Intergenerational altruism with future bias*

Francisco M. Gonzalez, Itziar Lazkano and Sjak A. Smulders

June 2018

Abstract

We show that standard preferences of altruistic overlapping generations exhibit future bias, which involves preference reversals associated with increasing impatience. This underlies a conflict of interest between successive generations. We explore the implications of this conflict for intergenerational redistribution when there is a sequence of utilitarian governments representing living generations and choosing policies independently over time. We argue that future bias creates incentives to legislate and sustain a pay-as-you-go pension system, which every government views as a self-enforcing commitment mechanism to increase future old-age transfers.

JEL classification: D71; D72; H55.
Keywords: intergenerational altruism; future bias; time inconsistency; β-δ discounting; pay-as-you-go pension plans.

*Gonzalez (corresponding author): University of Waterloo (francisco.gonzalez@uwaterloo.ca). Lazkano: University of Wisconsin-Milwaukee (lazkano@uwm.edu). Smulders: Tilburg University (J.A.Smulders@uvt.nl). We have benefited from comments by Stefan Ambec, John Burbidge, Hippolyte d'Albis, Matt Doyle, Jean Guillaume Forand, Reyer Gerlagh, John Hassler, Per Krusell, David Levine, Fabien Postel-Vinay, Debraj Ray, Victor Rios-Rull, Mike Veall and Randy Wright. Gonzalez gratefully acknowledges financial support from the Social Sciences and Humanities Research Council of Canada.

1 Introduction

In this paper we show that standard preferences of altruistic overlapping generations (Barro 1974, Kimball 1987) exhibit future bias. The corresponding time inconsistency can explain political support for a pay-as-you-go pension system at the expense of economic growth.

To understand the nature of the future bias, consider a sequence of overlapping generations. For the sake of concreteness, let the total utility of the date-t generation be

$$U_t = u^y\left(c_t^y\right) + u^o\left(c_{t+1}^o\right) + \delta U_{t+1},$$

with $0 < \delta < 1$, where c_t^y and c_{t+1}^o are the tth generation's consumption when young and when old, respectively, and where $u^y\left(c_t^y\right) + u^o\left(c_{t+1}^o\right)$ is the utility associated with the tth generation's lifetime consumption. It is well known that the above system of interrelated utilities generates utility functions defined over streams of consumption allocations:

$$F_t\left(\left\{\left(c_{t+s}^y, c_{t+s}^o\right)\right\}_{s=0}^{\infty}\right) = \sum_{s=0}^{\infty}\delta^s\left(u^y\left(c_{t+s}^y\right) + u^o\left(c_{t+s+1}^o\right)\right).$$

Here, we highlight the fact that F_t exhibits future bias, in that the date-t generation is more willing to give up consumption at date t than at any future date. To see why, note that the tth generation weighs the utilities from $c_t^y, c_{t+1}^y, c_{t+2}^y, \ldots$ according to the sequence $1, \delta, \delta^2, \ldots$, but it weighs the utilities from $c_t^o, c_{t+1}^o, c_{t+2}^o, c_{t+3}^o, \ldots$ according to $0, 1, \delta, \delta^2, \ldots$. Date t is special for the date-t generation, because its preferences put zero weight on the consumption of the old at date t, but positive weight on the consumption of the old at any future date. Thus, the relative discount factor between $u^o\left(c_t^o\right)$ and $u^o\left(c_{t+1}^o\right)$ is (infinitely) larger than δ, which is the discount factor between $u^o\left(c_{t+s}^o\right)$ and $u^o\left(c_{t+s+1}^o\right)$ for any $s > 0$. In other words, the date-t generation exhibits increasing impatience.

Now, suppose that $u^y(c) = u^o(c)$ and restrict attention to streams of symmetric consumption allocations, in the sense that the distribution of consumption across living generations has the property that $c_t^y = c_t^o = c_t$ for all t. The utilities associated with $c_t, c_{t+1}, c_{t+2}, c_{t+3}, \ldots$ are in effect weighted according to $1, 1+\delta, (1+\delta)\delta, (1+\delta)\delta^2, \ldots$. One can see that the discount factor between periods t and $t+1$ is equal to $1+\delta = (1+\delta^{-1})\delta$, while the discount

factor between periods $t+s$ and $t+s+1$ is equal to δ, for all $s > 0$. This resembles the well known β-δ discounting, except that the short-term discount factor $\beta = 1 + \delta^{-1}$ is greater than one, reflecting future bias, rather than the familiar present bias of quasi-hyperbolic preferences (Phelps and Pollak 1968). In the paper we provide a formal definition of future bias as a property of utility functions defined over streams of consumption allocations, where an allocation is an arbitrary distribution of consumption across co-existing generations.

One might think that the mere introduction of backward altruism could eliminate the above future bias, but this is not the case. To see why, consider the case where each generation cares about its lifetime consumption and also about the total utility of its immediate ancestors and descendants. Further, suppose that preferences are stationary across generations. It is well known that, under natural assumptions, the preferences of each generation can be represented by a utility function over consumption streams such that the consumption of other generations, specifically past generations, is discounted (Kimball 1987, Hori and Kanaya 1989). What has not been noted before, to the best of our knowledge, is that such a discounting causes future bias. Intuitively, while the date-t young weighs the utility from her date-$t+1$ consumption as "her own utility", it discounts the utility from the consumption of the old generation at date-t, weighing it as "someone else's utility". Thus, even when each generation also cares about the utility of its parent generation, the date-t generation is more willing to postpone consumption at date t than at any future date.

Because the future bias we emphasize is rooted in preferences over consumption across, as opposed to within, generations, it has non-trivial consequences for the political demands for intergenerational redistribution. We analyze this issue by considering a sequence of one-period governments seeking to maximize a weighted sum of the utilities of the two living generations. Others before us have noted that intergenerational disagreements render plausible social preferences time inconsistent, but they have addressed neither the essential role of future bias nor its link to pensions.[1] To track the source of time inconsistency, note that while the date-t generation weighs the utility from her date-$t+1$ consumption as "her

[1] Burbidge (1983), Kimball (1987), Calvo and Obstfeld (1988) and Hori and Kanaya (1989) recognize the potential time inconsistency, and Bernheim (1989) emphasizes it as the source of Pareto inefficiency.

own utility", the date-$t+1$ generation will discount it, weighing it as "someone else's utility". Accordingly, the date-$t+1$ generation is more willing than the date-t generation to postpone date-$t+1$ consumption. We show that such time inconsistency is driven by the future bias of individual preferences. Furthermore, since the preferences of the old exhibit no bias, the time inconsistency of social preferences arises because they inherit the future bias of individual preferences, not because they aggregate the preferences of different generations.

To address the equilibrium implications of future bias, we consider the case where consumption utilities have a constant elasticity of marginal utility, which implies that the optimal allocation of consumption across generations within a period is independent of aggregate consumption. In this setting, preferences can be expressed over aggregate consumption streams and, conditional on a stationary sharing rule, exhibit β-δ discounting: the discount factor between the current period and the next is $\beta\delta$, while the discount factor between any two future, consecutive periods is δ. The short-term discount factor β, however, is both a function of the sharing rule and strictly greater than one. Thus, the model delivers an endogenous future bias rather than the exogenous present bias of the quasi-hyperbolic preferences proposed by Phelps and Pollak (1968).

It is straightforward to replicate Phelps and Pollak's analysis of stationary Markov equilibria in the isoelastic setting. The tractability of this setting allows us to show that future equilibrium old-age transfers are insufficient from the viewpoint of the government at any given date. In this context, we argue that pay-as-you-go social security legislation can be understood as a commitment to increase future old-age transfers.

