Wages as signals of worker mobility*

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Abstract

We analyze a model in which workers direct their search on and off the job and employer-worker match productivities are private information. Employers can commit neither to post contracts such that wages are a function of tenure nor to disregard counteroffers. In this context, potential employers who do not observe workers' productivity in their current matches use wages as a signal of workers' willingness to switch jobs. In turn, this implies that the wage contracts that employers post in the market for entry jobs — the jobs unemployed workers search for — not only direct job search but also signal future worker mobility. When the costs of creating entry jobs are sufficiently small, the unique equilibrium supports the efficient allocation under full information. When the costs of creating entry jobs are sufficiently large, the efficient equilibrium may break down and there may exist a competitive search equilibrium that is non-revealing and exhibits inefficient turnover. From the perspective of our model, posted wages and counteroffers can be understood as complementary parts of a second-best market solution to holdup problems.

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1 Introduction

Economists increasingly recognize that worker mobility plays a key role in the allocation of labor in decentralized markets. Matching the right workers to the right jobs is a complex process and turnover provides a potential solution to otherwise large misallocation problems.[1] In this paper, we analyze a competitive search model of turnover. Like standard competitive search models, we assume that employers post contracts that are legally enforceable and direct search (e.g., Menzio and Shi, 2011). Unlike standard competitive search models, we assume that worker-employer match productivity is private information and that there is limited commitment.

We assume that commitment is limited in two important respects. First, firms cannot commit to disregard outside offers. Second, they cannot commit to post bilaterally efficient contracts. We allow firms to commit to contracts under which the wage is conditional on match productivity, which can take only one of two values. Thus, well matched and poorly matched workers may earn different wages. However, we assume that firms cannot commit to contracts under which the wage is a function of tenure. In this context, we show that the efficiency of turnover depends on the informational content of wage contracts in the market for entry jobs — which are the jobs that unemployed workers search for — which not only direct the job search of unemployed workers but also signal future worker mobility. The central issue is whether equilibrium wages reveal whether a worker is willing to move or is instead seeking to elicit a retention offer.

Our main results concern the existence and properties of competitive search equilibria. First, if the costs of creating entry jobs are small enough, there exists a revealing equilibrium with positive job-to-job quits that exhibits efficient turnover. This equilibrium simultaneously solves both the problem of directing the search of unemployed workers and the problem of signaling the potential mobility of employed workers. Efficient turnover requires that firms pay poorly matched workers their marginal productivity. This allows for maximization of the match surplus as it induces poorly matched workers to quit exactly when it is efficient to do so. Paying the match surplus to poorly matched workers, however, requires that firms recover the full costs of creating entry jobs from well matched workers, while poorly matched workers bear none of these costs. Workers who are well matched cannot avoid these costs via search on the job because wages reveal productivity realizations to poaching firms, which renders well matched workers immobile.

The efficient equilibrium may break down when the costs of creating entry jobs is large enough. The reason is that match-specific risk gives rise to a holdup problem when contracts cannot be made contingent on tenure. This is because up front job creation costs are an *ex ante* investment on the part of firms. Workers, however, can credibly reject matches that deliver less than the value of unemployment, and can make this decision after observing the productivity realization. This implies that the delivery of positive *expected* surplus is not sufficient to ensure the creation of a job. Sufficiently high job creation costs reduce the value of high productivity jobs to workers to a level below the value of unemployment, so workers will reject such offers. Hence, the efficient equilibrium

[1] Mortensen (1978) and Jovanovic (1979) are two early papers.

solves the holdup problem, but only when job creation costs are sufficiently low. The existence of the efficient equilibrium depends on the ability of firms to offload all job creation costs to well matched workers. This implies that the efficiency of competitive search cannot be understood independently of the division of the match surplus.[2]

Next we show that if the costs of creating entry jobs are sufficiently large, there may exist an equilibrium that is both non-revealing and exhibits inefficient worker mobility. In a non-revealing equilibrium, firms post contracts under which wages are not contingent on productivity realizations, so that all entry jobs pay identical wages. The non-revealing equilibrium spreads the costs of creating entry jobs equally across matches. It solves the holdup problem in the market for entry jobs but at the expense of distorting the market for non-entry jobs. Because entry jobs pay wages that fail to reveal match productivity, potential poaching firms cannot distinguish poorly matched from well matched workers. This creates adverse selection in the market for non-entry jobs: well matched workers have an incentive to search on the job, but solely to elicit a retention offer from their current employers rather than to change jobs. This congestion reduces the return to job creation in the market for non-entry jobs and, as a result, generates inefficient worker mobility.

That pooling wage contracts are an equilibrium outcome does not mean that the non-revealing equilibrium relies on unreasonable off-equilibrium beliefs. To rule out equilibria that rely on unduly pessimistic off-equilibrium beliefs, we propose a refinement that extends the one proposed by Guerrieri et al. (2010) in order to accommodate the possibility of non-revealing wages. This possibility arises in our setting because wage contracts in the market for entry jobs play both an informational and an allocative role. Our refinement restricts off-equilibrium beliefs in two distinct ways, in the spirit of the Intuitive Criterion and the D1 criterion, respectively (Cho and Kreps, 1987). First, they must place zero weight on types that can never gain from deviating from a fixed equilibrium outcome. Second, they must be supported on types that have the most to gain from deviating from a fixed equilibrium.[3]

The central message of the paper is that competitive search equilibria do not ensure efficient turnover. The problem with posted wages — i.e., pooling wage contracts — is that they are the incorrect tool to retain workers unless the distribution of match productivities is non-degenerate. Accordingly, one might expect that choosing to commit to contracts that make wages contingent on productivity must always be better than committing to pooling contracts. However, this disregards the possibility of holdups and the (endogenous) informational content of wages. It also disregards the complementary role of counteroffers. Once the signaling role of wages is taken into account, we show that posting pooling contracts in the market for entry jobs and subsequently responding to outside offers may in fact be an equilibrium outcome. This provides a novel perspective on both

[2]Mortensen (1979) argues that turnover is independent of the division of the surplus if bilaterally efficient contracts are available. Stevens (2004) shows the first best contract specifies a hiring fee that workers pay on being hired. The worker is then paid marginal product while employed, which ensures any subsequent quit decision is jointly efficient.

[3]Chang (2018) proposes a similar refinement in order to analyze separating and pooling competitive search equilibria in a model with multidimensional asymmetric information. Kurlat and Scheuer (2021) propose a refinement in the same spirit and apply it to a signaling model in which firms have heterogeneous information about workers' types.

posted wages and counteroffers: they can be understood as complementary parts of a second-best solution to holdup problems.

Empirical evidence suggests that wage posting and counteroffers play a role in some markets. For example, Hall and Krueger (2012), in a survey of employed workers, document that nearly one third of workers knew exactly how much the job would pay at the time they were first interviewed. This finding is suggestive of wage posting. In a similar vein, Barron, Berger and Black (2006) find evidence from a survey of employers that suggests that employers are willing to match outside offers for roughly 41% of workers. We are not aware of any empirical work that examines the possible link between posted wages and counteroffers. From the perspective of our model, however, the two observations can be better understood jointly, rather than separately: by posting wages, firms anticipate that they will respond to outside offers. Our analysis illustrates how these observations arise from a common commitment failure, the source of which is twofold: firms can commit neither to post bilaterally efficient contracts nor to disregard counteroffers *ex post*.

1.1 Related literature

Our core analytical framework builds on previous work on competitive search equilibria with adverse selection (Guerrieri et al. 2010) to address turnover under incomplete information about workers' match productivity.[4] Our specification of search on the job combines elements of directed search that are standard in competitive search models (Menzio and Shi 2011) and elements of bargaining that are standard in random matching models (Postel-Vinay and Robin 2002). In particular, we allow employers to counter outside offers, which, combined with the fact that search on the job is directed, plays a crucial role in generating an adverse selection problem. The possibility of counteroffers renders our framework remarkably tractable by limiting the scope for job quits, thereby eliminating the sorts of wage ladders found in Delacroix and Shi (2006). This enables us to characterize pooling equilibria, which are neither block-recursive nor constrained efficient.[5]

Our paper is related to an existing literature that studies the signaling role of prices in directed search equilibria. For example, Delacroix and Shi (2013) show that competitive search equilibria can be inefficient when sellers post prices that not only direct buyers' search, but also signal product quality.[6] Here, in contrast, employers' decisions to create jobs and workers' job search decisions take place *before* they posses any private information. Furthermore, match productivity is known to both parties *before* matching takes place and such information is contractible.

The importance of asymmetric information between employers is widely recognized.[7] Waldman

[4]Faig and Jerez (2005), Guerrieri (2008) and Moen and Rosen (2011) analyze competitive search equilibria in models with match-specific private information. Guerrieri and Shimer (2014) and Chang (2018) analyze competitive search equilibria with adverse selection in asset markets. Wright et al. (2021) provide an insightful survey of competitive search applications in economics.

[5]Shi (2009) analyzes block-recursive competitive search equilibria with search on the job.

[6]Menzio (2007) and Kim and Kircher (2015) examine the signaling role of prices when firms and workers can engage in pre-match communication, or cheap talk. Other work emphasizes the signaling role of promotions (Waldman, 1984), price announcements that differ from actual prices (Lester et al., 2017), or types of contracts (Stacey, 2016).

[7]Kahn (2013) is an interesting empirical study.

(1984) and Greenwald (1986) are two early examples. More recently, Carrillo-Tudela and Kaas (2015) consider on-the-job search with adverse selection under random matching, but they assume that wage contracts are not renegotiated when workers receive outside offers. By contrast, our focus is on the interaction between directed search and *ex post* renegotiation.

Our paper is also related to the literature that examines the efficiency properties of both wage posting and competitive search. It is well known that if employers can commit to general enough contracts, competitive search equilibrium is able to internalize a variety of externalities, including those that play an important role in our setting. For example, Acemoglu and Shimer (1999) show that competitive search solves holdup problems associated with pre-match investments. The interest in the efficiency properties of competitive search extends to the literature that studies on-the-job search and worker mobility. Burdett and Coles (2003) and Shi (2009) argue that on-the-job search induces firms to backload wages, thus making wages increase and quit rates decrease with tenure. Both papers, however, assume that the productivity of a match is public information and that a firm does not respond to outside offers. These restrictions hold in some markets, but they are clearly violated in others. Our paper seeks to understand the factors that determine when equilibrium matching offers do and do not arise.

The search literature tends to view commitment to wage posting as an efficient contract that exploits the role of directed search in attracting workers to the right jobs. Under this view, ex post bargaining tends to be seen as an alternative mechanism that is used to make ex post pay contingent on productivity. For example, Michelacci and Suarez (2006) argue that posting dominates bargaining when the allocative inefficiency of ex post bargaining is large enough that the benefits of posting wages in order to direct job search are correspondingly large. Conversely, when the benefits of directing job search are relatively small and the benefits from bargaining in terms of making pay responsive to productivity are large enough, then bargaining dominates posting. Our analysis provides a very different perspective. Productivity-based pay requires commitment to contingent wages and it is an efficient contract in terms of attracting as well as retaining workers. Posted wages, by contrast, are in fact an inefficient way to solve holdup problems. Furthermore, ex post bargaining and posted wages are not substitutes, but rather complementary tools that employers can use to attract and retain workers.

Finally, our paper is related to a literature that examines the existence of counteroffers. Golan (2005) argues that counteroffers help achieve efficient job assignment in a model where there is no turnover. By contrast, we focus on worker mobility but disregard the issue of job assignment. Yamaguchi (2010) considers alternative sources of wage growth in the context of a standard random matching model with counteroffers. Postel-Vinay and Robin (2004) consider the incentives of employers to commit to non-matching of outside wage offers within a random matching model. By contrast, our focus is on the interaction between directed search and *ex post* renegotiation.

2 The model

2.1 Environment

Time is discrete. There are *ex ante* homogeneous workers and *ex ante* homogeneous employers. All agents are risk neutral and discount the future at a rate $r > 0$. There is a unit measure of workers who are either employed or unemployed. An unemployed worker searches for a job and receives a flow benefit from unemployment equal to $b \geq 0$. All worker-employer matches produce y_h units of output with probability $\alpha \in (0, 1)$ and y_l units of output with probability $1 - \alpha$, where $b < y_l < y_h$. Subsequently, a separation shock makes an employed worker become unemployed with probability $\delta > 0$. An employed worker can search for a different job while employed.

It will be convenient to assume that firms can post a vacancy for one of two types of jobs, indexed by $j = u, e$. We label them *entry* jobs, which cater to the unemployed ($j = u$), and *non-entry* jobs, which cater to employed workers ($j = e$). There is free entry of jobs and employers incur a cost $k_j > 0$ in order to post a vacancy. We allow the two types of vacancies to have different creation costs. For example, hiring employed versus unemployed workers may involve different screening costs.

We assume that only unemployed workers can fill entry jobs and only employed workers can fill non-entry jobs. One interpretation is that employers do not want to hire unemployed workers for non-entry jobs, because they lack the necessary experience, while they do not want to hire employed workers for entry-level positions, because overqualified workers are more likely to become dissatisfied with those jobs. It would be natural to assume that entry and non-entry jobs have different production technologies. For simplicity, however, we abstract away from such differences.

