'}) \neq G(K^*_{t'-1}, B^*_{t'-1})$ on the equilibrium path of an arbitrary subgame perfect Nash equilibrium. However, in this case we rely on the fact that σ^* has nontrivial elections.

□

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