1.1 Related literature

Our paper contributes to the literature on time inconsistency of preferences.[2] Galperti and Strulovici (2017) consider a setting with non-overlapping generations and show that forward altruism exhibits a present bias whenever altruism is nonpaternalistic and incorporates directly the utilities of not only the immediate descendants, but *all* future descendants. In

[2]See, for example, Strotz (1956), Phelps and Pollak (1968), Laibson (1997), Caplin and Leahy (2004), Gollier and Zeckhauser (2005), Pearce (2008), Halevy (2008, 2015), Gollier and Weitzman (2010), Jackson and Yariv (2014), Galperti and Strulovici (2017) and Ray (2018).

this case, grandparents care about their grandchildren directly as well as indirectly through their own children. However, parents care about their own children only directly, ignoring the grandparents' direct concern about their grandchildren. Consequently, from the grandparents' standpoint, parents appear to care too little about their children. Like Galperti and Strulovici, we challenge the view that nonpaternalistic altruism and time consistent preferences are two sides of the same coin (Phelps and Pollak 1968, Barro 1974). Our analysis complements theirs by showing that the emerging bias depends on the demographic structure as well as the specific model of intergenerational altruism.

Jackson and Yariv (2014) show that utilitarian aggregation of time-consistent preferences results in a present bias when discount factors are heterogeneous. This is because, when aggregating time-consistent preferences, those with higher discount factors gain increasingly more weight when evaluating intertemporal rates of substitution further in the future. We show that a future bias can be expected in the intergenerational context, where time horizons are heterogeneous and social preferences inherit the future bias of individual preferences.

Our analysis offers a novel perspective on the widespread legislation of pay-as-you-go social security despite recognition of its negative effects on capital accumulation (Auerbach and Kotlikoff 1987). Our argument is clearly different from the idea that social security legislation is socially optimal (Samuelson 1975). It is also different from the idea that it occurs because current generations do not internalize the costs to future generations. Furthermore, our argument does not require the preferences of politicians to be biased towards the old (Grossman and Helpman 1998).

Others have argued that governments' self-control problems matter for fiscal policy. For instance, Phelps and Pollak (1968) and Krusell et al. (2002) consider the impact of policy on capital accumulation under the assumption that governments' preferences have a present bias. This assumption seems natural in their non-overlapping generations setting, which is the standard setting in analyses of equilibrium growth with intergenerational disagreement.[3]

To the best of our knowledge, we are the first to identify the role of future bias in redistributive politics. Veall (1986) and Hansson and Stuart (1989) presume that children

[3]Kohlberg (1976), Bernheim and Ray (1987), Ray (1987) and Barro (1999) are well known examples.

place sufficiently large weight on their parents' consumption and, in the absence of social security, the current young would have an incentive to undersave in order to elicit old-age transfers from the future young.[4] By contrast, we argue that young generations have an incentive to oversave in order to increase future old-age transfers precisely because the next generation will place insufficient weight on the parents' consumption.

In a similar spirit, the literature on strategic debt views government debt as a way to tie the hands of future governments that have different preferences from the current one (e.g., Persson and Svensson 1989 and Alesina and Tabellini 1990). Their focus is on discretionary policy when current voters have an incentive to lower strategically the size of the future pie to be distributed.[5] By contrast, we argue that policymakers would have a strategic incentive to increase the size of the future pie in the absence of social security legislation. Instead, it is this legislation that leads to a decrease in the size of the future pie.

In the next section, we introduce preferences. Section 3 shows that these preferences exhibit future bias. Section 4 considers a sequence of utilitarian governments in the tractable case of isoelastic utility and linear production technology, characterizes the unique Markov perfect equilibrium in linear strategies and formalizes our theory of pay-as-you-go social security. Section 5 concludes. Proofs of all propositions are in the Appendix.

2 Individual and social preferences

2.1 Overlapping altruistic generations

Consider an economy with overlapping generations. Each generation lives for two periods and the utility of the tth generation is given by

$$U_t = u^y\left(c_t^y\right) + u^o\left(c_{t+1}^o\right) + \mu U_{t-1} + \lambda U_{t+1}, \quad \text{with} \quad \mu > 0 \quad \text{and} \quad \lambda > 0, \tag{1}$$

where c_t^y and c_{t+1}^o are the tth generation's consumption when young and when old, respectively. To distinguish the overall level of utility U_t from the utility $u^y\left(c_t^y\right) + u^o\left(c_{t+1}^o\right)$

[4] Tabellini (2000) also presumes that children are sufficiently altruistic towards parents, but he stresses the fact that social security redistributes both across and within generations.

[5] Also see Azzimonti (2011) and Halac and Yared (2014).

associated with the tth generation's lifetime consumption, we refer to U_t as its well-being.

Suppose (i) preferences do not vary across generations, (ii) they put non-negative weight on the utility associated with each generation's lifetime consumption and (iii) altruism is bounded, in the sense that concern for infinitely distant ancestors and descendants becomes negligible. Under these assumptions, Kimball (1987) and Hori and Kanaya (1989) show that $\mu + \lambda < 1$ is a necessary and sufficient condition for the preferences of the tth generation over consumption streams to be represented by a utility function F_t such that

$$F_t\left(\left\{\left(c_j^y, c_{j+1}^o\right)\right\}_{j=-\infty}^{\infty}\right) = \sum_{s=1}^{\infty}\theta^s v\left(c_{t-s}^y, c_{t-s+1}^o\right) + v\left(c_t^y, c_{t+1}^o\right) + \sum_{s=1}^{\infty}\delta^s v\left(c_{t+s}^y, c_{t+s+1}^o\right), \tag{2}$$

with $v\left(c^y, c^o\right) = u^y\left(c^y\right) + u^o\left(c^o\right)$, where $0 < \theta < 1$ and $0 < \delta < 1$, and where

$$\theta = \frac{1 - \sqrt{1 - 4\mu\lambda}}{2\lambda} \quad \text{and} \quad \delta = \frac{1 - \sqrt{1 - 4\mu\lambda}}{2\mu} \quad .$$

It is worth noting up front that $\theta < 1$ is a sufficient condition for the future bias we identify below. Kimball shows that this property must be satisfied if concern for infinitely distant ancestors is to be negligible. To understand the relationship between (1) and (2), note that non-negativity (assumption (ii) above) requires that $\mu\lambda < 1/4$, whereas bounded altruism (assumption (iii)) requires the stronger condition $\mu + \lambda < 1$. The latter implies positive discounting of the consumption of others ($\theta < 1 < \delta^{-1}$), where $\theta = \delta = 1$ is a limiting case as $\mu\lambda$ approaches $1/4$. Importantly, the utility function given by (2) involves a geometric sequence backward as well as a geometric sequence forward, but a single geometric sequence forward and backward is impossible, since $\theta < \delta^{-1}$.[6]

2.2 Governments

Suppose there is a sequence of governments and the date-t government seeks to maximize

$$V_t = U_{t-1}^t + aU_t, \quad \text{with} \quad a > 0, \tag{3}$$

[6]See Bergstrom (1999) for further discussion of the relationship between (1) and (2).

where U_{t-1}^{t} denotes the well-being of the date-$t-1$ generation from the viewpoint of date t. The utilitarian welfare objective captures in a simple manner the fact that democratic governments are unlikely to be immune to disagreement between coexisting generations. It can be interpreted as the outcome of political competition in a probabilistic voting model (Lindbeck and Weibull 1987, Grossman and Helpman 1998).

3 Future bias

This section offers a novel perspective on the above familiar preferences by showing that they involve an inherent future bias. It also tracks a conflict of interest between current and future utilitarian governments representing living generations to the fact that each government inherits the future bias of individual preferences.

Let (c_t^y, c_t^o) denote a consumption allocation at date t and let $C^t = \left\{\left(c_j^y, c_j^o\right)\right\}_{j=t}^{\infty}$ denote a stream of consumption allocations starting at date t. Disregarding past consumption allocations irrelevant for the behavior of the tth generation, we can express its well-being as

$$H_t\left(C^t\right) = -(\delta^{-1} - \theta)u^o\left(c_t^o\right) + \sum_{s=0}^{\infty}\delta^s[u^y(c_{t+s}^y) + \delta^{-1}u^o\left(c_{t+s}^o\right)]. \tag{4}$$

Similarly, the well-being of the date-$t-1$ generation from the viewpoint of date t is given by

$$H_{t-1}^t\left(C^t\right) = \delta\sum_{s=0}^{\infty}\delta^s[u^y(c_{t+s}^y) + \delta^{-1}u^o\left(c_{t+s}^o\right)]. \tag{5}$$

Letting $G_t\left(C^t\right) = H_{t-1}^t\left(C^t\right) + aH_t\left(C^t\right)$, the date-$t$ government's preferences are given by

$$G_t\left(C^t\right) = -(\delta^{-1} - \theta)au^o\left(c_t^o\right) + (\delta + a)\sum_{s=0}^{\infty}\delta^s\left[u^y\left(c_{t+s}^y\right) + \delta^{-1}u^o\left(c_{t+s}^o\right)\right]. \tag{6}$$

This captures the idea that democratically elected governments care about future generations only to the extent that the current electorate does so.