Each employer can post any feasible job, where a job x is defined below as a wage contract together with a job type. A submarket is defined by the job x posted in that submarket and the corresponding queue length q, which is defined as the ratio of workers searching for x to employers posting x. We refer to the submarket offering job x simply as "submarket x". Workers can direct their search across submarkets. Employed and unemployed workers have the same search intensity. Meetings are bilateral, so each employer meets at most one worker and vice versa. Workers who search in submarket x with queue length q meet an employer with probability $f(q)$ and employers in the same market meet a worker with probability $qf(q)$. We assume that $f(q)$ is twice differentiable, strictly decreasing and convex, with $f(0) = 1$ and $f(\infty) = 0$. We also assume that $qf(q)$ is strictly increasing and concave, approaching 1 as q converges to ∞. These assumptions ensure that the job-finding elasticity, given by $\eta(q) = -qf'(q)/f(q)$, is such that $0 = \eta(0) < \eta(1) \leq 1$, with $\eta'(q) > 0$. For simplicity, we also assume that $\eta(q)$ is concave, with $\eta(\infty) = 1$.[8]

When a worker (employed or not) and a potential employer meet, the latter observes the worker's labor market status and, if currently employed, her wage and her job type. Then, the productivity of the potential match is drawn randomly and observed by both parties to the match. The match productivity, however, is not observed by outsiders to the match. For example, the current match

[8] An example of a meeting technology that satisfies these assumptions is $M(u, v) = uv/(u + v)$.

productivity of an employed worker is not observed by potential new employers. Vice versa, the new match productivity at the poaching firm is not observed by the incumbent employer. Subsequently, employers decide whether or not to make formal offers. We assume that all employers make take-it-or-leave-it offers, that incumbent employers can counter outside offers, and that wages can only be renegotiated by mutual agreement. Finally, workers decide whether or not to accept any offers. To break ties, we assume that workers reject any outside offers that are matched by the incumbent employer. New matches start producing in the next period.

Contracts are specified in terms of fixed, though possibly productivity-contingent wages. The hiring and retention policies are not included as part of the contract, but firms and workers do take into account that fixed wages can be renegotiated by mutual agreement. A worker who gets a credible outside offer can choose to terminate the current wage contract and agree to a "new" contract with a different wage, which lasts until another outside offer arrives. If the outside offer is credible and if a better counteroffer is feasible, the incumbent employer then commits to make a new wage contract. Retention policies will depend on history only through the worker's current wage, which is a sufficient statistic for the payoff-relevant history of the current contract.

A job $x = (j, w_l, w_h)$ specifies the job type, $j \in \{u, e\}$, and a wage contract, $(w_l, w_h) \in [0, y_l] \times [0, y_h]$, both of which are observable. The job type (entry jobs versus non-entry jobs) conveys the necessary information to attract the right workers (employed versus unemployed workers).[9] The wage contract consists of the pair of fixed wages, w_l and w_h, to be paid when the realizations of match productivity in the new match are $y' = y_l$ and $y' = y_h$, respectively.

Neither workers nor employers can be forced to participate in a match before observing match productivity. That is, employers cannot commit to make a formal job offer and workers cannot commit to accept such an offer before observing the realized match productivity. In this sense, matches are pure inspection goods, rather than experience goods. These assumptions are made to highlight the role of incomplete information about workers' outside options.

The adverse selection problem that arises from the combination of limited commitment and asymmetric information will play an important role in our analysis. Since match productivity is unobserved by third parties, a worker's current labor productivity is private information to the worker *vis-a-vis* potential new employers. Consequently, poaching offers cannot discriminate between workers with different outside options, unless (equilibrium) wages reveal match productivity. Since workers are unable to commit not to search on the job and employers are unable to commit to not countering outside offers, workers in high productivity matches have an incentive to seek outside offers solely to elicit retention offers from their current employers.

We also assume that employers face a small cost of making a credible offer, so they will never make offers that they know will be rejected with certainty. For simplicity, we assume these costs are negligible and so are not explicit about them.

[9] Marinescu and Wolthoff (2020) use data from CareerBuilder.com to argue that the informational content of job titles refers to the hierarchy, level of experience, and specialization of different jobs.

2.2 Competitive search equilibrium

Let $s \in S$ denote a worker's payoff-relevant state. By convention, unemployed workers are associated with the state s_u. For all $s \neq s_u$, we let $s \equiv (w, y)$, where $w \in [0, y_h]$ denotes the worker's current wage and $y \in \{y_l, y_h\}$ denotes current match productivity. Thus, the feasible state space is given by $S = \{s_u\} \cup S_e$, where $S_e = [0, y_h] \times \{y_l, y_h\}$. To minimize clutter, we do not include the type j of the job an employed worker has in her payoff-relevant state. It will become clear that the worker's job type conveys no payoff-relevant information to potential employers, conditional on the worker's wage.

A stationary competitive search equilibrium (Moen, 1997) specifies a mapping Q from feasible jobs to submarket queues. Workers direct their search across all feasible jobs, taking as given the submarket queue length $Q(x)$ for all $x \in X$, where X is the set of feasible jobs. Workers' decisions must be optimal at any information set. This information set includes the worker's own state $s \in S$ as well as the distribution of workers across states, that is, the aggregate state of the economy $\psi : S \to [0, 1]$, where $\psi(s)$ is the proportion of state-s workers in the economy. It will become clear that competitive search equilibria need not be *block recursive*. That is, the agents' value functions and therefore equilibrium strategies may be a function of the aggregate state. However, in order to minimize clutter, we are not explicit about the potential dependence of the agents' value functions on the aggregate state ψ. We restrict attention to stationary equilibria throughout the paper.

For all $s \in S$, the value function of a worker in state s, denoted by $V(s)$, satisfies the following:

$$V(s) = w + \frac{1}{1+r}\left[\delta V(s_u) + (1-\delta)\left(V(s) + \max_{x \in X \cup \{x_\varnothing\}} U(s, x, Q(x))\right)\right], \tag{1}$$

where $x = x_\varnothing$ denotes the choice of not searching and where we use the convention that $w = b$ for unemployed workers. For employed workers, the current match is destroyed with probability δ, in which case the worker becomes unemployed. The term $U(s, x, Q(x))$ denotes the expected surplus to a worker with current state s from searching in submarket x, with associated queue length $Q(x)$, evaluated next period:

$$U(s, x, Q(x)) = \begin{cases} f(Q(x))\,\mathbb{E}_{y'}\left[g_h(x, w, s_o)\max\left(0, V(s_o) - V(s)\right)\right] & \text{if} \quad s = s_u \\ f(Q(x))\,\mathbb{E}_{y'}\left[g_h(x, w, s_o)\max\left(0, V(s_o) - V(s), V(s_c) - V(s)\right)\right] & \text{if} \quad s \neq s_u. \end{cases}$$

where $\mathbb{E}_{y'}$ denotes the expectation taken with respect to the random productivity draw y' and $g_h(x, w, s_o)$ denotes the hiring policy of the firm.

To understand these expressions, note that when searching for a job $x = (j, w'_l, w'_h)$, a worker meets an employer with probability $f(Q(x))$ in which case y' is realized and the wage offer w' is made, where $w' = w'_l$ if $y' = y_l$ and $w' = w'_h$ if $y' = y_h$. If the offer is accepted, then the worker's state becomes $s_o = (w', y')$. Workers with current state s reject any offer w' such that $V(s) > V(s_o)$. If $V(s) < V(s_o)$, an unemployed worker accepts the offer and her state becomes

s_o, while an employed worker may be able to elicit a counteroffer from her current employer. When a counteroffer w_c is made, the worker decides whether to accept the outside offer w', in which case she her state becomes $s_o = (w', y')$, or to accept the counteroffer, in which case her state becomes $s_c = (w_c, y)$. Workers searching in submarket x anticipate the hiring policy $g_h(x, w, s_o)$ and the retention policy g_r such that $w_c = g_r(s, x, w')$. The employers' policies g_h and g_r are specified later.

A worker's optimal search policy is given by

$$g_x(s) \in \arg \max_{x \in X \cup \{x_\varnothing\}} U(s, x, Q(x)). \tag{2}$$

We now turn our attention to the workers' decision about whether or to not accept an offer. For simplicity, we restrict attention to pure acceptance policies and let g_a be such that

$$g_a(s, s_o, w_c) \in \begin{cases} \arg \max_{a \in \{0,1\}} \{aV(s_o) + (1-a)V(s)\} & \text{if} \quad s = s_u \\ \\ \arg \max_{a \in \{0,1\}} \{aV(s_o) + (1-a)\max\{V(s), V(s_c)\}\} & \text{if} \quad s \neq s_u \end{cases} \tag{3}$$

for all $s \in S$, $s_o \in S_e$ and $w_c \in [0, y_h]$. For an employed worker in state $s = (w, y)$, $g_a(s, s_o, w_c) = 1$ if the worker accepts an offer from a new employer at the wage w', given that w_c is her current employer's counteroffer, with $s_o = (w', y')$. If the worker accepts the retention offer, then $g_a(s, s_o, w_c) = 0$ and her state becomes $s_c = (w_c, y)$. To break ties, we assume that the worker stays with her current employer and $g_a(s, s_o, w_c) = 0$ if $w_c = w'$. That is, matching outside offers is sufficient to retain workers.

It is easy to verify that for all $s = (w, y) \in S_e$ the present value of an ongoing match to the employer, denoted by $J_f(s)$, solves

$$\begin{aligned} \frac{J_f(s)}{1+r} &= \frac{y - w}{r + \delta + (1-\delta) f(Q(g_x(s))) \mathbb{E}_{y'}\{g_h(x, w, s_o)\}} \\ &+ \frac{(1-\delta) f(Q(g_x(s)))}{r + \delta + (1-\delta) f(Q(g_x(s))) \mathbb{E}_{y'}\{g_h(x, w, s_o)\}} \\ &\times \; \mathbb{E}_{y'}\left\{g_h(x, w, s_o) \max_{w_c}\left\{(1 - g_a(s, s_o, w_c)) \frac{J_f(s_c)}{1+r}\right\}\right\} \end{aligned} \tag{4}$$

subject to $w_c \geq w$,

where $s_o = (w', y')$ and $s_c = (w_c, y)$. The denominator on the right side reflects the three sources of discounting: the discount rate, the exogenous probability of job destruction and the probability that the worker receives an outside offer from a poaching firm. The incumbent employer keeps the worker if the counteroffer w_c is accepted, in which case the value of the match becomes $J_f(s_c)$.

Note that the maximization problem in (4) has been written as if the incumbent employer knows the match realization y' associated with an outside offer. To see why, note that the employer needs to anticipate the worker's search policy (g_x), her acceptance policy (g_a) and the hiring policy of the worker's potential new employer (g_h). If $g_x(s) = x_\varnothing$, then the worker does not search on the job and $Q(x_\varnothing) = \infty$. If $g_x(s) \neq x_\varnothing$, then the worker searches in some submarket $x = (j, w'_l, w'_h)$. The employer anticipates that she searches in the market for non-entry jobs, that is, $j = e$, and also that when she meets a potential employer, she will get an outside offer w', where $w' = w'_l$ if $y' = y_l$ and $w' = w'_h$ if $y' = y_h$. Furthermore, the incumbent employer understands that all credible offers in the market for non-entry jobs must be such that $w'_h > y_l \geq w'_l$. This is because outside offers are made after the potential employer observes the productivity of the new match and because the incumbent employer can retain the worker by matching the outside offer. Hence, credible offers in the market for non-entry jobs always reveal the productivity realization in the new match, y'. In effect wage counteroffers are a function of y'. We have used this fact to write the max operator inside the expectation operator in the above equation, even though the incumbent firm does not observe the realization y' directly.

Let $g_r(s, x, w')$ denote a solution to problem (4), for any $s = (w, y) \in S_e$, $x = (j, w'_l, w'_h) \in X$ and $w' \in \{w'_l, w'_h\}$, where $g_r(s, x, w')$ specifies the best response w_c in order to retain a state-s worker who receives an offer w' after searching in submarket x.

Given $Q(x)$, workers searching for x do not need to account for the composition of workers in that submarket. By contrast, employers posting x need to anticipate not only the likelihood of meeting a worker, given by $Q(x) f(Q(x))$, but also the composition of the pool of workers searching for that contract. We let $\mu(\cdot|x)$ denote a probability distribution on S, for each $x \in X$. An employer posting x meets a worker with probability $Q(x) f(Q(x))$, in which case the expected surplus to the employer is given by $\mathbb{E}_s J(s, x)$, where $J(s, x)$ is the expected value of the employer's surplus conditional on meeting a state-s applicant and $\mathbb{E}_s$ is taken with respect to $\mu(\cdot|x)$. Thus, an employer is willing to post $x = (j, w'_l, w'_h)$ only if

$$k_j \leq Q(x) f(Q(x)) \mathbb{E}_s \{J(s, x) | x\},$$

where k_j is the cost of creating a type-j job, for $j \in \{u, e\}$, and where, for any $s = (w, y)$ and any $x = (j, w'_l, w'_h)$,

$$J(s, x) = \mathbb{E}_{y'} \left\{ \max_{h \in \{0,1\}} \left\{ h g_a\left(s, s_o, g_r\left(s, x, w'\right)\right) \frac{J_f(s_o)}{1 + r} \right\} \right\}, \tag{5}$$

where $J_f(s_o)$ satisfies equation (4) and $s_o = (w', y')$, with $w' = w'_l$ if $y' = y_l$ and $w' = w'_h$ if $y' = y_h$. Equation (5) reflects the fact that potential employers anticipate the current acceptance policies of the workers they attract (g_a) and the retention policies of their current employers (g_r), if the worker is employed. In order to minimize clutter, we do not include these explicitly as arguments in the value function J.

Let $g_h(x, w, s_o)$ denote a solution to the problem in (5), for $s \neq s_u$, and write $g_h(x, b, s_o)$ for $s = s_u$, by convention. Note that employers posting non-entry jobs never make offers to unemployed workers, that is, $g_h((e, w'_l, w'_h), w, s_o) = 0$ if $w = b$. Similarly, employers posting entry jobs never make offers to employed workers, that is, $g_h\left((u, w_l', w_h'), w, s_o\right) = 0$ if $w \neq b$.