Definition 1 below builds on Jackson and Yariv's (2014) definition of present bias. Although our focus here is on future bias, it is instructive to contrast the notion of future bias with the more familiar notion of present bias. Let $W_t\left(C^t\right) = \sum_{s=0}^{\infty} w(c_{t+s}^y, c_{t+s}^o, s)$ be

an agent's well-being associated with the stream of consumption allocations C^t. The definition of present bias in Part (i) of Definition 1 captures a notion of diminishing impatience. Part (1) states that if the allocation (c_1', c_2') at some time $t+s+k$ is preferred to another allocation (c_1, c_2) at an earlier time $t+s$ (i.e., $w(c_1', c_2', s+k) \geq w(c_1, c_2, s)$), then the same preference ordering holds when both allocations are equally delayed (i.e., $w(c_1', c_2', s+k+1) \geq w(c_1, c_2, s+1)$). This implies that future preferences are at least as patient as current preferences, ruling out preference reversals associated with future bias (see below). By contrast, Part (2) emphasizes the possibility of preference reversals associated with diminishing impatience. When preferences are present biased, it is possible that as of date t, an allocation is preferred to another one at some later time $t+k$ (i.e., $w(c_1, c_2, 0) > w(c_1', c_2', k)$), but the reverse is true when both allocations are equally delayed (i.e., $w(c_1, c_2, s) < w(c_1', c_2', s+k)$).

Instead, the definition of future bias in Part (ii) captures a notion of *increasing* impatience. Part (1) implies that current preferences are at least as patient as future preferences: if the allocation (c_1, c_2) at time $t+s$ is preferred to another allocation (c_1', c_2') at some later time $t+s+k$ (i.e., $w(c_1, c_2, s) \geq w(c_1', c_2', s+k)$), then the same preference ordering holds when both allocations are equally delayed (i.e., $w(c_1, c_2, s+1) \geq w(c_1', c_2', s+k+1)$). Note that this condition rules out preference reversals associated with present bias. By contrast, Part (2) emphasizes the possibility of preference reversals associated with increasing impatience. When preferences are future biased, it is possible that, as of date t, an allocation at some time $t+k$ is preferred to another allocation at time t (i.e., $w(c_1', c_2', k) > w(c_1, c_2, 0)$), but the reverse is true when both allocations are equally delayed (i.e., $w(c_1', c_2', s+k) < w(c_1, c_2, s)$).

Definition 1 *(i) W_t has a present bias if: (1) for any (c_1, c_2), (c_1', c_2') and $s \geq 0$, $k \geq 1$, $w(c_1, c_2, s) \leq w(c_1', c_2', s+k)$ implies $w(c_1, c_2, s+1) \leq w(c_1', c_2', s+k+1)$ and (2) for any $s \geq 1$ and $k \geq 1$, there exist (c_1, c_2) and (c_1', c_2') such that $w(c_1, c_2, 0) > w(c_1', c_2', k)$ and $w(c_1, c_2, s) < w(c_1', c_2', s+k)$.*
(ii) W_t has a future bias if: (1) for any (c_1, c_2), (c_1', c_2') and $s \geq 0$, $k \geq 1$, $w(c_1, c_2, s) \geq w(c_1', c_2', s+k)$ implies $w(c_1, c_2, s+1) \geq w(c_1', c_2', s+k+1)$ and (2) for any $s \geq 1$ and $k \geq 1$, there exist (c_1, c_2) and (c_1', c_2') such that $w(c_1, c_2', k) > w(c_1, c_2, 0)$ and $w(c_1', c_2', s+k) <$

$w(c_1, c_2, s)$.

It is straightforward to prove the following.

Proposition 1 *Both H_t and G_t have a future bias.*

The preferences of the date-t generation, given by H_t, exhibit future bias because the generation discounts the consumption of its ancestors, which in turn implies that young individuals are more reluctant to transfer resources from themselves to the living old than from the young to the old at any future date. To see why, note that the date-t young weighs the utility from her date-t consumption as "her own utility", but she weighs that from the date-t old's consumption as "someone else's utility" and so she discounts it. By contrast, the date-t young discounts the consumption of the young relative to that of the coexisting old at any future date. This implies that the date-t generation is more willing to postpone consumption at date t than at any future date. To see this, note that the date-t generation discounts the consumption stream $c_t^y, c_{t+1}^y, c_{t+2}^y \ldots$ according to the sequence $1, \delta, \delta^2 \ldots$, but it discounts the stream $c_t^o, c_{t+1}^o, c_{t+2}^o \ldots$ according to the sequence $\theta, 1, \delta, \delta^2 \ldots$. Hence, the discount factor between c_t^o and c_{t+1}^o is θ^{-1}, whereas the discount factor between any two future, consecutive periods is δ. Future bias arises because $\theta^{-1} > \delta$.

Note that $\theta < 1$ is a necessary condition for the utility function given in (2) to represent the preferences in (1), but it is only a sufficient condition for H_t and G_t to have a future bias. Alternatively, if one viewed altruism as paternalistic and H_t and G_t as primitives, then they would have a future bias if and only if $\theta < \delta^{-1}$.

It is also worth noting that future bias in H_t does not stem from an assumed asymmetry in the way ancestors and descendants are treated relative to each other. It arises whether $\mu > \lambda$, $\mu = \lambda$, or $\mu < \lambda$. It is therefore consistent with $\theta < \delta$, $\theta = \delta$ and $\theta > \delta$, since $\theta/\delta = \mu/\lambda$.

It is easy to see that H_t exhibits future bias even in the absence of backward altruism, that is, even if $\mu = 0$, in which case one has $\theta = 0$ and $\delta = \lambda$. Furthermore, in this case future bias arises even if $\delta = 1$, which is well defined if, for example, utility is isoelastic and the elasticity of intertemporal substitution is less than one.

The overlap of generations is necessary for the total utility of a generation to exhibit future bias. If generations did not overlap, the preferences of a single generation would not exhibit a future bias, regardless of the weight they put on the consumption of past generations (i.e., regardless of the value of θ). To see why, consider briefly the case where every generation lives for one period. Note that equation (2) implies that the total utility of the date-t generation would be given then by

$$F_t = \sum_{s=1}^{\infty} \theta^s u^y \left(c_{t-s}^y\right) + u^y \left(c_t^y\right) + \sum_{s=1}^{\infty} \delta^s u^y \left(c_{t+s}^y\right) \ , \tag{7}$$

with $0 < \theta < 1$ and $0 < \delta < 1$, which does not exhibit future bias because, in this case, the value of θ only affects how the consumption of dead generations is weighted.

The future bias of the social welfare function G_t is not the result of aggregation of individual preferences. Rather, it is driven by the overlapping generations demographic structure. Note that the social welfare function adds up the total utility of co-existing generations, which includes current and past generations, but not future, unborn generations. For a comparison, consider the case of *non-overlapping* generations and consider a planner that adds up the utility of the date-t generation and the utility of the date-$t-1$ generation. In this case one has the following date-t social welfare function (ignoring the consumption of dead generations):

$$F_{t-1} + aF_t = (\delta + a) \sum_{s=0}^{\infty} \delta^s u^y \left(c_{t+s}^y\right), \tag{8}$$

which exhibits no future bias, regardless of whether there is backward altruism or not. In this sense, aggregation of individual preferences is not by itself sufficient to generate future bias.

Furthermore, since the preferences of the old exhibit no bias, the time inconsistency of social preferences in the case of overlapping generations arises because social preferences inherit the future bias of individual preferences, not because they aggregate individual preferences. Adding the total utility of the date-t old does not influence the future bias at all because it amounts to adding a stream of utilities discounted using a geometric sequence.

Importantly, future bias is the reason why social preferences are time inconsistent. Note

that the two notions are logically distinct. The biases formalized in Definition 1 refer to the particular utility function W_t and so they are associated with static preference reversals: the ranking of consumption allocations depends on the distance from t (i.e., preferences are non-stationary). By contrast, time inconsistency is associated with dynamic preference reversals: the ranking of consumption allocations changes with the evaluation date (i.e., with t).

Formally, for any fixed date t', consider the sequence $\{W_t(C^t)\}_{t=t'}^{\infty}$, where $W_t(C^t) = \sum_{s=0}^{\infty} w_t(c_{t+s}^y, c_{t+s}^o, s)$, for all $t \geq t'$. Dynamic preference reversals consistent with static future bias imply that for any $s \geq 1$ and $k \geq 1$, there exist (c_1, c_2) and (c_1', c_2') such that $w_t(c_1, c_2, s) > w_t(c_1', c_2', s+k)$ and $w_{t+s}(c_1, c_2, s) < w_{t+s}(c_1', c_2', s+k)$. Thus, it is possible that as of date t, an allocation at some later date-$t+s$ is preferred to another one at an even later date-$t+s+k$, but the reverse is true when date $t+s$ arrives. When $W_t = G_t$, the exact condition for such dynamic preference reversals is given by Part (2) of the definition of future bias. This is because $w_t = w$, for all t (i.e., social preferences are time invariant), hence $w_t(c_1, c_2, s+k) = w(c_1, c_2, s+k)$ and $w_{t+s}(c_1, c_2, s+k) = w(c_1, c_2, k)$, for all $s \geq 1$ and $k \geq 0$.[7]

Unlike social preferences, individual preferences are time varying. This is because the ranking of consumption allocations is a function of age, which also explains why preferences from the viewpoint of old age exhibit no bias — the old do not age any further, they care about the well-being of their children and all their ancestors are dead. Consequently, social preferences would be time consistent if the old were dictators.

4 Equilibrium intergenerational redistribution

To focus on the equilibrium interaction between present and future governments, assume that the date-t government has sufficient instruments to implement its desired date-t allocation. Thus, one can treat the date-t government as if it can choose both the allocation of date-t aggregate resources between consumption and investment and the allocation of date-t aggregate consumption between the young and the old directly, without decentralizing the

[7] Halevy (2015) shows that any two properties among stationarity, time invariance and time consistency imply the third.

equilibrium allocations at date-t. Specifically, this implies that each government can control current, but not future, old-age transfers and investment levels.

4.1 Preferences and production technology

For the remainder of the paper, we will focus directly on social preferences over streams of consumption allocations, as given by equation (6). Furthermore, we will suppose that $u^y(c) = u^o(c) = u(c)$, where u is given by

$$u(c) = \begin{cases} \frac{c^{1-\sigma}}{1-\sigma} & \text{if} \quad \sigma \neq 1 \quad , \qquad \sigma > 0 \\ \ln(c) & \text{if} \quad \sigma = 1. \end{cases} \tag{9}$$

The aggregate resources constraint in the economy is given by

$$Ak_t \geq c_t^y + c_t^o + k_{t+1} - k_t, \tag{10}$$

where k_t units of capital produce Ak_t units of output, with $A > 0$, for all $t \geq 0$.

It is easy to see that the results stated in Proposition 1 hold regardless of the individuals' rate of time preference. Accordingly, we have assumed a zero time discount rate, for simplicity. The assumption of isoelastic utility is restrictive, but it facilitates the equilibrium analysis. Specifically, it implies that the optimal static allocation of consumption across generations is independent of the level of aggregate consumption. It is also needed for the economy to converge to a balanced growth path. Furthermore, we assume that $\delta(A+1) > 1$ in order to ensure positive equilibrium growth rates and $\delta(A+1)^{1-\sigma} < 1$ in order to ensure that growth is not so fast that it leads to unbounded utility.

4.2 Markov perfect equilibrium

Consider the following problem. Every period t the government chooses investment $(k_{t+1} - k_t)$, and consumption (c_t^y and c_t^o) in order to maximize (6), with $u^y(c) = u^o(c) = u(c)$, subject to (9) and (10), taking as given the strategies of all other governments. A Markov strategy of the date-t government consists of an investment policy $i^t(k_t)$ and consumption policies $c_y^t(k_t)$ and $c_o^t(k_t)$ that are only functions of the payoff-relevant state variable k_t. A sequence

of Markov strategies $\left\{\left(i^t\left(k_t\right), c_y^t\left(k_t\right), c_o^t\left(k_t\right)\right)\right\}_{t=0}^{\infty}$ is a symmetric Markov perfect equilibrium if it is a subgame perfect equilibrium for every realization of the state variable k_t, and all governments follow the same strategy, that is, if $\left(i^t\left(k_t\right), c_y^t\left(k_t\right), c_o^t\left(k_t\right)\right) = \left(i\left(k_t\right), c_y\left(k_t\right), c_o\left(k_t\right)\right)$, for all t.

Our focus on Markov equilibria helps to understand how future bias distorts optimal intergenerational redistribution when the possibility of commitment is ruled out. This case is interesting on its own and we also build on it below to highlight the potential role of a pay-as-you-go pension system as a commitment mechanism to increase future old-age transfers.

Following Laibson (1997) and Krusell et al. (2002), we focus on the unique Markov perfect equilibrium in linear strategies, which is the unique Markov perfect equilibrium that is a limit of finite-horizon equilibria.

It will be convenient to let $c_t^y = \tau_t c_t$ and $c_t^o = \left(1-\tau_t\right) c_t$, where c_t denotes aggregate consumption at date t and τ_t denotes the share that is allocated to the young generation. In equilibrium, each government recognizes that all governments solve the static problem

$$\max_{\tau\in[0,1]} \left\{\left(\delta+a\right) u\left(\tau c\right) + \left(1+\theta a\right) u\left(\left(1-\tau\right) c\right)\right\} \quad \text{for given} \quad c \geq 0, \tag{11}$$

and so the young's share of aggregate consumption is given by τ^* every period, where

$$\tau^* = \frac{1}{1+\left(\frac{1+\theta a}{\delta+a}\right)^{1/\sigma}}. \tag{12}$$

Hence, one can express the relevant preferences for the date-t government in terms of aggregate consumption streams as

$$\widehat{G}_t\left(\left\{c_j\right\}_{j=t}^{\infty}\right) = q\left(\tau^*, a\right) u\left(c_t\right) + q\left(\tau^*, 0\right) \sum_{s=1}^{\infty} \delta^s u\left(c_{t+s}\right), \tag{13}$$

where

$$q\left(\tau, a\right) = \tau^{1-\sigma} + \frac{1+\theta a}{\delta+a}\left(1-\tau\right)^{1-\sigma}. \tag{14}$$

In order to apply Definition 1, let $w(\tau_{t+s}, c_{t+s}, s) = d(s)\, u(c_{t+s})$, with

$$d(s) = \begin{cases} 1 & \text{if} \quad s = 0 \\ \beta\delta^s & \text{if} \quad s \geq 1 \end{cases}$$

where $\beta \equiv \frac{q(\tau^*,0)}{q(\tau^*,a)}$, in which case each government discounts consumption streams starting then according to the sequence 1, $\beta\delta$, $\beta\delta^2$, $\beta\delta^3$, ... The discount factor between the current period and the next is $\beta\delta$ whereas the discount factor between any two future periods is δ.

The social preferences given in (13) would have a present bias if $\beta < 1$, in which case governments would have quasi-hyperbolic preferences over aggregate consumption streams of the form proposed by Phelps and Pollak (1968). Here, however, not only is β endogenous, but also greater than one, because $q(\tau^*, a) < q(\tau^*, 0)$, since $\theta\delta < 1$.