Definition 1 *A stationary equilibrium* $\mathcal{E} = (X^*, S^*, V, J, g_x, g_a, g_h, g_r, Q, \mu, \psi)$ *consists of a set of posted jobs* $X^* \subseteq X$*, a set of workers' states* $S^* \subseteq S$*, value functions* $V : S \to R_+$ *and* $J : S \times X \to R_+$*, policy functions* $g_x : S \to X \cup \{x_\varnothing\}$*,* $g_a : S \times S_e \times [0, y_h] \to \{0, 1\}$*,* $g_h : X \times [0, y_h] \times S \to \{0, 1\}$ *and* $g_r : S_e \times X \times [0, y_h] \to [0, y_h]$*, a function* $Q : X \to R_+$*, a distribution* $\mu : S \times X \to [0, 1]$ *and a distribution* $\psi : S \to [0, 1]$*, such that:*

(A) Workers' optimal search and acceptance: V *satisfies (1);* g_x *satisfies (2);* g_a *satisfies (3).*

(B) Optimal job posting and retention with free entry: g_h*,* g_r *and* J *solve (4) and (5). Moreover, for any* $x = (j, w_l, w_h) \in X$*,* $Q(x) f(Q(x)) \int_S J(s, x)\, d\mu(s|x) \leq k_j$*, with equality if* $x \in X^*$*.*

(C) Consistent beliefs: For any $x \in X^*$*,*

$$\mu(s|x) = \frac{\psi(s)\, \mathbb{I}_x(g_x(s))}{\int_S \mathbb{I}_x(g_x(s))\, d\psi(s)}, \text{ with } \int_S \mathbb{I}_x(g_x(s))\, d\psi(s) > 0,$$

for all $s \in S$*, where* $\mathbb{I}_x(g_x(s)) = 1$ *if* $g_x(s) = x$ *and* $\mathbb{I}_x(g_x(s)) = 0$ *if* $g_x(s) \neq x$*.*

(D) Consistent allocations: for every $x \in X^*$*, the mass of workers visiting submarket* x *divided by the mass of employers posting* x *is equal to* $Q(x)$*.*

Part (A) of Definition 1 ensures that workers' search and acceptance policies are optimal for all states, taking as given the market queue length for all jobs. Part (B) ensures that employers' posting behavior and their subsequent retention policies are optimal, and employers posting equilibrium contracts make zero profits. Part (C) ensures that employers' beliefs are consistent with the workers' equilibrium search policies through Bayes' rule. It ensures that any contract that is posted in equilibrium attracts a positive mass of workers and that the distribution of workers searching for any equilibrium contract is exactly what the employers posting those contracts expect. Part (D) ensures the correct market clearing queue. We refer to the pair (X^*, ψ), where S^* is the support of ψ, as an *equilibrium allocation.*

We have deliberately excluded the characterization of the distribution of workers across states from the equilibrium definition. This serves to highlight the property that in order to analyze individual decisions it is sufficient to know the function Q and the function μ. Once the analysis is completed, one can simply aggregate individuals' decisions to find the equilibrium distributions of workers and wages.

2.3 Equilibrium refinement

The definition of a competitive search equilibrium allows for more or less arbitrary off-equilibrium beliefs and so it allows for many equilibria, each of which is supported by particular beliefs in the markets where no trade takes place. The issue is that some contracts may not be traded because employers fear they would attract only undesirable types of workers. If workers expect the labor market queue associated with those contracts to be sufficiently high then those contracts would in fact not attract any workers and so the employers' pessimistic beliefs are never contradicted. To address this issue, we propose the following equilibrium refinement.

Definition 2 *An equilibrium* $\mathcal{E} = (X^*, S^*, V, J, g_x, g_a, g_h, g_r, Q, \mu, \psi)$ *is a refined equilibrium if:*

(i) $\mu(s|x) = 0$ *if* $U(s, x, q) < U(s, g_x(s), Q(g_x(s)))$*, for any* $x \notin X^*$*,* $q \in R_+ \cup \{\infty\}$ *and* $s \in S$*;*

(ii) there does not exist a job $x = (j, w_l, w_h) \notin X^*$ *and queue* $q \in R_+ \cup \{\infty\}$ *such that* $qf(q) \int J(s, x) \, d\mu(s|x) > k_j$*.*

Intuitively, a competitive search equilibrium is a refined equilibrium if there is no labor market queue such that a firm posting a deviating job can make positive profits and attract solely workers for whom this action is not equilibrium dominated. Part (i) is in the spirit of the Intuitive Criterion proposed in Cho and Kreps (1987), in that it requires off-equilibrium beliefs to place zero weight on types that can never gain from deviating from a fixed equilibrium outcome. Part (ii) is in the spirit of the D1 criterion (Cho and Kreps, 1987) in that it requires that off-equilibrium beliefs be supported on types that have the most to gain from deviating from a fixed equilibrium. Specifically, our refinement builds on the concepts proposed in Gale (1992, 1996) and Guerrieri et al. (2010), by requiring that deviating firms anticipate that an off-equilibrium contract would attract only those workers who are willing to endure the highest labor market queue. Our refinement is a natural extension to allow for both separating and pooling equilibria.

Note that Part (i) of the refinement implies that employers posting an off–equilibrium job must believe that the only workers the job would ever attract must be indifferent between the off-equilibrium job and their preferred equilibrium job. Moreover, Part (ii) implies that the employer's belief about the distribution of workers who would search for the deviating contract places all its weight on types that are indifferent about searching for the job. These two restrictions uniquely pin down $\mu(\cdot|x)$ and $Q(x)$, for all $x \notin X^*$. Importantly, they also allow for the possibility that there may be more than one type of worker searching for an off-equilibrium job.

3 Equilibrium worker mobility

In this section we analyze the equilibrium properties of the model presented in Section 2. Our analysis highlights the role of wages as a public signal of worker mobility.

We begin by noting that our assumptions impose a lot of structure on the problem. Recall that we assume that incumbent firms can retain workers by matching outside offers and that poaching firms never make offers that will be rejected with certainty. Hence, since outside offers are made after observing the productivity of the new match, poaching firms never make an offer to a worker with whom they would form a low productivity match. Furthermore, our equilibrium refinement implies that workers in high productivity matches can only profit from on-the-job search if their match productivity is not revealed in equilibrium. The reason is that there are no gains from trade between workers in high productivity matches and potential poaching firms.

This shapes the set of possible job and wage transitions as follows: all job switches occur when a worker in a low productivity match meets a firm with which she has a high productivity match. A worker in a high productivity match (who can search on the job by pooling) who meets a firm with which she would also form a high productivity match will elicit both a job offer from the poaching firm and a retention offer from the incumbent firm. Therefore, all job switches and wage changes reveal that a worker is now employed in a high productivity match. Hence, workers in non-entry jobs will no longer be the target of poaching firms and so the job ladder has at most two rungs. Similarly, workers in entry jobs whose wage reveals that they have a high match productivity will not get any outside offers.

Our model shares the well known property that the allocation supported by a competitive search equilibrium can be characterized as the solution of a corresponding dynamic programming problem. However, it will become clear below that equilibrium wages may or may not reveal match productivity. While each equilibrium outcome corresponds to the solution of a distinct dynamic programming problem, here we present one set of Bellman equations to characterize the equilibrium allocations corresponding to two distinct equilibria with positive job-to-job quits. One is the efficient equilibrium, which involves separating wage contracts in the markets for entry and non-entry jobs. The other one is an equilibrium that involves pooling contracts in the market for entry jobs.[10]

Let $\rho \in \{1-\alpha, 1\}$ denote the fraction of poorly matched workers among all on-the-job searchers, where we can restrict attention to two types of situations: one where wages are revealing and, consequently, only poorly matched workers search on the job ($\rho = 1$), and another where wages are non-revealing and, consequently, both well matched and poorly matched workers search on the job ($\rho = 1-\alpha$). For a given value of $\rho \in \{1-\alpha, 1\}$, the value of employment to a worker in state $s = (w, y) \neq s_u$ is given by

$$V(s) = w + \frac{1}{1+r}\left[\delta V(s_u) + (1-\delta)\Big(V(s) + U_0(s)\Big)\right], \tag{P1}$$

where

$$\frac{U_0(s)}{1+r} = \max_{(w',q')}\left\{f(q')\,\alpha \max\left\{0, \frac{w'}{r+\delta} + \left(\frac{\delta}{r+\delta}\right)\frac{V(s_u)}{1+r} - \frac{V(s)}{1+r}\right\}\right\}$$

subject to

$$k_e \leq q' f(q')\,\alpha\rho\left(\frac{y_h - w'}{r+\delta}\right),$$
$$w' > y_l,$$
$$U_0(s) = 0 \text{ if } s = (w, y_h) \text{ and } \rho = 1.$$

Denote a solution to Problem (P1) by $(w_e(s), q_e(s))$, where we let $w_e(s) = 0$ and $q_e(s) = \infty$, for

[10] An inefficient equilibrium with separating contracts in all markets may exist for some parameter values, but our focus in this paper is on the existence and the properties of an equilibrium with pooling contracts.

$s = (w, y_h)$, if $\rho = 1$. This normalization simply captures the fact that workers searching on the job from high productivity matches do not crowd out workers searching from low productivity matches in an equilibrium where entry wages reveal match productivities.

$U_0(s)/(1+r)$ represents the option value to an employed worker of on-the-job search in the market for non-entry jobs. As discussed above, the structure of our counteroffer game implies that an employed worker whose wage reveals her to be in a high productivity match cannot profit from on-the-job search. Workers in low productivity matches and workers who are indistinguishable from them can search on the job, and the option value of this search is given by the constrained optimization problem above.

Workers who can profit from on-the-job search face a relatively straightforward competitive search problem. The first constraint imposes that firms that post a non-entry job must make non-negative expected profits. In this constraint, $\rho \in \{1-\alpha, 1\}$ is used to index the two problems. When $\rho = 1$, the non-negative profit constraint is written as if all poaching offers are accepted by workers. This version of the problem corresponds to the equilibrium where wages reveal match productivity, in which case workers in high productivity matches cannot profit from on-the-job search. When $\rho = 1 - \alpha$, the non-negative profit constraint is written as if poaching offers are accepted by workers with probability $1 - \alpha$. This version of the problem corresponds to the equilibrium in which wages do not reveal productivity, high productivity workers search on the job, and a fraction $1 - \alpha$ of applicants to poaching firms reject job offers in favor of retention offers. The other constraint, $w' > y_l$, reflects the assumption that poachers recognize the fact that employed workers can only be recruited if the poaching offer exceeds the worker's current productivity.

One can verify that a solution to Problem (P1) is such that, if $\rho = 1 - \alpha$, then $w_e((w, y_h)) = w_e((w, y_l))$ and $q_e((w, y_h)) = q_e((w, y_l))$. This reflects the fact that workers searching on the job from high and low productivity matches have identical incentives in an equilibrium where wages in entry jobs do not reveal match productivities. Both types of workers compete for the same outside offers, where subsequent retention offers elicited by well matched workers will just match the outside offers that will be accepted by poorly matched workers. Thus, the usual single crossing condition does not hold in this case.

Letting $s_l = (w_l, y_l)$ and $s_h = (w_h, y_h)$, the value of unemployment to a worker is given by:

$$V(s_u) = b + \frac{1}{1+r}\left[V(s_u) + \max_{(w_l, w_h, q)} f(q)\Big(\alpha V(s_h) + (1-\alpha)V(s_l) - V(s_u)\Big)\right] \tag{P2}$$

subject to

$$k_u \leq q f(q)\left[\alpha\left(\frac{y_h}{r+\delta} - \frac{w_e(s_h)}{r+\delta} + \frac{w_e(s_h) - w_h}{r+\delta+(1-\delta)\alpha f(q_e(s_h))}\right)\right.$$
$$\left. + (1-\alpha)\left(\frac{y_l - w_l}{r+\delta+(1-\delta)\alpha f(q_e(s_l))}\right)\right],$$

$$w_l \leq y_l \ , \ w_h \leq y_h \text{ and } w_h \begin{cases} = w_l & \text{if } \rho = 1-\alpha \\ \neq w_l & \text{if } \rho = 1. \end{cases}$$

Denote a solution to Problem (P2) by $\left(w_u^l, w_u^h, q_u\right)$.

The last constraint in Problem (P2) reflects the facts that employers cannot commit to pay wages that exceed the worker's marginal product and that wages reveal a worker's current match productivity if and only if entry wages vary across realizations of match productivity.

The first constraint is the non-negative profits constraint, incorporating all possibilities for on-the-job search allowed under our assumptions. The two terms within the parentheses in the first line reflect the profits an employer enjoys when it forms a high productivity match with an unemployed job seeker. The first term is the expected discounted value of the profits received if the employer were to pay the future retention offer $w_e(s_h)$. The second term reflects the temporary extra profits due to the fact that the entry wage w_h of a high productivity worker is lower than the retention offer the worker will elicit as soon as she receives an outside offer. The denominator reflects the sources of discounting: the discount rate (r), the exogenous probability of job destruction (δ), and the probability that such a worker receives an outside offer ($(1-\delta)\alpha f(q_e(s_h))$), in which case the incumbent firm will match and the worker's wage will change. The term within the parentheses in the second line represents the profits a firm enjoys when if forms a low productivity match with an unemployed job seeker. Such workers are always able to search on the job, never elicit retention offers, and quit whenever they meet a poaching firm with which they form a high productivity match.

The objective function in problem (P2) is not generally concave in (w_l, w_h, q). The main complication arises because employers do not take workers' future quit rates as given, but rather they understand that workers' future quit rates are a function of their current wages. To see why, consider how a worker' current wage affects her trade-off between quit rates and future wages. For a given current wage, a worker is willing to quit at a relatively slower rate only in exchange for relatively higher future wages. The higher her current wage, the lower the *ex post* surplus she can obtain from a given wage and thus, the lower the worker's quit rate. Since a given (future) wage represents a smaller proportional share of the wage gain in the worker expected surplus for workers with higher current wages, a worker's quit rate declines with her current wage at a decreasing rate. While this property is as one would expect, it implies that the worker's value function $V(s)$, for $s = (w, y)$, may not be a concave function of w, which is problematic. In general, it is unclear whether or not the properties of $q_e(s)$ ensure that both the worker's surplus and the employer's surplus are well-behaved with respect to w.