To understand the source of β-δ discounting, note that equation (13) characterizes preferences in terms of only two objects: the flow utility from current aggregate consumption and the discounted sum of utilities from the stream of future aggregate consumption using the constant discount factor δ. The trade-off between these two objects is characterized by the distribution of consumption across living generations today relative to the distribution at any other future date. Future bias emerges because the date-t government is more reluctant to transfer resources from the young to the old at date t than at any future date.

Given τ^*, our problem has the same structure as the one originally analyzed by Phelps and Pollak (1968) and more recently Laibson (1997) and Krusell et al. (2002). Accordingly, there is a unique Markov perfect equilibrium that is the limit of finite-horizon equilibria. Here, one can verify by explicit backward induction that the corresponding investment strategies are linear.

To construct such an equilibrium, suppose that the current government anticipates that every future government follows the linear investment policy $i' = \widehat{g}k'$, with $\delta(1+\widehat{g})^{1-\sigma} < 1$. Then, the current investment decision solves the following dynamic programming problem:

$$V_0(k) = \max_{0 \leq k' \leq Ak} \left\{ q(\tau^*, a)\, u(Ak - k' + k) + \delta V(k') \right\}, \tag{15}$$

with

$$V(k) = q(\tau^*, 0)\, u\left(Ak - (1+\widehat{g})\, k + k\right) + \delta V\left((1+\widehat{g})\, k\right), \tag{16}$$

where $q(\tau, a)$ is given by equation (14). An investment policy $i(k) = gk$ that is part of a symmetric Markov perfect equilibrium must have $g = \widehat{g}$.

In the Appendix, we show that the above dynamic programming problem implies

$$g = \frac{A+1}{1 + \left(\frac{q(\tau^*,a)}{q(\tau^*,0)} \frac{\delta^{-1} - (1+\widehat{g})^{1-\sigma}}{(A-\widehat{g})^{1-\sigma}}\right)^{1/\sigma}} - 1 \equiv B(\tau^*, \tau^*, \widehat{g}). \tag{17}$$

The best response mapping $g = B(\tau, \tau', \widehat{g})$ characterizes the best investment response by a government that allocates a share τ of current consumption to the current young and anticipates that future governments will allocate a share τ' of consumption to the young and invest according to $i(k') = \widehat{g}k'$.

Proposition 2 *There is a unique symmetric Markov perfect equilibrium in linear strategies. It is characterized by $i(k) = g^* k$, $c_y(k) = \tau^*(A - g^*)\, k$, and $c_o(k) = (1-\tau^*)(A - g^*)\, k$, where g^* is given by the solution to $g^* = B(\tau^*, \tau^*, g^*)$ and τ^* is given by equation (12).*

To understand the equilibrium implications of future bias, consider the benchmark case where the date-t government can commit to all allocations from date $t+1$ onwards. In such a first-best scenario, the date-t government would choose the optimal sharing rule $\overline{\tau} = \left(1 + \delta^{-1/\sigma}\right)^{-1}$ starting at date $t+1$, which solves the problem

$$\max_{\tau \in [0,1]} \left\{ u(\tau c) + \delta^{-1} u((1-\tau)\, c) \right\} \quad \text{for given} \quad c \geq 0. \tag{18}$$

Moreover, given $\overline{\tau}$, optimal investment from date $t+1$ onwards would solve

$$W(k) = \max_{0 \leq k' \leq Ak} \left\{ q(\overline{\tau}, 0)\, u(Ak - k' + k) + \delta W(k') \right\}. \tag{19}$$

It is easy to verify that the solution to this standard dynamic programming problem implies that investment from date $t+1$ onwards is given by $k' - k = \overline{g}k$, where $\overline{g} = [\delta(A+1)]^{1/\sigma} - 1$.

In the Appendix, we verify that $\tau^* > \overline{\tau}$, and $g^* > \overline{g}$. In this sense, future bias implies

that future equilibrium old-age transfers are insufficient and future equilibrium growth is excessive from the viewpoint of the government at any given date.

4.3 Pay-as-you-go social security

> "We put those pay roll contributions there so as to give the contributors a legal, moral, and political right to collect their pensions and their unemployment benefits. With those taxes in there, no damn politician can ever scrap my social security program. Those taxes aren't a matter of economics, they're straight politics." [President Franklin Roosevelt, 1941]

The above quote is FDR's well known remark that the U.S. payroll tax system was designed to ensure the political sustainability of the social security program. This remark can be understood through the lens of future bias, which implies that every generation tends to favor, temporarily, growth at the expense of old-age transfers. Below we show that the corresponding time inconsistency can explain both FDR's fear of a constituency to scrap the social security program in the future and his own support for pay-as-you-go social security in the first place. It also explains why committing future government to support social security is necessary for the survival of the program.

Here, in addition to tying the hands of future governments, commitment plays a role because future bias implies that the date-t government, if it could, would rather postpone the legislation to date-$t+1$, even if it expected every future government to abide by the original legislation. Rather than modeling the commitment to pensions as the equilibrium outcome of a game, in this section we simply assume that commitment is possible and focus instead on the hypothesis that the future bias of social preferences gives rise to a demand for sustainable old-age transfers. Specifically, we show that, starting from the no-commitment solution characterized in Proposition 2, the date-t government will be better off legislating a stationary sequence of old-age transfers, which is sustainable in the sense that no later government has an incentive to change the legislated scheme. Restricting attention to stationary sequences of old-age transfers allows for a simple commitment device, which could be formalized in a number of ways. For instance, the date-t government may be able

to legislate a transfer at date t that is sufficiently costly to change in the future (Boadway and Wildasin 1989). Alternatively, a stationary sequence of transfers can be supported by the threat of collapse of the system if any government repeals the legislation (Cooley and Soares 1999).[8]

Formally, suppose that the date-t government can precommit future transfers, provided that it treats current and future generations symmetrically, by making proportional transfers identical at all points in time. Further, suppose that the date-t government can control *current,* but not future, investment. Thus, once transfers are legislated, consecutive governments will choose investment unilaterally, taking into account the investment strategies of future governments. Conditional on the share τ, our previous analysis implies that there is a symmetric Markov perfect equilibrium in linear strategies such that equilibrium growth solves $g = B(\tau, \tau, g)$, where B is given by (17). Letting $g(\tau)$ be a solution to this equation, one can easily verify that our previous analysis implies that the date-t government's optimal choice of τ solves the following problem:

$$\max_{\tau \in [0,1]} \left\{ q(\tau, a)\, u((A - g(\tau))\, k) + \frac{\delta q(\tau, 0)}{1 - \delta (1 + g(\tau))^{1-\sigma}} u((A - g(\tau))(1 + g(\tau))\, k) \right\}. \quad (20)$$

Let τ^P denote a solution to this problem. The following proposition compares τ^P, and the corresponding growth rate $g(\tau^P)$, to the Markov perfect equilibrium share τ^*, and the corresponding growth rate g^*, characterized in Proposition 2.

Proposition 3 *(i) $\tau^P < \tau^*$ for all $\sigma > 0$. (ii) $g(\tau^P) < g^*$ if and only if $\sigma > 1$.*

Part (i) of the proposition implies that the optimal old-age transfers under commitment to the pension system, given by $1 - \tau^P$, always exceed the Markov perfect equilibrium transfers given by $1 - \tau^*$. Hence, the date-t government always has an incentive to legislate old-age transfers. Part (ii) implies that the legislation will depress equilibrium growth if and only if $\sigma > 1$. More generally, one can verify that the relationship between transfers and growth is such that $g(\tau)$ is increasing in τ if and only if $\sigma > 1$. Thus, when $\sigma > 1$ old-age transfers,

[8] Azariadis and Galasso (2002) argue that giving current voters or policymakers some veto power over changes in future policies acts like a commitment device.

$1-\tau$, and growth, $g(\tau)$, are negatively related, which is regarded as the empirically relevant case (Auerbach and Kotlikoff 1987). Furthermore, there is widespread agreement that $\sigma > 1$ is the empirically relevant case in macroeconomics and public finance applications.