It is well known that the above problem complicates significantly the analysis of competitive search on the job (e.g., Delacroix and Shi, 2006). In the appendix, we address this problem by viewing the solution to (P1) as a mapping from the workers' quit rates to their current wages, rather than the reverse.

3.1 Revealing equilibrium

In this section we characterize the constrained efficient equilibrium, in which all employers post separating contracts and so wages in both entry and non-entry jobs reveal match productivity. We employ the terminology of the traditional rational expectations equilibrium literature and refer to this kind of equilibrium as (fully) revealing. Propositions 1 and 2 below provide conditions under which a constrained efficient allocation with positive job quits can be supported by a refined equilibrium as well as conditions under which it cannot.

An allocation is constrained efficient (efficient, for short) if it maximizes the present value of aggregate production net of search costs under full information. In the Appendix we characterize the efficient steady-state allocation as the solution to a planning problem. Proposition 1 provides sufficient conditions under which the unique refined equilibrium with positive job quits supports the efficient allocation.

Proposition 1 *Let* $\frac{y_h-b}{y_h-y_l} \geq \frac{r+\delta+\alpha}{r+\delta}$. *For any* $k_e \in \left(0, \alpha\frac{y_h-y_l}{r+\delta}\right)$, *there is a number* $\kappa \in \left(0, \alpha\frac{y_h-y_l}{r+\delta}\right)$ *such that (i) there is a unique refined equilibrium with positive quits, for all* $k_u \in (0, \kappa)$; *(ii) the equilibrium supports the efficient allocation; (iii) the equilibrium allocation solves problems (P1) and (P2) with* $\rho = 1$.

Lemma 4 in the Appendix provides necessary and sufficient conditions for existence of an efficient steady-state allocation with positive job quits. Intuitively, the flow benefit from unemployment and the costs of creating entry and non-entry jobs must all be sufficiently small. We then show that $\frac{y_h-b}{y_h-y_l} \geq \frac{r+\delta+\alpha}{r+\delta}$ is a sufficient condition for the efficient allocation to exhibit positive job quits for all (k_u, k_e) such that $k_u \leq \alpha\frac{y_h-y_l}{r+\delta} + \frac{y_l-b}{r+\delta}$ and $k_e < \alpha\frac{y_h-y_l}{r+\delta}$.

Not only must the efficient equilibrium allocation solve problems (P1) and (P2) subject to $\rho = 1$, but it must maximize the value of unemployment across *all* feasible contracts (both pooling and separating) subject to all firms making zero profits. Hence, there is no deviating contract that would attract workers and make positive profits. To see why, note that the candidate equilibrium does support the efficient allocation and that the planner can always replicate the job and worker flows associated with both pooling and separating contracts.

Intuitively, a refined equilibrium with positive job quits that supports the efficient allocation must be fully revealing. In the Appendix, we show that if an efficient allocation has positive job quits, then the solution to problems (P1) and (P2) with $\rho = 1$ must be such that $V(w_l, y_l) \geq V(s_u)$. We also show that it must be such that $V(w_h, y_h) > V(w_l, y_l)$ if the cost of creating entry jobs, k_u, is sufficiently small, as assumed in the proposition.[11] In turn, this ensures that $V(w_h, y_h) \geq V(s_u)$, which is needed for unemployed workers to be willing to accept equilibrium job offers regardless of the realization of match productivity.

Having established the existence of an equilibrium with positive job quits that supports the efficient allocation, we find sufficient conditions under which it is the unique refined equilibrium. In

[11] Our arguments in the proof of Proposition 1 also imply that one can find values of k_u for which there exists an efficient equilibrium such that $V(w_h, y_h) < V(w_l, y_l)$.

the Appendix, we show that the solution of problems (P1) and (P2) is unique under the assumptions in Proposition 1. Hence, a refined equilibrium that supports an efficient allocation with positive job quits must be unique within the class of revealing equilibria. In principle, a non-revealing equilibrium may also exist under the same parameter values, because the continuation value of unemployment is endogenous. For example, if $V\big((w_h, y_h)\big) - V(s_u) > 0$ for high values of $V(s_u)$, but $V\big((w_h, y_h)\big) - V(s_u) < 0$ for relatively low values of $V(s_u)$, then a pooling contract may in fact be the optimal contract in the market for entry jobs when everyone expects the equilibrium to be non-revealing. In the Appendix, we show that this cannot be the case when the costs of creating entry jobs are sufficiently low.

In the efficient equilibrium with positive job quits the wage distribution has three mass points: one wage for each productivity realization for workers who find jobs out of unemployment, and one wage for workers who find jobs via on-the-job search. Equilibrium transitions are as follows: All job offers made to unemployed workers are accepted. Unemployed workers who meet a firm with which they form a low productivity match conduct on-the-job search. These workers change jobs upon meeting another firm with which they form a high productivity match. Our equilibrium refinement implies that workers in high productivity matches will not crowd out poorly matched workers in a revealing equilibrium. Accordingly, such workers do not profit from search on the job and never change jobs. Jobs are destroyed both exogenously and, for low productivity matches, endogenously by quits.

Low productivity workers are paid their marginal product, that is, $w_l = y_l$. This is needed if the equilibrium is to support the efficient allocation. Intuitively, the *ex ante* match surplus is maximized when the employer assigns all of the match surplus to poorly matched workers *ex post*, in which case they quit exactly when it is efficient to do so. Such a surplus division is optimal from the viewpoint of employers, because they are able to maximize surplus extraction when workers are well matched *ex post*. Here, it is worth stressing the role of directed search. For example, if search were undirected, a firm may offer a starting wage equal to the flow value of unemployment (b) and then wait to match the worker's outside offers later. By contrast, directed search induces $w_l > b$ precisely because the wage $w_l = b$ is suboptimal for an unemployed applicant's tradeoff between the wage and the meeting probability.

The revealing equilibrium is such that (w_e, q_e) is the unique pair (w', q') that satisfies the usual zero profit and matching efficiency conditions in the market for employed workers. That is, the expected value of a vacancy to potential poachers equals the cost of posting the vacancy, which implies that employers are willing to offer higher wages and suffer reductions in the net present value of their profits only if they expect to fill their vacancies at a faster rate. The matching-efficiency condition implies that the ratio of the worker's surplus to the firm's surplus in new matches equals the ratio of their matching elasticities.

Similarly, we show in the Appendix that $\left(w_u^h, q_u\right)$ is the unique pair (w, q) that satisfies the usual zero profit and matching efficiency conditions in the market for unemployed workers, together with the Bellman equation in Problem (P2). It should be noted that the latter is completely standard because of the result that, in equilibrium, firms earn no profit from low productivity workers. Consequently, the firm's match surplus is entirely a function of the profits it makes when employing high productivity workers. Since these workers cannot profit from on-the-job search in a revealing equilibrium, employers have no incentive to set wages in order manipulate their quit rates.

The existence of an efficient equilibrium with positive job quits rests on the fact that the costs of creating entry jobs are sufficiently small that unemployed workers are willing to accept good matches. Proposition 1 above provides sufficient conditions both for the efficient allocation to exhibit positive job quits and for the value of entry jobs to unemployed workers to be higher when match productivity is high rather than low. However, if the cost of creating entry jobs is sufficiently high, then there may be no separating contract that implements the efficient allocation.

Proposition 2 *For each $\alpha \in (0,1)$, there is a number $\beta \in (0, y_l)$ such that for any (b, k_u, k_e) with $b \in (\beta, y_l)$, $k_u \in \left[\alpha \frac{y_h - y_l}{r+\delta}, \alpha \frac{y_h - y_l}{r+\delta} + \frac{y_l - b}{r+\delta}\right]$ and $k_e \in \left(0, \alpha \frac{y_h - y_l}{r+\delta}\right)$: (i) the efficient allocation exhibits positive job quits and (ii) there is no refined equilibrium that supports the efficient allocation.*

The reason why the constrained-efficient equilibrium can break down when the costs of creating entry jobs are sufficiently high is as follows. Efficient on-the-job search requires that workers in low productivity matches fully internalize the effect of their search decisions on the joint surplus of the match. This will be the case only when those workers enjoy the full surplus of the match. Consequently, the corresponding equilibrium separating contract must be such that the cost of job creation is not shared across types of matches. Instead, the employer must recover the entire cost of job creation by extracting surplus from high productivity matches. But this may lower the value of employment in good matches below the value of unemployment when the costs of creating entry jobs are sufficiently high.

The key problem is that contracts cannot be made contingent on tenure as well as match productivity and outside offers. Otherwise, one can verify that the efficient allocation with positive job flows can always be supported as an equilibrium. This is because an efficient allocation with positive job flows is such that the expected value of entry jobs exceeds the expected costs of job creation. However, if contracts can only be made contingent on match productivity and outside offers, entry jobs are subject to a holdup problem. The reason is that unemployed workers are able to credibly reject any offer associated with good matches whose value does not exceed the value of unemployment. If the cost of creating entry jobs is sufficiently high, employers will not be able to solve the holdup problem without distorting worker mobility across matches. Then, there is a trade-off between the costs associated with inefficient job quits and the costs associated with the holdup problem.

In principle, there may be a refined equilibrium that is revealing and is such that $w_l < y_l$ and $V(w_h, y_h) = V(s_u)$. The equilibrium allocation would solve problems (P1) and (P2), with $\rho = 1$, subject to $V(w_h, y_h) \geq V(s_u)$. It would solve the holdup problem, but job quits would be inefficient. Next, however, we show that the competitive search equilibrium may take the form of a non-revealing equilibrium, which also solves the holdup problem and distorts job quits. Below we argue that the existence of the non-revealing equilibrium is particularly interesting, because it sheds new light on the role of both wage posting and counteroffers.

3.2 Non-revealing equilibrium

In this section we provide conditions under which there exists a refined equilibrium such that pooling contracts, which do not reveal match productivity, are posted in the market for entry jobs. We refer to this kind of equilibria as non-revealing.[12]

In the non-revealing equilibrium the wage distribution has two mass points. Since wages do not differ across productivity realizations, all entry jobs pay an identical wage. In principle, there could be two wages in the on-the-job search market, as poorly matched workers accept poaching offers whereas well matched workers transition to retention wages. However, since both types of workers have the same current wage, their incentives to search on the job are identical, so the equilibrium poaching and retention wages are also identical.

Equilibrium transitions are as follows: All job offers made to unemployed workers are accepted, and all workers employed in entry jobs search on the job, with workers who are well matched *ex post* mimicking the on-the-job search behavior of workers who are poorly matched. As a result of pooling, all workers searching on the job face the same matching probabilities. Workers with low productivity realizations in an entry job change jobs upon meeting another employer with which they form a high productivity match. Workers with high productivity realizations in an entry job receive retention offers upon meeting another employer with which they form a high productivity match. Jobs are destroyed both exogenously (at rate δ) and, in the case of workers in low productivity matches, endogenously by quits.

Consider the search problem of a worker who is currently employed in an entry-level job earning a wage w and searching for a non-entry job. It is easy to verify that an interior solution of Problem (P1), (w', q'), satisfies the familiar zero-profit and matching efficiency conditions in the market for employed workers. The latter condition is identical to the corresponding matching efficiency condition in the revealing equilibrium, though the entry wage (w) is generally different. Furthermore, in a non-revealing equilibrium, potential poachers need to anticipate that a fraction $1 - \alpha$ of their pool of applicants are poorly matched in their current jobs, and so a fraction α will turn down their job offers because they are only searching to elicit a retention offer.

Recall that solving Problem (P2) is non-trivial because the objective function is not generally concave in (w, q). This problem can be addressed by viewing the solution to Problem (P1) as a mapping from the workers' quit rates to their entry wages, rather than the reverse, and then treat current and future quit rates as the relevant choice variables in Problem (P2). We followed this approach in the proof of Proposition 1 to characterize the equilibrium allocation in the constrained efficient equilibrium. In the Appendix, we show that this approach can be followed more generally to characterize the allocation in the non-revealing equilibrium and prove Proposition 3 below.

[12] Examples of pooling equilibria are found in Shi (2002) and Shimer (2005) in the context of labor markets and Chang (2018) and Guerrieri and Shimer (2014) in the context of asset markets.

Observe that employers understand i) that all workers with entry jobs search on the job, and ii) that job finding probabilities in the market for non-entry jobs depend on the wages earned by workers in entry jobs. Employers take this effect into account and set current wages, in part, in order to influence future quit rates. In an interior non-revealing equilibrium the match surplus is not maximized, except in the special case where the first-order conditions hold at the corner, so entry wages are such that $w = y_l$. The problem is that while employers can lower the workers' future quit rates by raising the entry wages they offer in the first place, they also have an impact on the outside offers the workers will get, because workers with higher wages have an incentive to elicit higher outside offers. Since well matched workers cannot be prevented from seeking outside offers, the allocation of surplus at the margin is allocated disproportionately to the worker, and employers do not typically have an incentive to raise entry wages all the way to y_l.

The following proposition provides sufficient conditions for the existence of a refined equilibrium that is non-revealing.

Proposition 3 *There is a number $\alpha_0 \in (0,1)$ such that for each $\alpha \in (\alpha_0, 1)$ there are numbers $\beta_0 \in (0, y_l)$ and $\kappa_0 \in \left(0, (1-\alpha)\,\alpha\frac{y_h - y_l}{r+\delta}\right)$ such that there exists a refined, non-revealing equilibrium with positive job quits, for all (b, k_u, k_e) such that $b \in (\beta_0, y_l)$, $k_u \in \left[\alpha\frac{y_h-y_l}{r+\delta}, \alpha\frac{y_h-b}{r+\delta}\right)$ and $k_e \in (0, \kappa_0)$. It is the unique refined equilibrium within the class of non-revealing equilibria. The equilibrium allocation solves problems (P1) and (P2) with $\rho = 1-\alpha$.*

The proof of the proposition proceeds as follows. First, we provide sufficient conditions under which there is a unique interior solution to problems (P1) and (P2) with $\rho = 1 - \alpha$. This solution maximizes the value of unemployment subject to non-negative profits, under the assumption that entry jobs are restricted to those offering pooling contracts. Specifically, we show that there is a number $\kappa_0 \in \left(0, (1-\alpha)\,\alpha\frac{y_h-y_l}{r+\delta}\right)$ that ensures that the relevant wage in the market for non-entry jobs is such that $w_e^* > y_l$ for all $k_e \in (0, \kappa_0)$. Hence, the solution to Problem (P1) exhibits positive quits for all $k_e \in (0, \kappa_0)$. Furthermore, we show that there is a unique pooling contract that solves Problem (P2) with $\rho = 1-\alpha$, for all $k_u \in \left[\alpha\frac{y_h-y_l}{r+\delta}, \alpha\frac{y_h-b}{r+\delta}\right)$ and that the posted wage is such that $w_u^* < y_l$. Intuitively, the costs of creating entry jobs is sufficiently high that the optimal pooling contract within the class of pooling contracts is such that the costs of job creation are optimally shared between good and bad matches, that is, $w_u^* < y_l$.

The second part of the proof provides conditions under which the above non-revealing allocation can be supported by a refined equilibrium. We first consider deviations in the market for non-entry jobs, taking as given that entry jobs are characterized by pooling contracts. It should be clear that the allocation that solves Problem (P1) with $\rho = 1 - \alpha$ is the only one that is consistent with a refined equilibrium that exhibits pooling contracts in the market for entry jobs. This is because it is the unique allocation that maximizes the value of search on the job subject to potential employers making non-negative profits and having reasonable off-equilibrium beliefs in the sense of our refinement. In particular, note that the beliefs of a deviating firm in the market for non-entry jobs must be consistent with the fact that workers hired under the same pooling contract not only are observationally equivalent, but they also have identical incentives, so they cannot be separated.

Hence, the relevant issue here is whether a pooling contract in the market for entry jobs can ever arise along the equilibrium path that is consistent with a refined equilibrium. To address this issue, consider a deviation in the market for entry jobs. Clearly, the pooling contract that solves Problem (P2) with $\rho = 1-\alpha$ cannot be dominated by any other pooling contract. If the deviating contract is a separating contract, then there are several cases to consider. We begin by considering a deviating contract $\left(w_l^d, w_h^d\right)$ such that $w_h^d \neq w_l^d = y_l$. This is the separating contract that maximizes the joint surplus of the match, since it is such that workers in bad matches enjoy the full surplus of the match and so they make bilaterally efficient job search and quit decisions. Formally, we show that for each $\alpha \in (0,1)$ there is a number $\beta_0 \in (0, y_l)$ such that the deviating entry job will attract no unemployed workers for all $b \in (\beta_0, y_l)$ and all $k_u \geq \alpha \frac{y_h - y_l}{r+\delta}$. The logic is the same as the one underlying the breakdown of an efficient revealing equilibrium. When the cost of job creation is sufficiently high, then it is possible that the value of a high productivity match to a worker is dominated by the value of unemployment. When that happens, unemployed workers reject high productivity matches unless the costs of job creation are shared across types of matches, which is inefficient.

Now consider a deviating contract $\left(w_l^d, w_h^d\right)$ such that $w_h^d \neq w_l^d \neq y_l$, so the costs of job creation are shared across match realizations. There are two cases to consider. The deviating contract may be such that $V\left(\left(w_h^d, y_h\right)\right) < V\left(s_u\right)$, in which case workers who are attracted to this contract will reject the job when the match productivity realization is high. Alternatively, if $V\left(\left(w_h^d, y_h\right)\right) = V\left(s_u\right)$, then workers who meet a deviating contract will accept the job for all match realizations, but they will be indifferent between accepting a job offer associated with a high productivity match and staying unemployed. In either case, however, the surplus associated with a deviating contract becomes negligible (even negative) as the probability of a high productivity match approaches one. Note that this is indeed the case for reasonable off-equilibrium beliefs. By contrast, we show that the surplus associated with the pooling contract that maximizes the value of unemployment subject to non-negative profits remains bounded away from zero, provided that the cost of creating non-entry jobs, k_e, is sufficiently close to zero. The latter condition is needed because otherwise the gains from trade in the market for non-entry jobs necessarily vanish as the probability of a high productivity match approaches one. Formally, we show that there is a number $\alpha_0 \in (0,1)$ such that for each $\alpha \in (\alpha_0, 1)$ there are numbers $\beta_0 \in (0, y_l)$ and $\kappa_0 \in \left(0, (1-\alpha)\,\alpha \frac{y_h - y_l}{r+\delta}\right)$ such that the most profitable pooling contract dominates all other feasible contracts, for all (b, k_u, k_e) such that $b \in (\beta_0, y_l)$, $k_e \in (0, \kappa_0)$ and $k_u \in \left[\alpha \frac{y_h - y_l}{r+\delta}, \alpha \frac{y_h - b}{r+\delta}\right)$.

The equilibrium characterized in Proposition 3 is the unique refined equilibrium within the class of non-revealing equilibria. However, the conditions given in the proposition are not sufficient to rule out the existence of a revealing equilibrium. To understand why, first note the existence of an informational externality: employers in the market for unemployed workers do not take into account the value of the informational content of their wages to future employers. Then, note the existence of a second externality: future employers do not internalize their effect on the outside option of workers hired in previous periods. Guerrieri (2008) was the first to show that competitive search fails to internalize the latter externality. Here, the interaction of the two externalities generates dynamic strategic complementarities, because employers have no direct incentive to post revealing wages. This means that non-revealing wages can be equilibrium wages as long as unemployed workers choose to search for non-revealing contracts when revealing contracts are feasible. This occurs because the option to search on the job constitutes an important component of the value of a job, but the value of this option depends on the beliefs of both workers and potential poaching firms. It is interesting to note that the above externalities are across submarkets, not across matches.

The presence of strategic complementarities is not a sufficient condition for multiple equilibria.[13] However, it helps to understand the self-fulfilling nature of the non-revealing equilibrium. Recruiting firms expect that it is likely that an applicant is already employed in a good match and, hence, will not accept the offer. This reduces the job-filling probability and the expected profit of recruiting, which induces firms to offer low (and non-revealing) wages. Such low wages induce employed workers in good jobs to search on the job only to obtain offers for the incumbent employers to match, thus tying the self-fulfilling loop.

4 Discussion and conclusion

The central message of this paper is that non-revealing wages and counteroffers can be understood as complementary features of a second-best market solution to a holdup problem that is associated with revealing wages when there is limited commitment. Holdup problems can arise when employers incur costs to create entry jobs but must share the surplus of the worker-employer match with the worker. The holdups we emphasize in this paper are associated with the impact of match-specific risk on (future) worker mobility. Bilaterally efficient worker mobility requires that workers who are expected to leave their current job enjoy the full surplus of the worker-employer match. This implies that the costs of creating entry jobs (e.g., capital costs, job training costs) must fall disproportionately on those workers whom employers expect to retain, that is, those employed in good matches. If these costs are sufficiently large, then employers must share the costs of creating entry jobs across types of matches, because if they do not, unemployed workers will refuse to participate in good matches. This solves the holdup problem, but necessarily distorts turnover.

Our analysis illustrates the potential for inefficient turnover when wages must both direct search and signal worker mobility. It also provides a novel perspective on the role of both posted wages and counteroffers. Posted wages, which are characterized as pooling wage contracts, create adverse selection in the market for non-entry jobs. The problem is that, under non-revealing wages, well matched workers cannot be identified and, therefore, have an incentive to search on the job in order to elicit retention offers from their current employers. Thus, non-revealing wages increase the value of on-the-job search to well matched workers, relative to the case where wages reveal match quality. The overall effect of this adverse selection problem is to depress the returns to poaching firms, which reduces the entry of poachers and, therefore, depresses the returns to on-the-job search. This

[13] See Gale (1995) for an analysis of basic dynamic coordination games.

reduction is concentrated on poorly matched workers.

Commitment to a bilaterally efficient long-term contract would allow the firms to share job creation costs across types of matches without distorting wage mobility. This could be done by reducing the wages workers earn in low productivity matches during the first period and then paying them their marginal product so their on-the-job search decisions maximize the joint surplus of the match. In this context, internships might be viewed as one mechanism to backload worker compensation. Employer-provided pension plans can also be understood as a solution to a version of this problem. More generally, mechanisms that allow the firm to credibly promise future increases in worker compensation provide a potential solution to holdup problems in the market of entry jobs. This view is different from, but complementary with, the common view that backloading compensation via increasing wage-tenure profiles and pension plans is a mechanism to retain workers.

In our setting, non-revealing equilibria are the only refined equilibria in which employers counter outside offers along the equilibrium path. Accordingly, they are the only refined equilibria that exhibit wage mobility both within and across employers. Between-employer wage mobility is associated with productivity increases. By contrast, within-employer wage mobility is driven by retention offers, without the need for productivity increases, employer learning, or new public information. To the extent that counteroffers are not uncommon in many markets, disregarding them leads to overestimating the contribution of, for example, employer learning to wage growth. In our model, counteroffers are associated with informational problems, so they only arise as part of inefficient equilibria. However, they also have a productive role. They allow for an equilibrium with worker mobility when job creation costs are sufficiently high that the efficient equilibrium breaks down.

The existence of non-revealing equilibria depends on whether the costs of creating entry jobs are large *relative* to the expected surplus of good versus bad matches, as reflected in the term $\frac{\alpha(y_h - y_l)}{r+\delta}$. Thus, all else equal, revealing equilibria are more likely when the costs of creating entry jobs are larger, or when the dispersion in match productivities is larger. One implication is that counteroffers are not necessarily restricted to high-paying jobs. Our analysis also suggests that younger workers — who are more likely to be employed in entry-level jobs[14] — and workers at smaller establishments — where employers may be unable to commit to bilaterally efficient wage contracts — are likely to be paid lower wages but are more likely to receive counteroffers. Similarly, jobs with higher up front creation costs, such as capital intensive jobs, are more likely to suffer from the holdup problem we identify. This is consistent with available evidence on counteroffers (Barron et al., 2006) and on performance pay (MacLeod and Parent, 1999).

Our results imply that posted wages and counteroffers can be understood as resulting from

[14] In October 2016, the U.S. Department of Justice and the Federal Trade Commission issued joint antitrust guidance warning HR professionals that no-poaching and wage-fixing agreements violate federal law. In July 2018, Washington Attorney General Bob Ferguson announced that seven chains — including Arby's, Carl's Jr. and McDonald's — had agreed to no longer enforce no-poaching agreements. In March 2020, Attorney General Josh Stein announced a multi-state settlement with Burger King, Popeye's, and Tim Horton's over no-poaching agreements. The fact that no-poaching clauses preventing managers from hiring employees that have worked elsewhere in the same chain have been the norm in the U.S. fast-food industry until recently is suggestive of the relevance of counteroffers in these markets.

a common commitment failure. In particular, the inability of firms to commit to not making counteroffers is central to the existence of the non-revealing equilibrium. If firms can commit to not matching outside offers, then recruiting firms know that they will be able to attract an employed applicant by posting a higher wage. But then the holdup and adverse selection problems both disappear.

From the perspective of our model, the lack of counteroffers in some markets is not the result of firms' ability to commit to not making counteroffers. Rather, it is a symptom of efficient turnover, which is in fact supported by the inability of firms to commit to not making counteroffers. The reason why counteroffers are not observed is that they serve no purpose along the equilibrium path. The insight here is that the *threat* of retention offers precludes on-the-job search for workers in entry jobs with high productivity. This allows firms to recover the costs of creating entry jobs exclusively from high productivity matches, which is what allows for the efficient turnover of workers in entry jobs with low productivity.

Appendix

Proof of Proposition 1

We first characterize the solution to problems (P1) and (P2) with $\rho = 1$. Then we characterize the efficient steady-state allocation with positive job quits and provide conditions under which it can be supported by a refined equilibrium. Finally, we provide conditions under which the efficient equilibrium is the unique refined equilibrium.

We begin by characterizing the solution to Problem (P1) as a function of a worker's wage.

Lemma 1 *Let $k_e \in \left(0, \alpha \frac{y_h - y_l}{r+\delta}\right)$. For any $w \in [0, y_l]$, $(w_e(s), q_e(s))$, for $s = (w, y_l)$, is given by the unique pair (w', q') with $y_l < w' < y_h$ and $0 < q_a < q' \leq q_b < \infty$ that solves the following conditions:*

$$q' f(q') \alpha \left(\frac{y_h - w'}{r + \delta}\right) = k_e,$$

$$\frac{\frac{y_h - w'}{r+\delta}}{\frac{y_h - w'}{r+\delta} + \frac{w' - w}{r+\delta+(1-\delta)\alpha f(q')}} \leq \eta(q')$$

and $q' \leq q_b$ with complementary slackness, where q_a is given by

$$q_a f(q_a) \alpha \left(\frac{y_h - y_l}{r + \delta}\right) = k_e$$

and $q_b > q_a$ is given by

$$\frac{y_h - y_l}{r + \delta} = \left(\frac{k_e}{q_b f(q_b) \alpha}\right) \left(1 + \left(\frac{1 - \eta(q_b)}{\eta(q_b)}\right) \left(\frac{r + \delta + (1 - \delta) \alpha f(q_b)}{r + \delta}\right)\right). \tag{6}$$

Proof: The first-order conditions for an interior solution of problem (P1) with $\rho = 1$ are given by

$$\lambda q' = 1,$$

where λ is the relevant Lagrange multiplier, and

$$\frac{w'}{r + \delta} + \left(\frac{\delta}{r + \delta}\right) \frac{V(s_u)}{1 + r} - \frac{V((w, y_l))}{1 + r} = \lambda q' \left(\frac{1 - \eta(q')}{\eta(q')}\right) \left(\frac{y_h - w'}{r + \delta}\right),$$

together with the zero-profit constraint

$$q' f(q') \alpha \left(\frac{y_h - w'}{r + \delta}\right) = k_e.$$

This is the first condition stated in the lemma. The second condition follows from combining the first two first-order conditions above and the fact that the Bellman equation implies that a solution to the problem must be such that

$$\frac{w'}{r+\delta}+\left(\frac{\delta}{r+\delta}\right)\frac{V\left(s_{u}\right)}{1+r}-\frac{V\left(\left(w,y_{l}\right)\right)}{1+r}=\frac{w'-w}{r+\delta+\left(1-\delta\right)\alpha f\left(q'\right)}.$$

Clearly, $w_e\left(\left(w,y_l\right)\right)>y_l$ if and only if $q_e\left(\left(w,y_l\right)\right)>q_a$. The assumption in the lemma ensures that $0<q_a<\infty$.