To understand why old-age transfers and growth are negatively related when $\sigma > 1$, it is useful to consider first the best investment response underlying the equilibrium characterized in Proposition 2. Without commitment to pensions, the date-t government expects that future governments will allocate a share τ^* of consumption to the young and invest according to $i(k') = \widehat{g}k'$. Whenever it anticipates $(\tau^*, \widehat{g})$ to deviate from the first-best allocation $(\overline{\tau}, \overline{g})$ in the future, it anticipates a welfare loss. The discrepancy between $(\tau^*, \widehat{g})$ and $(\overline{\tau}, \overline{g})$ gives rise to two opposing effects. On the one hand, for given aggregate levels of future consumption, utility is anticipated to be lower because consumption will be misallocated every period over the two living generations ($\tau^* > \overline{\tau}$). The date-t government can compensate for this loss by strategically raising investment in order to increase future consumption. On the other hand, transferring wealth to the future has a lower return, because it will be misallocated: by strategically lowering investment the date-t government can substitute intertemporally away from future misallocation. When $\sigma > 1$, the former effect dominates the latter and each government has an incentive to mitigate the loss from misallocation by raising investment. Formally, the best response mapping (17) indicates how each government will attempt to manipulate investment next period. One can easily verify that locally around the equilibrium, current and next-period investments are strategic complements ($\partial B/\partial g > 0$) if $\sigma > 1$ and strategic substitutes ($\partial B/\partial g < 0$) if $\sigma < 1$.

Now consider the situation in which date-t government can legislate time-independent transfers. Note that, since this government can still choose $\tau = \tau^*$, committing to an optimal stationary sequence of transfers cannot lower welfare.

Note that $\tau^P < \tau^*$ because the date-t government has an incentive to commit old-age transfers in order to mitigate the misallocation associated with the fact that $\tau^* > \overline{\tau}$. However, increasing old-age transfers starting at date $t+1$ comes at a cost, from the viewpoint of the date-t government, since it means it has to increase old-age transfers at date t as well. Accordingly, one can also verify that $\tau^P > \overline{\tau}$.

Next, note that $\tau^P < \tau^*$ implies that the date-t legislation reduces the misallocation of aggregate consumption between the young and the old starting at date $t+1$, for all $\sigma > 0$. If $\sigma > 1$, the income effect arising from misallocation dominates the substitution effect and so, with less misallocation there is less need to compensate by increasing investment at date t. Accordingly, pensions lower growth if $\sigma > 1$.

Instead, if $\sigma < 1$, the substitution effect dominates the income effect and so the date-t government prefers to compensate for the future misallocation by making the pie larger today rather than tomorrow. Since pensions reduce misallocation, they raise the return to investment. Accordingly, pensions raise growth if $\sigma < 1$. Recall that future growth is viewed as excessive by each generation in the Markov perfect equilibrium (recall that $g^* > \overline{g}$ for all $\sigma > 0$). Thus, if the government could choose future growth and pensions, it would choose lower growth. However, commitment to future growth rates is not possible here and the date-t government anticipates that future governments will choose future growth according to their best response. The welfare effect of a higher growth rate in combination with an improved consumption distribution along the best response is positive in the case with $\sigma < 1$ for the same reason that current and future investments are strategic substitutes. Misallocation in the future makes governments reduce investment if $\sigma < 1$. Since pensions reduce future misallocation, governments then benefit from increasing investment. Hence, while pensions have opposite effects on growth depending on whether $\sigma < 1$ or $\sigma > 1$, the welfare effect of the corresponding change in growth is positive for all $\sigma > 0$.

Proposition 3 provides a positive theory of intergenerational redistribution. In the empirically relevant case where $\sigma > 1$, institutions that will necessarily harm future generations, by lowering growth, are supported because future old-age transfers are too low and future growth is too high *from the perspective of currently living generations*. It is important to recognize that this does *not* arise because equilibrium investment is dynamically inefficient (e.g., Samuelson 1975). As usual, we say that an investment allocation is dynamically efficient if there is no alternative allocation that provides more aggregate consumption in one period and at least the same consumption in every future period. It is not difficult to verify that equilibrium investment is dynamically efficient. To see why, note that this is the case

if the growth rate is lower than the *social* return to investment, that is, if $g^* < A$.[9] That this condition must hold, for all $c_t > 0$, follows immediately from the aggregate resources constraint: $Ak_t \geq c_t + k_{t+1} - k_t$.

While equilibrium investment is dynamically efficient, the equilibrium is not Pareto efficient, because the private return to investment, which is given by $A - \partial (g^* k) / \partial k = A - g^*$, is lower than its social return, which is given by A. A Pareto improvement would result from investing optimally from the viewpoint of the date-t young generation at the socially optimal rate of return, without changing the allocation for any other generation. This is in contrast with the view that nonpaternalistic altruism towards the following generation must lead to Pareto efficiency (Streufert, 1993), which is the case if nonpaternalistic altruism amounts to time-consistent preferences. However, with time-inconsistent preferences, as is the case here, the private return to investment is necessarily lower than the social return, because the incentive to manipulate future investment does not disappear.[10] The same argument continues to apply after the legislation of old-age transfers.

When $\sigma > 1$ old-age transfers and growth are negatively related, hence the above social security legislation hurts future generations. Yet, it is self-enforcing in the sense that once stationary old-age transfers given by τ^P are legislated, no government will ever support a proposal to switch to an alternative stationary sequence of old-age transfers. Each future government chooses to sustain the current legislation because it faces essentially the same problem as the government that enacted the original legislation.

[9] The following proof replicates the argument in Saint Paul (1992). Consider an allocation $\left\{\tilde{k}_t\right\}$ with $\tilde{k}_s < k_s$, for some s, with $\tilde{c}_t \geq c_t$ for $t \geq s$. Since $\tilde{k}_{t+1} = (A+1)\tilde{k}_t - \tilde{c}_t$ and $k_{t+1} = (A+1)k_t - c_t$, for $t \geq s$, it must be that $k_{t+1} - \tilde{k}_{t+1} \geq (A+1)\left(k_t - \tilde{k}_t\right)$, for $t \geq s$. In turn this implies that $k_{s+T} - \tilde{k}_{s+T} \geq (A+1)^T \left(k_s - \tilde{k}_s\right)$, and thus $\tilde{k}_{s+T} \leq (1+g^*)^T k_s - (A+1)^T \left(k_s - \tilde{k}_s\right)$, for any $T \geq 1$. Clearly, if $g^* < A$, the right side of the inequality becomes negative for T sufficiently large, contradicting the hypothesis that there is a feasible deviation $\tilde{k}_s < k_s$, for some s, with $\tilde{c}_t \geq c_t$ for $t \geq s$. This concludes the proof.

[10] Pearce (2008) emphasizes the inefficiency that arises when the well-being of a generation increases with the well-being of not-yet-born future generations and the latter fail to internalize the impact of their equilibrium choices on the well-being of dead ancestors, either because generations are not altruistic toward their ancestors, or because they treat the well-being of dead ancestors parametrically. Under natural assumptions, equations (4)-(6) continue to apply in both cases (for the latter case, see Hori and Kanaya 1989 and Bergstrom 1999) and our main results go through under these alternative assumptions.

5 Conclusion

We have shown that standard preferences of altruistic overlapping generations (e.g., Barro 1974, Kimball 1987) exhibit future bias. We have traced a conflict of interest between current and future utilitarian governments representing living generations to the fact that every government inherits the future bias of individual preferences and so it is more reluctant to transfer resources from the young to the old in the present than at any future date. We have analyzed the implications of this conflict for intergenerational redistribution in a tractable setting, where the optimal sharing rule to allocate consumption between coexisting generations every date is independent of aggregate consumption. Conditional on a stationary sharing rule, preferences over aggregate consumption streams exhibit β-δ discounting with $\beta > 1$, where the value of the short-term discount factor β is a function of the consumption sharing rule.

Here, β-δ discounting is a reflection of indirect, nonpaternalistic altruism. In Galperti and Strulovici's (2017) non-overlapping generations model, β-δ discounting reflects direct, as opposed to indirect, nonpaternalistic altruism. Both papers challenge the view that nonpaternalistic altruism and time consistency are two sides of the same coin (e.g., Phelps and Pollak 1968, Barro 1974). Yet, whether altruism leads to present or future bias depends on the demographic structure and the specific model of altruism.

Our analysis further suggests that pay-as-you-go social security legislation can be understood as a commitment to increase future old-age transfers. Even when the legislation systematically favors current generations at the expense of future generations, by lowering growth, future governments do not have an incentive to repeal the legislation, because each future government faces essentially the same problem as the government before it. This sheds new light on the widespread legislation of pay-as-you-go social security despite concerns about its negative effects on capital accumulation (Auerbach and Kotlikoff 1987).

Appendix

Proof of Proposition 1

Applying Definition 1 with

$$w(c_{t+s}^{y}, c_{t+s}^{o}, s) = \begin{cases} u^{y}\left(c_{t}^{y}\right) + \theta u^{o}\left(c_{t}^{o}\right) & \text{if} \quad s = 0 \\ \delta^{s}[u^{y}\left(c_{t+s}^{y}\right) + \delta^{1} u^{o}\left(c_{t+s}^{o}\right)] & \text{if} \quad s \geq 1 \end{cases},$$

one can easily verify Part (i) of the proposition. Similarly, applying Definition 1 with

$$w(c_{t+s}^{y}, c_{t+s}^{o}, s) = \begin{cases} u^{y}\left(c_{t}^{y}\right) + \left(\frac{1+\theta a}{\delta+a}\right) u^{o}\left(c_{t}^{o}\right) & \text{if} \quad s = 0 \\ \delta^{s}[u^{y}\left(c_{t+s}^{y}\right) + \delta^{-1} u^{o}\left(c_{t+s}^{o}\right)] & \text{if} \quad s \geq 1 \end{cases},$$

Part (ii) of the proposition can be readily verified. **QED**

Proof of Proposition 2

To derive the best response mapping (17), first note that the first-order condition with respect to k' at date t is given by

$$-q\left(\tau^{*}, a\right) \frac{\partial u\left(c\right)}{\partial c} \frac{\partial c}{\partial k'} = \delta \frac{\partial V\left(k'\right)}{\partial k'}. \tag{21}$$

Solving the recursion in equation (16) it can be verified that

$$V\left(k'\right) = \left(\frac{q\left(\tau^{*}, 0\right)}{1 - \delta\left(1 + \widehat{g}\right)^{1-\sigma}}\right) u\left(c'\right)$$

and so we have

$$\frac{\partial V\left(k'\right)}{\partial k'} = \left(\frac{q\left(\tau^{*}, 0\right)}{1 - \delta\left(1 + \widehat{g}\right)^{1-\sigma}}\right) \frac{\partial u\left(c'\right)}{\partial c'} \frac{\partial c'}{\partial k'}. \tag{22}$$

Combining equations (21) and (22), the relevant Euler equation is given by

$$\frac{\partial u\left(c\right)/\partial c}{\delta\left(\partial u\left(c'\right)/\partial c'\right)} = \left(\frac{q\left(\tau^{*}, 0\right)/q\left(\tau^{*}, a\right)}{1 - \delta\left(1 + \widehat{g}\right)^{1-\sigma}}\right) \frac{-\partial c'/\partial k'}{\partial c/\partial k'}. \tag{23}$$

Recognizing that

$$\frac{\partial c'}{\partial k'} = A - \frac{\partial i\left(k'\right)}{\partial k'} = A - \widehat{g} \ ,$$

since $c' = Ak' - i\left(k'\right)$ and $i\left(k'\right) = \widehat{g}k'$, and using the facts that the instantaneous utility functions are isoelastic and the aggregate resources constraint holds with equality, it is

straightforward to write the above Euler equation as

$$\left(\frac{k'}{Ak-k'+k}\right)^{\sigma}=\frac{q\left(\tau^{*},0\right)\left(A-\widehat{g}\right)^{1-\sigma}}{q\left(\tau^{*},a\right)\left(\delta^{-1}-\left(1+\widehat{g}\right)^{1-\sigma}\right)} \quad ,$$

which describes the best response k' to the anticipation of $\widehat{g}$, for given k. Clearly, the best response to any given $\widehat{g}$ is linear in k and we obtain the best-response mapping (17).

Lemma Let $\bar{g}=\left[\delta\left(A+1\right)\right]^{1/\sigma}-1$. For all $\widehat{g}\in\left(\bar{g},A\right)$, $\partial B\left(\tau^{*},\tau^{*},\widehat{g}\right)/\partial\widehat{g}\geq 0$ if and only if $\sigma\geq 1$, with equality if and only if $\sigma=1$.

From equation (17), $B(\tau^{*},\tau^{*},\hat{g})$ can be written as $\tilde{B}(\hat{g},Q^{*})$, where

$$1+\tilde{B}(\hat{g},Q)\equiv\frac{A+1}{1+Q\left(\frac{\delta^{-1}-(1+\widehat{g})^{1-\sigma}}{(A-\widehat{g})^{1-\sigma}}\right)^{1/\sigma}} \quad ,$$

and

$$Q^{*}\equiv\left(\frac{q\left(\tau^{*},a\right)}{q\left(\tau^{*},0\right)}\right)^{1/\sigma} \quad ,$$

One can verify that

$$Q^{*}=\left(\frac{1+\left(\frac{1+\theta a}{\delta+a}\right)^{1/\sigma}}{1+\delta^{-1}\left(\frac{1+\theta a}{\delta+a}\right)^{-1}\left(\frac{1+\theta a}{\delta+a}\right)^{1/\sigma}}\right)^{1/\sigma} \quad ,$$

hence, $Q^{*}<1$ if and only if $\theta\delta<1$. Next, note that the sign of $\partial\tilde{B}(\hat{g},Q)/\partial\hat{g}$ is given by the sign of $(\sigma-1)\left[(1+\widehat{g})^{\sigma}-\delta(A+1)\right]$. It is easy to verify that, for given Q^{*}, $\tilde{B}(\hat{g},Q^{*})$ has a global minimum at $\widehat{g}=\bar{g}$ if $\sigma>1$; it has a global maximum at $\widehat{g}=\bar{g}$ if $\sigma<1$; and it is flat at $B\left(\tau^{*},\tau^{*},\bar{g}\right)>\bar{g}$ if $\sigma=1$. This proves the lemma. **QED**

To prove existence of a unique fixed point $g^{*}\in(\bar{g},A)$, evaluate $g=\tilde{B}(\hat{g},Q)$ at $\hat{g}=g$ and rewrite it as

$$\delta^{-1}Q^{\sigma}(1+g)^{\sigma}+(1-Q^{\sigma})(1+g)-(A+1)=0. \tag{24}$$

As long as $Q\leq 1$, the left side of the equation is increasing in g. Moreover, it is negative when $g=-1$ and positive when $g=A$. Hence, there is exactly one fixed point, $\tilde{g}(Q)<A$, where $g^{*}=\tilde{g}(Q^{*})$. Differentiating equation (24), one can verify that $\partial\tilde{g}(Q)/\partial Q<0$ if and only if $1>\delta\left(1+\tilde{g}(Q)\right)^{1-\sigma}$. For $\sigma\leq 1$, the latter inequality holds since $g\leq A$ and we have assumed $1>(A+1)^{1-\sigma}\delta$. For $\sigma>1$, first note that $\tilde{g}(1)=\bar{g}$ and $1>\delta\left(1+\tilde{g}(1)\right)^{1-\sigma}$, so $\partial\tilde{g}(1)/\partial Q<0$; hence for all $Q\leq 1$ the inequality $1>\delta\left(1+\tilde{g}(Q)\right)^{1-\sigma}$ holds. Since $Q^{*}<1$, for all $\theta\delta<1$, we have $\tilde{g}(Q^{*})=g^{*}>\tilde{g}(1)=\bar{g}$. Therefore, $A>g^{*}>\bar{g}$. This concludes the proof. **QED**