Combining the two conditions stated in the proposition implies that an interior solution $q_e\left(\left(w,y_l\right)\right)$ is the unique value of q' that solves

$$\frac{y_h-w}{r+\delta}=\left(\frac{k_e}{q'f\left(q'\right)\alpha}\right)\left(1+\left(\frac{1-\eta\left(q'\right)}{\eta\left(q'\right)}\right)\left(\frac{r+\delta+\left(1-\delta\right)\alpha f\left(q'\right)}{r+\delta}\right)\right). \tag{7}$$

It follows that $w\leq y_l$ implies that $q_e\left(\left(w,y_l\right)\right)\leq q_b$. Clearly, $\infty>q_b>q_a>0$. **QED**

Invert (7) to express the worker's current wage as a function of q':

$$W\left(q'\right)\equiv y_h-\left(\frac{k_e}{q'f\left(q'\right)\alpha}\right)\left(r+\delta+\left(\frac{1-\eta\left(q'\right)}{\eta\left(q'\right)}\right)\left(r+\delta+\left(1-\delta\right)\alpha f\left(q'\right)\right)\right), \tag{8}$$

for all $q'\in\left(q_a,q_b\right]$, and note the following.

Lemma 2 *$W\left(q'\right)$ and $V\left(\left(W\left(q'\right),y_l\right)\right)$ are strictly increasing and concave functions of q' on $\left(q_a,q_b\right]$.*

Proof: It is easy to verify that the Bellman equation for $V\left(\left(w,y_l\right)\right)$ implies that

$$\begin{aligned}\frac{V\left(\left(W\left(q'\right),y_{l}\right)\right)}{1+r}=&\left(\frac{\delta}{r+\delta}\right)\frac{V\left(s_{u}\right)}{1+r}+\left(\frac{r+\delta}{r+\delta+\left(1-\delta\right)\alpha f\left(q'\right)}\right)\frac{W\left(q'\right)}{r+\delta}\\&+\left(1-\frac{r+\delta}{r+\delta+\left(1-\delta\right)\alpha f\left(q'\right)}\right)\frac{w_{e}\left(\left(W\left(q'\right),y_{l}\right)\right)}{r+\delta}\end{aligned} \tag{9}$$

and, using the first-order conditions stated in Lemma 1, one can write

$$\frac{V\left(\left(W\left(q'\right),y_{l}\right)\right)}{1+r}=\frac{y_h}{r+\delta}+\left(\frac{\delta}{r+\delta}\right)\frac{V\left(s_{u}\right)}{1+r}-\frac{k_e}{\eta\left(q'\right)q'f\left(q'\right)\alpha}. \tag{10}$$

One can verify that

$$\frac{\partial}{\partial q}\left(\frac{V\left(\left(W\left(q'\right),y_{l}\right)\right)}{1+r}\right)=\frac{k_e}{q'f\left(q'\right)\alpha}\left(\frac{\eta'\left(q'\right)}{\left(\eta\left(q'\right)\right)^{2}}+\frac{1}{q'}\left(\frac{1-\eta\left(q'\right)}{\eta\left(q'\right)}\right)\right),$$

which is positive on $\left(q_a,q_b\right]$. A sufficient condition for it to be strictly decreasing on $\left(q_a,q_b\right]$ is that $\eta'\left(q'\right)/\left(\eta\left(q'\right)\right)^2$ is a decreasing function, which follows from the concavity of η. Hence $V\left(\left(W\left(q'\right),y_l\right)\right)$ is strictly concave on $\left(q_a,q_b\right]$, as required.

Next, differentiating equation (7) with respect to w and q' one can verify that

$$\frac{\partial W\left(q'\right)}{\partial q'}=\left(r+\delta+\left(1-\delta\right)\alpha f\left(q'\right)\right)\frac{\partial}{\partial q'}\left(\frac{V\left(\left(W\left(q'\right),y_{l}\right)\right)}{1+r}\right),$$

which is positive and strictly decreasing on $(q_a, q_b]$, because both f and $\partial V/\partial q'$ are positive and strictly decreasing on $(q_a, q_b]$. Hence, W is strictly increasing and concave *on* $(q_a, q_b]$. **QED**

Let $M(s)$ denote the match surplus as a function of the worker's state and note that

$$\frac{M((w, y_h))}{1+r} = \frac{V((w, y_h))}{1+r} - \frac{V(s_u)}{1+r} + \frac{y_h - w}{r+\delta} \tag{11}$$

and

$$\frac{M((W(q'), y_l))}{1+r} = \frac{V((W(q'), y_l))}{1+r} - \frac{V(s_u)}{1+r} + \frac{y_l - W(q')}{r+\delta+(1-\delta)\alpha f(q')}. \tag{12}$$

Lemma 3 *$M((w, y_h))$ is independent of w; $M((W(q'), y_l))$ is a strictly concave function of q' on $(q_a, q_b]$ and it is maximized at $q' = q_b$; $M((W(q'), y_l)) - V((W(q'), y_l))$ is a strictly decreasing and convex function of q' on $(q_a, q_b]$.*

Proof: Fix $V(s_u)$. Noting that

$$\frac{V((w, y_h))}{1+r} = \frac{w}{r+\delta} + \frac{\delta}{r+\delta}\frac{V(s_u)}{1+r}$$

one can write

$$\frac{M((w, y_h))}{1+r} = \frac{y_h}{r+\delta} - \frac{r}{r+\delta}\frac{V(s_u)}{1+r},$$

which is independent of q'. Using (10), together with (8) and (12), one can write

$$\begin{aligned}\frac{M((W(q'), y_l))}{1+r} &= \frac{y_h}{r+\delta} - \frac{r}{r+\delta}\frac{V(s_u)}{1+r} - \frac{r+\delta}{r+\delta+(1-\delta)\alpha f(q')}\left(\frac{y_h - y_l}{r+\delta}\right) \\ &\quad - \left(1 - \frac{r+\delta}{r+\delta+(1-\delta)\alpha f(q')}\right)\left(\frac{k_e}{q' f(q')\alpha}\right).\end{aligned} \tag{13}$$

where $M((w, y_h)) > M((W(q'), y_l))$ whenever $y_h > y_l$. Differentiating equation (13) one can verify that

$$\begin{aligned}\frac{\partial}{\partial q'}\left(\frac{M((W(q'), y_l))}{1+r}\right) &= \left(\frac{1-\delta}{(q')^2\left[r+\delta+(1-\delta)\alpha f(q')\right]}\right) \\ &\quad \times \left(\left(1-\eta(q')\right)k_e - \left(\frac{(r+\delta)\eta(q')}{r+\delta+(1-\delta)\alpha f(q')}\right)\left(q' f(q')\alpha\left(\frac{y_h - y_l}{r+\delta}\right) - k_e\right)\right).\end{aligned}$$

The term in the first line is decreasing in q' since $q' f(q')$ is strictly increasing on $(q_a, q_b]$. The terms in the second line are also decreasing in q', since f is decreasing, η is increasing and $q' f(q')$ is increasing on $(q_a, q_b]$, and $q' f(q')\alpha(y_h - y_l) \geq (r+\delta)k_e$ for $q' > q_a$. Hence $M((W(q'), y_l))$ is strictly concave on $(q_a, q_b]$. It is now easy to verify that equation (6) is a necessary and sufficient condition for $\partial M\left(\left(W\left(q'\right), y_l\right)\right)/\partial q' = 0$. Hence $M\left(\left(W\left(q'\right), y_l\right)\right)$ is maximized at $q' = q_b$.

Using equations (10) and (13) one can write

$$\frac{M(s) - \Big(V(s) - V(s_u)\Big)}{1+r} = \left(\frac{k_e}{q' f(q') \alpha}\right)\left(\frac{r+\delta}{r+\delta+(1-\delta)\alpha f(q')} + \frac{1-\eta(q')}{\eta(q')}\right) - \left(\frac{y_h - y_l}{r+\delta+(1-\delta)\alpha f(q')}\right),$$

for $s = \left(W\left(q'\right), y_l\right)$, and differentiating this equation one can verify that

$$\frac{\partial}{\partial q}\left(\frac{M(s) - \Big(V(s) - V(s_u)\Big)}{1+r}\right) = \left(\frac{(1-\delta)\alpha f'(q')}{\left[r+\delta+(1-\delta)\alpha f(q')\right]^2}\right)\left(y_h - y_l - \frac{(r+\delta)k_e}{q' f(q')\alpha}\right) - \left(\frac{k_e}{q' f(q')\alpha}\right)\left(\left(\frac{1-\eta(q')}{q'}\right)\left(\frac{r+\delta}{r+\delta+(1-\delta)\alpha f(q')} + \frac{1-\eta(q')}{\eta(q')}\right) + \frac{\eta'(q')}{\left(\eta(q')\right)^2}\right),$$

for $s = \left(W(q), y_l\right)$. The term in the first line of the right side is negative since $f' < 0$ and $q' f(q') \alpha (y_h - y_l) \geq (r+\delta) k_e$ for $q > q_a$. The term subtracted in the second line is positive since $\eta(q) < 1$ and $\eta' > 0$. Hence $M(s) - (V(s) - V(s_u))$, for $s = \left(W\left(q'\right), y_l\right)$, is a strictly decreasing function of q' on $(q_a, q_b]$. Moreover, the term in the first line of the right side is an increasing function of q', because f' is increasing, $q' f(q')$ is increasing and f is decreasing. The term subtracted in the second line is a decreasing function of q', since $q' f(q')$ and η are increasing and f and $\eta'(q')/(\eta(q'))^2$ are decreasing. Hence, $M(s) - (V(s) - V(s_u))$, for $s = \left(W\left(q'\right), y_l\right)$, is a strictly convex function of q'. **QED**

Consider a candidate revealing equilibrium. Note that Problem (P2) can be formulated as

$$V(s_u) = b + \frac{1}{1+r}\left[V(s_u) + \max_{w,q,q'} f(q)\Big(V_0\left(w, q'\right) - V(s_u)\Big)\right], \tag{P3}$$

subject to

$$k_u \leq q f(q)\left(\frac{M_0(w,q') - \Big(V_0(w,q') - V(s_u)\Big)}{1+r}\right),$$

$$q' \in (q_a, q_b]\ , w \leq y_h,\ w \neq W\left(q'\right)$$

where

$$V_0\left(w, q'\right) = \alpha V\left((w, y_h)\right) + (1-\alpha) V\left(\left(W\left(q'\right), y_l\right)\right),$$

and

$$M_0\left(w,q'\right)=\alpha M\left(\left(w,y_h\right)\right)+\left(1-\alpha\right)M\left(\left(W\left(q'\right),y_l\right)\right).$$

With a slight abuse of notation, we let $\left(w_u^h,q_u,q_e^l\right)$ denote a solution to Problem (P3) while disregarding the constraint $w\neq W\left(q'\right)$. Even though the objective is not concave in (w,q,q'), we prove below that the solution is unique (and it is such that $V(s_u)<\min\left\{V\left(\left(W\left(q'\right),y_l\right)\right),V\left(w,y_h\right)\right\}$ and $w_u^h\neq W\left(q_e^l\right)$). It is then easy to see that $\left(w_u^h,W\left(q_e^l\right),q_u\right)$ solves problem (P2), since $q_e^l=q_e\left(\left(W\left(q_e^l\right),y_l\right)\right)$.

One can readily verify that an *interior* solution to Problem (P3) is such that the total surplus of the match is maximized. Specifically, it must be that $\partial M_0\left(w,q'\right)/\partial q'=0$, which requires that $\partial M\left(\left(W\left(q'\right),y_l\right)\right)/\partial q'=0$. Hence, Lemma 3 implies that $q_e^l=q_b$, thus

$$q_e\left(\left(W\left(q_b\right),y_l\right)\right)=q_b, \tag{14}$$

and Lemma 1 then implies that

$$q_b f\left(q_b\right)\alpha\left(\frac{y_h-w_e\left(\left(W\left(q_b\right),y_l\right)\right)}{r+\delta}\right)=k_e, \tag{15}$$

where q_b is given by equation (6). Comparing (6) and (7), it follows that $W\left(q_e^l\right)=y_l$, thus

$$w_u^l=y_l. \tag{16}$$

Next note that $\left(w_u^h,q_u\right)$ is given by any pair (w,q) that satisfies the zero-profit condition

$$qf(q)\alpha\left(\frac{y_h-w}{r+\delta}\right)=k_u \tag{17}$$

and the standard condition for matching efficiency in the market for unemployed workers, given by

$$\frac{\alpha\left(\frac{y_h-w}{r+\delta}\right)}{\alpha\left(\frac{y_h-w}{r+\delta}\right)+\left(\alpha\frac{V\left(\left(w,y_h\right)\right)}{1+r}+\left(1-\alpha\right)\frac{V\left(\left(y_l,y_l\right)\right)}{1+r}-\frac{V\left(s_u\right)}{1+r}\right)}=\eta\left(q\right) \tag{18}$$

together with the Bellman equation in Problem (P2).