Proof of Proposition 3

Let $U(\tau, g(\tau))$ denote the objective function in (20). Since $U(\tau, g(\tau))$ is a continuous function of τ on $[0,1]$, it must have a maximum. Since $g(\tau)$ satisfies $g = B(\tau, \tau, g)$, we have

$$\frac{q(\tau, a)}{q(\tau, 0)} = \frac{(1+g)^{-\sigma}}{\delta^{-1} - (1+g)^{1-\sigma}} (A - g) \ . \tag{25}$$

First, suppose $\sigma \neq 1$. Substituting (25) in (20) we find

$$U(\tau, g(\tau)) \propto q(\tau, a)(1-\sigma)^{-1}(A - g(\tau))^{-\sigma},$$

which implies that U is differentiable on $(0,1)$ with

$$\frac{dU}{d\tau} \propto \frac{1}{(A-g)^{\sigma}} \left(\frac{1}{1-\sigma}\right) \frac{\partial q(\tau, a)}{\partial \tau} + \frac{q(\tau, a)\sigma}{(A-g)^{\sigma+1}} \left(\frac{1}{1-\sigma}\right) \frac{dg}{d\tau} \ .$$

We now determine the sign of this derivative for $\tau \geq \tau^*$. First, (14) implies $\left(\frac{1}{1-\sigma}\right) \frac{\partial q(\tau,a)}{\partial \tau} < 0$ if and only if $\tau > \tau^*$. Second, from total differentiation of (25) we find that $\left(\frac{1}{1-\sigma}\right) \frac{dg}{d\tau} \leq 0$ for all τ, with equality only if $\sigma = 1$. Hence, $dU/d\tau < 0$ for all $\tau \geq \tau^*$, which implies that $\tau^P < \tau^*$, for all $\sigma \neq 1$.

Now suppose that $\sigma = 1$. In this case we have $dg/d\tau = 0$, hence $dU/d\tau = \partial U/\partial \tau$. It is straightforward to verify that $\partial U/\partial \tau < 0$ for $\tau \geq \tau^*$, which implies that $\tau^P < \tau^*$, as required. This concludes the proof that $\tau^P < \tau^*$ for all $\sigma > 0$.

It follows immediately from our previous arguments that $g\left(\tau^P\right) < g^*$ if $\sigma > 1$, $g(\tau^P) = g^*$ if $\sigma = 1$, and $g(\tau^P) > g^*$ if $\sigma < 1$. **QED**

References

Alesina, A. and G. Tabellini (1990): "A positive theory of fiscal deficits and government debt", *Review of Economic Studies* 57, 403-414.

Auerbach, A.J. and L.J. Kotlikoff (1987): *Dynamic Fiscal Policy*, Cambridge: Cambridge University Press.

Azzimonti, M. (2011): "Barriers to investment in polarized societies", *American Economic Review* 101, 2182-2204.

Azariadis C. and V. Galasso (2002): "Fiscal Constitutions", *Journal of Economic Theory* 103, 255-281.

Barro, R.J. (1974): "Are government bonds net wealth?", *Journal of Political Economy* 82, 1095-1117.

Barro, R.J. (1999): "Ramsey meets Laibson in the Neoclassical growth model", *Quarterly Journal of Economics* 114, 1125-1152.

Bergstrom, T.C. (1999): "Systems of benevolent utility functions", *Journal of Public Economic Theory* 1, 71-100.

Bernheim, B.D. (1989): "Intergenerational altruism, dynastic equilibria and social welfare", *Review of Economic Studies* 56, 119-128.

Bernheim, B.D. and D. Ray (1987): "Economic growth with intergenerational altruism", *Review of Economic Studies* 54, 227-241.

Boadway, R.W. and D.E. Wildasin (1989): "A median voter model of social security", *International Economic Review* 30, 307-328.

Burbidge, J.B. (1983): "Government debt in an overlapping-generations model with bequests and gifts", *American Economic Review* 73, 222-227.

Calvo, G.A. and M. Obstfeld (1988): "Optimal time-consistent fiscal policy with finite lifetimes", *Econometrica*, 56, 411-432.

Caplin, A. and J. Leahy (2004): "The social discount rate", *Journal of Political Economy* 112, 1257-1268.

Cooley, T.F. and J. Soares (1999): "A positive theory of social security based on reputation", *Journal of Political Economy* 107, 135-160.

Galperti, S. and B. Strulovici (2017): "A theory of intergenerational altruism", *Econometrica*, forthcoming.

Gollier, C. and R. Zeckhauser (2005): "Aggregation of heterogeneous time preferences", *Journal of Political Economy* 113, 878-896.

Gollier, C. and M. Weitzman (2010): "How should the distant future be discounted when discount rates are uncertain?" *Economic Letters* 107, 350-353.

Grossman, G.M., Helpman, E. (1998): "Intergenerational redistribution with short-lived governments", *Economic Journal* 108, 1299-1329.

Halac M. and P. Yared (2014): "Fiscal rules and discretion under persistent shocks", *Econometrica* 82, 1557-1614.

Halevy, Y. (2008): "Strotz meets Allais: diminishing impatience and the certainty effect", *American Economic Review* 98, 1145-1162.

Halevy, Y. (2015): "Time consistency: stationarity and time invariance", *Econometrica* 83, 335-352.

Hansson, I. and C. Stuart (1989): "Social security as trade among living generations", *American Economic Review* 79, 1182-1195.

Hori, H. and S. Kanaya (1989): "Utility functionals with nonpaternalistic intergenerational altruism", *Journal of Economic Theory* 49, 241-265.

Jackson, M.O. and L. Yariv (2014): "Present bias and collective dynamic choice in the lab", *American Economic Review* 104, 4184-4204.

Kimball, M.S. (1987): "Making sense of two-sided altruism", *Journal of Monetary Economics* 20, 301-326.

Kohlberg, E. (1976): "A model of economic growth with altruism between generations", *Journal of Economic Theory* 13, 1-13.

Krusell, P., B. Kuruşçu and A.A. Smith (2002): "Equilibrium welfare and government policy with quasi-geometric discounting", *Journal of Economic Theory* 105, 42-72.

Laibson, D. (1997): "Golden eggs and hyperbolic discounting", *Quarterly Journal of Economics* 112, 443-447.

Lindbeck, A. and J.W. Weibull (1987): "Balanced-budget redistribution as the outcome of political competition", *Public Choice* 52, 273-297.

Pearce, D. (2008): "Nonpaternalistic Sympathy and the inefficiency of consistent intertemporal plans", in Foundations in Economic Theory: A Volume in Honor of Hugo F. Sonnenschein, edited by M. O. Jackson and A. McLennan, Springer.

Persson, T. and L.E.O. Svensson (1989): "Why a stubborn conservative would run a deficit: policy with time-inconsistent preferences", *Quarterly Journal of Economics* 104, 325-345.

Phelps, E. and R. Pollak (1968): "On second-best national saving and game-equilibrium growth", *Review of Economic Studies* 35, 185-199.

Ray, D. (1987): "Nonpaternalistic intergenerational altruism", *Journal of Economic Theory* 41, 112-132.

Ray, D. (2018): "Hedonistic altruism and welfare economics", mimeo, New York University.

Saint Paul, G. (1992): "Fiscal policy in an endogenous growth model", *Quarterly Journal of Economics* 107, 1243-1259.

Samuelson, P.A. (1975): "Optimum social security in a life-cycle growth model", *International Economic Review* 16, 539-544.

Streufert, P.A. (1993): "Consistent preferences and intergenerational equilibria in Markov strategies," in B. Dutta, S. Gangopadhyay, D. Mookerjee, and D. Ray (eds.), *Theoretical Issues in Economic Development*, 263-287, Bombay: Oxford University Press.

Strotz, R.H. (1956): "Myopia and inconsistency in dynamic utility maximization", *Review of Economic Studies* 23, 165-180.

Tabellini, G. (2000): "A positive theory of social security", *Scandinavian Journal of Economics* 102, 523-545.

Veall, M.R. (1986): "Public pensions as optimal social contracts", *Journal of Public Economics* 31, 237-251.