Note that

$$\begin{aligned}V\left(s_u\right)&=b+\left(1-f\left(q_u\right)\right)\frac{V\left(s_u\right)}{1+r}+f\left(q_u\right)\frac{V_0\left(w_u^h,q_b\right)}{1+r}\\&=b+\left(1-f\left(q_u\right)\right)\frac{V\left(s_u\right)}{1+r}+f\left(q_u\right)\left(\frac{V\left(s_u\right)}{1+r}+\left(\frac{1-\eta\left(q_u\right)}{\eta\left(q_u\right)}\right)\frac{k_u}{q_u f\left(q_u\right)}\right),\end{aligned}$$

where the first equality comes from the Bellman equation in Problem (P3) and the second equality follows from (18) and (17). It follows that

$$\frac{rV(s_u)}{1+r} = b + \left(\frac{1-\eta(q_u)}{\eta(q_u)}\right)\frac{k_u}{q_u}. \tag{19}$$

Using this equation, together with equations (17) and (18) and the fact that

$$\frac{V_0\left(w_u^h, q_e^l\right)}{1+r} - \frac{V(s_u)}{1+r} = \alpha\frac{w_u^h}{r+\delta} + (1-\alpha)\left(\frac{y_h}{r+\delta} - \frac{k_e}{\eta(q_b)\, q_b f(q_b)\,\alpha}\right) - \left(\frac{r}{r+\delta}\right)\frac{V(s_u)}{1+r}, \tag{20}$$

it follows that q_u is given by a value of q that satisfies

$$\frac{y_h - b}{r+\delta} - \frac{(1-\alpha)\,k_e}{\eta(q_b)\, q_b f(q_b)\,\alpha} = \frac{k_u}{\eta(q)\, q f(q)} + \left(\frac{1-\eta(q)}{\eta(q)}\right)\frac{k_u}{(r+\delta)\, q}. \tag{21}$$

The right side of (21) is strictly decreasing in q, it converges to ∞ as q approaches 0 and it converges to k_u as q approaches ∞. Hence, there is a unique solution $q_u \in (0,\infty)$ that solves the equation if and only if

$$\frac{y_h - b}{r+\delta} - \frac{(1-\alpha)\,k_e}{\eta(q_b)\, q_b f(q_b)\,\alpha} > k_u.$$

In turn, this implies that $k_u < y_h/(r+\delta)$ and so there must be a number $w \in (0, y_h)$ that solves equation (17).

Next, we consider the constrained efficient allocation. We say an allocation is constrained efficient (efficient, for short) if it maximizes the present value of aggregate production net of search costs under full information. The following lemma characterizes the efficient allocation as the solution to a planning problem and provides necessary and sufficient conditions for an efficient allocation to have positive job quits.

Lemma 4 *There exists an efficient steady-state allocation with positive job quits if and only if (i)* $k_e < \alpha\left(\frac{y_h - y_l}{r+\delta}\right)$*, (ii)* $k_u < \frac{y_h - b}{r+\delta} - \frac{(1-\alpha)k_e}{\eta(q_b)q_b f(q_b)\alpha}$ *and (iii)* $\frac{k_u}{\eta(q_u)q_u f(q_u)} - \frac{k_e}{\eta(q_b)q_b f(q_b)} \geq 0$ *all hold, where* q_b *and* q_u *are the unique solutions to equations (6) and (21).*

Proof: First, note that the state of the economy at the beginning of each period can be summarized by $\{u, m\}$, where $u \in [0,1]$ is the measure of unemployed workers, and $m : \{y_l, y_h\} \to [0,1]$, where $m(y)$ denotes the measure of employed workers with match productivity y. Let $p(y)$ denote the probability with which a match has productivity realization y. Let $z_u(y)$ denote the probability with which a meeting between an unemployed worker and a job is turned into a match given the productivity realization y, and $z_e(y'|y)$ denote the probability with which a meeting between a worker and a job with productivity realization y' is turned into a match given that the worker is currently employed in a job with match productivity y. Finally, let q_u denote the labor market queue where unemployed workers search for jobs, and $q_e(y)$ denote the labor market queue where employed workers search given that they are currently employed in jobs with productivity y.

Aggregate output can be written as:

$$Y(u,m) = bu + \sum_y ym(y) - k_u \frac{u}{q_u} - (1-\delta)k_e \sum_y \frac{m(y)}{q_e(y)}. \tag{22}$$

Denote by $\widehat{u}$ the measure of unemployed workers one period ahead, and by $\widehat{m}(y)$ the measure of employed workers with match productivity y one period ahead. Then,

$$\widehat{u} = \left(1 - \sum_y f(q_u)z_u(y)\right) u + \delta \sum_y m(y) \tag{23}$$

and

$$\begin{aligned} \widehat{m}(y) &= p(y)f(q_u)z_u(y)u + (1-\delta)m(y)\left[1 - p(y')f(q_e(y))z_e(y'|y)\right] \\ &\quad + (1-\delta)m(y')p(y)f(q_e(y'))z_e(y|y'). \end{aligned} \tag{24}$$

The allocation that maximizes aggregate output net of search costs can be characterized as the solution to the planning problem:

$$J(u,m) = \max_{q_u,z_u,q_e,z_e} \left\{ Y(u,m) + \frac{J(\widehat{u},\widehat{m})}{1+r} \right\}, \tag{25}$$

subject to equations (22)–(24). $J(u,m)$ is the unique solution to the planner's problem and can be written as:

$$J(u,m) = J_u u + \sum_y m(y) J_e(y),$$

where

$$J_u = \max_{q_u,z_u} \left\{ b - \frac{k_u}{q_u} + \sum_y p(y)f(q_u)z_u(y)\frac{J_e(y)}{1+r} + \left(1 - \sum_y p(y)f(q_u)z_u(y)\right)\frac{J_u}{1+r} \right\} \tag{26}$$

and

$$\begin{aligned} J_e(y) = \max_{z_e,q_e} \Bigg\{ & y - (1-\delta)\frac{k_e}{q_e(y)} + \delta\frac{J_u}{1+r} \\ & + (1-\delta)\left(1 - \sum_{y'} p(y')f(q_e(y))z_e(y'|y)\right)\frac{J_e(y)}{1+r} \\ & + (1-\delta)\sum_{y'} p(y')f(q_e(y))z_e(y'|y)\frac{J_e(y')}{1+r} \Bigg\}. \end{aligned} \tag{27}$$

It is easy to verify that at the optimum $q_e(y_h) = \infty$. This implies:

$$J_e(y_h) = y_h + \delta \frac{J_u}{1+r} + (1-\delta)\frac{J_e(y_h)}{1+r} > J_e(y_l). \tag{28}$$

It is also easy to verify that $z_e(y_h|y_l) = 1$ and $z_e(y_l|y_l) \in [0,1]$ at the optimum. This means that the planner's problem has multiple solutions, all of which yield the same optimal value. The multiplicity concerns the probability with which the planner instructs workers to accept or reject lateral job moves. We characterize the solution when $z_e(y_l|y_l) = 0$.

The necessary condition of (27) with respect to $q_e(y_l)$ can be written:

$$\frac{J_e(y_h)}{1+r} - \frac{J_e(y_l)}{1+r} = \frac{k_e}{\alpha q_e(y_l) f(q_e(y_l))\eta(q_e(y_l))} \tag{29}$$

and the Bellman equation for $J_e(y_l)$ gives:

$$\frac{J_e(y_l)}{1+r} = \frac{1}{r+\delta+(1-\delta)f(q_e(y_l))\alpha}\left(y_l - y_h - (1-\delta)\frac{k_e}{q_e(y_l)}\right) + \frac{J_e(y_h)}{1+r}.$$

The above equations, along with the expression for $J_e(y_h)$, yield equation (6), which implies that $q_e(y_l) = q_b$. Lemma 1 implies that Condition (i) in this lemma is necessary and sufficient to ensure that there is a unique value of q_e that solves (6) with $q_e = q_b \in (q_a, \infty)$ and $q_a > 0$.

Conjecture that $z_u(y) = 1$ for $y = \{y_l, y_h\}$. The necessary condition of (26) with respect to q_u can be written:

$$\frac{r}{r+\delta}\frac{J_u}{1+r} = \frac{y_h}{r+\delta} - \frac{k_u}{q_u f(q_u)\eta(q_u)} - \frac{(1-\alpha)k_e}{\eta(q_b) q_b f(q_b)\alpha}. \tag{30}$$

From the Bellman equation for J_u:

$$\frac{rJ_u}{1+r} = b + \frac{k_u}{q_u}\left(\frac{1-\eta(q_u)}{\eta(q_u)}\right). \tag{31}$$

Combining these two equations yields equation (21), which characterizes the equilibrium value q_u. It follows from the arguments above that Condition (ii) in the lemma is necessary and sufficient for the existence of a unique value of q_u that solves (21), with $q_u \in (0, \infty)$.

To show that $z_u(y) = 1$ for $y = \{y_l, y_h\}$, combine equations (29) and (30) to obtain:

$$\frac{J_e(y_l)}{1+r} - \frac{J_u}{1+r} = \frac{k_u}{q_u f(q_u)\eta(q_u)} - \frac{k_e}{q_b f(q_b)\eta(q_b)}, \tag{32}$$

which is non-negative if and only if Condition (iii) in the lemma holds. Equation (29) then implies that Condition (iii) is also sufficient for $J_e(y_h) > J_u$. This concludes the proof of the lemma. **QED**

Lemma 5 *An efficient steady-state allocation with positive job quits can be supported by a refined equilibrium if and only if* $\frac{(1-\alpha)k_e}{\alpha q_b f(q_b)\eta(q_b)} + \frac{k_u}{q_u f(q_u)\eta(q_u)} \geq \frac{k_u}{\alpha q_u f(q_u)}$, *where* q_b *and* q_u *are the unique* *solutions to equations (6) and (21).*

Proof: Assume the conditions in Lemma 4 hold, so there exists an efficient steady-state allocation with positive job quits. Consider the solution to problems (P1) and (P2) with $\rho = 1$, as characterized above. Note that

$$V\left(\left(y_l, y_l\right)\right) - V\left(s_u\right) = V_0\left(w_u^h, q_b\right) - V\left(s_u\right) - \alpha\left[V\left(\left(w_u^h, y_h\right)\right) - V\left(\left(y_l, y_l\right)\right)\right]$$

and

$$V\left(\left(w_u^h, y_h\right)\right) - V\left(s_u\right) = V_0\left(w_u^h, q_b\right) - V\left(s_u\right) + (1-\alpha)\left[V\left(\left(w_u^h, y_h\right)\right) - V\left(\left(y_l, y_l\right)\right)\right],$$

where $V_0\left(w_u^h, q_b\right) = \alpha V\left(\left(w_u^h, y_h\right)\right) + (1-\alpha)\, V\left(\left(y_l, y_l\right)\right)$, and use the fact that

$$\frac{V_0\left(w_u^h, q_b\right)}{1+r} - \frac{V\left(s_u\right)}{1+r} = \left(\frac{1-\eta\left(q_u\right)}{\eta\left(q_u\right)}\right)\frac{k_u}{q_u f\left(q_u\right)}$$

and the fact that

$$\frac{V\left(\left(w_u^h, y_h\right)\right)}{1+r} - \frac{V\left(\left(y_l, y_l\right)\right)}{1+r} = \frac{k_e}{\eta\left(q_b\right) q_b f\left(q_b\right)\alpha} - \frac{k_u}{\alpha q_u f\left(q_u\right)} \tag{33}$$

to write

$$\frac{V\left(\left(y_l, y_l\right)\right)}{1+r} - \frac{V\left(s_u\right)}{1+r} = \frac{k_u}{\eta\left(q_u\right) q_u f\left(q_u\right)} - \frac{k_e}{\eta\left(q_b\right) q_b f\left(q_b\right)}. \tag{34}$$

Note that the right sides of (32) and (34) are identical and so

$$\frac{V\left(\left(y_l, y_l\right)\right)}{1+r} - \frac{V\left(s_u\right)}{1+r} = \frac{J_e(y_l)}{1+r} - \frac{J_u}{1+r}.$$

Using equations (29) and (32), it follows that

$$\frac{J_e(y_h)}{1+r} - \frac{J_u}{1+r} = \frac{(1-\alpha)\, k_e}{\alpha q_b f(q_b)\eta(q_b)} + \frac{k_u}{q_u f(q_u)\eta(q_u)} \tag{35}$$

and using (33) and (34) it follows that

$$\frac{V\left(\left(w_u^h, y_h\right)\right)}{1+r} - \frac{V\left(s_u\right)}{1+r} = \frac{J_e(y_h)}{1+r} - \frac{J_u}{1+r} - \frac{k_u}{\alpha q_u f\left(q_u\right)}. \tag{36}$$

Hence, unemployed workers will agree to form high productivity matches if and only if the condition stated in the lemma is satisfied. Since $J_e(y_h) > J_e(y_l)$ the same condition ensures that unemployed workers will agree to form low productivity matches.

The right sides of (31) and (19) are identical, so $V\left(s_u\right) = J_u$. It follows that the candidate equilibrium allocation maximizes the value of unemployment subject to firms making non-negative profits. Hence, there is no deviating contract that can attract workers and give the deviating firm non-negative profits, as required. **QED**

Differentiate equation (6) to verify that

$$\frac{\partial q_b}{\partial k_e} > 0 \text{ and } \frac{\partial}{\partial k_e}\left(\frac{k_e}{\eta\left(q_b\right) q_b f\left(q_b\right)}\right) > 0,$$

with

$$\frac{k_e}{\eta\left(q_b\right) q_b f\left(q_b\right)\alpha} \in \left(\frac{y_h - y_l}{r+\delta+(1-\delta)\alpha}, \frac{y_h - y_l}{r+\delta}\right), \tag{37}$$

for all $k_e < \alpha\left(\frac{y_h - y_l}{r+\delta}\right)$. It follows that

$$k_u \leq \frac{y_h - b}{r+\delta} - (1-\alpha)\left(\frac{y_h - y_l}{r+\delta}\right) \tag{38}$$

is sufficient for Condition (ii) in Lemma 4. Next, note that

$$\begin{aligned}
\frac{J_e(y_l)}{1+r} - \frac{J_u}{1+r} &= \frac{k_u}{\eta\left(q_u\right) q_u f\left(q_u\right)} - \frac{k_e}{\eta\left(q_b\right) q_b f\left(q_b\right)} \\
&> \frac{y_h - b}{1+r+\delta} - \frac{k_e}{\eta\left(q_b\right) q_b f\left(q_b\right)\alpha}\left(\frac{r+\delta}{1+r+\delta}\right) + \frac{\alpha k_e}{\eta\left(q_b\right) q_b f\left(q_b\right)\alpha}\left(\frac{r+\delta}{1+r+\delta} - 1\right) \\
&= \frac{y_h - b}{1+r+\delta} - \frac{k_e}{\eta\left(q_b\right) q_b f\left(q_b\right)\alpha}\left(\frac{\alpha+r+\delta}{1+r+\delta}\right) \\
&> \frac{1}{1+r+\delta}\left(y_h - b - (\alpha+r+\delta)\left(\frac{y_h - y_l}{r+\delta}\right)\right)
\end{aligned}$$

where the first inequality follows from the fact that the right side of (21) is increasing in k_u, with

$$\lim_{k_u \to 0} \frac{k_u}{\eta\left(q_u\right) q_u f\left(q_u\right)} = \left(\frac{y_h - b}{r+\delta} - \frac{(1-\alpha) k_e}{\eta\left(q_b\right) q_b f\left(q_b\right)\alpha}\right)\left(\frac{r+\delta}{1+r+\delta}\right),$$

and the last inequality follows from the fact that

$$\frac{k_e}{\eta\left(q_b\right) q_b f\left(q_b\right)\alpha} < \frac{y_h - y_l}{r+\delta},$$

for all $k_e < \alpha\left(\frac{y_h - y_l}{r+\delta}\right)$. It follows that

$$\frac{y_h - b}{y_h - y_l} \geq \frac{\alpha+r+\delta}{r+\delta},$$

as assumed in Proposition 1, is sufficient for Condition (iii) in Lemma 4.

Now, differentiate equation (21) to verify that

$$\frac{\partial q_u}{\partial k_u} > 0 \text{ and } \frac{\partial}{\partial k_u}\left(\frac{k_u}{q_u f\left(q_u\right)}\right) > 0,$$

for all $k_e < \alpha\left(\frac{y_h - y_l}{r+\delta}\right)$ and for all $k_u < \frac{y_h - b}{r+\delta} - (1-\alpha)\left(\frac{y_h - y_l}{r+\delta}\right)$, with

$$\lim_{k_u \to 0} \frac{k_u}{q_u f\left(q_u\right)} = 0.$$

It follows from (33) that for any $k_e \in \left(0, \alpha\frac{y_h - y_l}{r+\delta}\right)$ there exists a number $\kappa_0 \in \left(0, \alpha\frac{y_h - y_l}{r+\delta}\right)$ such that $V\left(\left(w_u^h, y_h\right)\right) - V\left(\left(y_l, y_l\right)\right) > 0$, with $w_l = y_l < w_h < y_h$, and, furthermore, k_u satisfies (38), for all $k_u \in (0, \kappa_0)$. Since $w_u^h \neq y_l$, equilibrium wages reveal the current productivity of employed workers, as required.

It follows from Lemmas 4 and 5 that there exists a refined equilibrium satisfying parts (ii) and (iii) of Proposition 1. It is straightforward to characterize ψ. The unemployment rate is given by

$$\psi\left(s_u\right) = \frac{\delta}{\delta + f\left(q_u\right)}.$$

The wage distribution has three mass points: $\left(w_u^l, w_u^h, w_e\left(\left(w_u^l, y_l\right)\right)\right)$, where $w_u^l = y_l$. The mass of workers earning the wage w_u^l is

$$\psi\left(\left(w_u^l, y_l\right)\right) = \left(\frac{(1-\alpha) f\left(q_u\right)}{\delta + (1-\delta)\alpha f\left(q_b\right)}\right)\psi\left(s_u\right),$$

where $q_b \equiv q_e\left(\left(w_u^l, y_l\right)\right)$. The mass of workers earning the wage w_u^h is

$$\psi\left(\left(w_u^h, y_h\right)\right) = \left(\frac{\alpha f\left(q_u\right)}{\delta}\right)\psi\left(s_u\right)$$

and the mass of workers earning the wage $w_e\left(\left(w_u^l, y_l\right)\right)$ is

$$\psi\left(\left(w_e\left(\left(w_u^l, y_l\right)\right), y_h\right)\right) = \left(\frac{(1-\delta)\alpha f\left(q_b\right)}{\delta}\right)\psi\left(\left(w_u^l, y_l\right)\right).$$

It remains to provide conditions under which the above equilibrium is the unique refined equilibrium. To that end, consider a solution to Problem (P3) for a fixed continuation value of unemployment $V^c < V(s_u)$, where $V(s_u)$ is the value of unemployment in the efficient equilibrium. Equations (17) and (18) and (20) imply that the most profitable deviation in the market for entry jobs is associated with the unique value q^d that solves

$$\frac{k_u}{\eta\left(q^d\right) q^d f\left(q^d\right)} = \alpha\frac{y_h}{r+\delta} + (1-\alpha)\left(\frac{y_h}{r+\delta} - \frac{k_e}{\eta\left(q_b\right) q_b f\left(q_b\right)\alpha}\right) - \left(\frac{r}{r+\delta}\right)\frac{V^c}{1+r}. \tag{39}$$

Note that there is a unique value of q^d that solves this equation for all $V^c < V(s_u)$ whenever there is a solution for $V^c = V(s_u)$, which is the relevant case. Then, differentiate the above equation and replicate our previous arguments to verify that

$$\frac{\partial q^d}{\partial k_u} > 0 \text{ and } \frac{\partial}{\partial k_u}\left(\frac{k_u}{q^d f\left(q^d\right)}\right) > 0,$$

for all $k_e < \alpha\left(\frac{y_h - y_l}{r+\delta}\right)$ and for all $k_u < \frac{y_h - b}{r+\delta} - (1-\alpha)\left(\frac{y_h - y_l}{r+\delta}\right)$, with

$$\lim_{k_u \to 0} \frac{k_u}{q^d f\left(q^d\right)} = 0.$$

Now, note that

$$\frac{V\left(\left(w^d, y_h\right)\right)}{1+r} - \frac{V^c}{1+r} = \frac{y_h}{r+\delta} - \frac{k_u}{\alpha q^d f\left(q^d\right)} - \left(\frac{r}{r+\delta}\right)\frac{V^c}{1+r}, \tag{40}$$

where $\left(w^d, q^d\right)$ solve (17) and (18), given $V^c < V(s_u)$. It follows that for any $k_e \in \left(0, \alpha\frac{y_h - y_l}{r+\delta}\right)$ there exists a number $\kappa_1 \in \left(0, \alpha\frac{y_h - y_l}{r+\delta}\right)$ such that $V\left(\left(w^d, y_h\right)\right) - V^c > 0$, for all $k_u \in (0, \kappa_1)$. Clearly, there is a number $\kappa_2 > 0$ such that this is the case for all $V^c \in \left[\frac{1+r}{r}b, V(s_u)\right]$ and so the most profitable deviating contract in the market for entry jobs is a separating contract that unemployed workers would never reject. It follows that the above equilibrium is the unique refined equilibrium within the class of revealing equilibria, for all $k_u \in (0, \kappa)$, with $\kappa = \min\{\kappa_0, \kappa_2\}$.

Now suppose, to the contrary, that there also exists a refined equilibrium that is non-revealing. Let $V^c < V(s_u)$ denote the value of unemployment in the candidate equilibrium. Consider the most profitable deviation among all separating contracts in the market for entry jobs. Note that (39) implies that $\partial q^d / \partial V^c > 0$. Then, (17)–(18) imply that $\alpha V\left(\left(w^d, y_h\right)\right) + (1-\alpha)V\left((y_l, y_l)\right) - V^c$ is a decreasing function of V^c. Since $f\left(q^d\right)$ is also a decreasing function of V^c, it follows that the candidate (non-revealing) equilibrium must have $V^c > V(s_u)$. Otherwise there would be a profitable deviating (separating) contract. But this contradicts the fact that the efficient equilibrium supports the efficient allocation, so $V^c < V(s_u)$. Hence, the efficient equilibrium is the unique refined equilibrium, for all $k_u \in (0, \kappa)$. This concludes the proof of Proposition 1. **QED**

Proof of Proposition 2

Recall that equation (38) is sufficient for Condition (ii) in Lemma 4. Then, note that $k_u > \alpha\left(\frac{y_h - y_l}{r+\delta}\right)$ is sufficient for Condition (iii) in Lemma 4. Part (i) of the proposition follows immediately.

To prove Part (ii), suppose that there is a refined equilibrium that supports an efficient allocation with positive job quits. From equation (17), $k_u > \alpha\left(\frac{y_h - y_l}{r+\delta}\right)$ implies that $w_u^h < y_l$. It follows that for each $\alpha \in (0,1)$, there is a number $\beta \in (0, y_l)$ such that $w_u^h < b$ for any $b \in (\beta, y_l)$. But then it must be that $V\big((w_h, y_h)\big) - V\left(s_u\right) < 0$; a contradiction. **QED**

Proof of Proposition 3

We first characterize the solution to problems (P1) and (P2) with $\rho = 1 - \alpha$. Then, we provide conditions under which this allocation can be supported by a refined equilibrium.

The first part of the proof parallels that of Proposition 1. An interior solution of Problem (P1) with $\rho = 1 - \alpha$ satisfies the familiar matching efficiency condition

$$\frac{\frac{y_h - w'}{r+\delta}}{\frac{y_h - w'}{r+\delta} + \frac{w' - w}{r+\delta+(1-\delta)\alpha f(q')}} = \eta\left(q'\right),$$

and the zero-profit condition

$$q' f\left(q'\right)(1-\alpha)\alpha\left(\frac{y_h - w'}{r+\delta}\right) = k_e, \tag{41}$$

where potential poachers anticipate that a fraction α will turn down their job offers.

Because the objective function in Problem (P2) is not generally concave in (w, q), we adopt the same strategy we followed in the proof of Proposition 1. That is, we view the solution to Problem (P1) as a mapping from the workers' quit rates to their entry wages, rather than the reverse, and then treat current and future quit rates as the relevant choice variables in Problem (P2). To that end, use the above first-order conditions to express the worker's entry wage as a function of q':

$$\widetilde{W}\left(q'\right) \equiv y_h - \left(\frac{k_e}{q' f\left(q'\right)(1-\alpha)\alpha}\right)\left(r+\delta+\left(\frac{1-\eta\left(q'\right)}{\eta\left(q'\right)}\right)\left(r+\delta+(1-\delta)\alpha f\left(q'\right)\right)\right). \tag{42}$$

We have that $q_e(s) = q_e((w, y_l)) = q_e((w, y_h))$ and $q_e(s) \in (\widehat{q}_a, \widehat{q}_b]$, where $w_e(s) > y_l$ if and only if $q_e(s) > \widehat{q}_a$ and $\widetilde{W}(q_e(s)) \leq y_l$ if and only if $q_e(s) \leq \widehat{q}_b$ and where $\widehat{q}_a$ and $\widehat{q}_b$ are given by

$$\widehat{q}_a f\left(\widehat{q}_a\right)(1-\alpha)\alpha\left(\frac{y_h - y_l}{r+\delta}\right) = k_e \tag{43}$$

and

$$\frac{y_h - y_l}{r+\delta} = \left(\frac{k_e}{\widehat{q}_b f\left(\widehat{q}_b\right)(1-\alpha)\alpha}\right)\left(1+\left(\frac{1-\eta\left(\widehat{q}_b\right)}{\eta\left(\widehat{q}_b\right)}\right)\left(\frac{r+\delta+(1-\delta)\alpha f\left(\widehat{q}_b\right)}{r+\delta}\right)\right). \tag{44}$$

Clearly, $\infty > \widehat{q}_b > \widehat{q}_a > 0$ if $k_e < (1-\alpha)\alpha\left(\frac{y_h - y_l}{r+\delta}\right)$.

Proceeding as before, Problem (P2) can be formulated in the present case as

$$V\left(s_u\right) = b + \frac{1}{1+r}\left[V\left(s_u\right) + \max_{q,q'} f(q)\left(\widetilde{V}_0\left(q'\right) - V\left(s_u\right)\right)\right], \tag{P4}$$

subject to

$$k_u \leq qf(q)\left(\frac{\widetilde{M}_0(q') - \left(\widetilde{V}_0(q') - V(s_u)\right)}{1+r}\right),$$

$$q' \in (\widehat{q}_a, \widehat{q}_b]\ , w \leq y_l.$$

Let $\left(q_u, q_e^l\right)$ denote a solution to Problem (P4). $\widetilde{V}_0(q')$ denotes the *ex ante* value of a match to a worker, expressed as a function of q':

$$\begin{aligned}\widetilde{V}_0(q') &\equiv \alpha V\left(\left(\widetilde{W}(q'), y_h\right)\right) + (1-\alpha) V\left(\left(\widetilde{W}(q'), y_l\right)\right) \\ &= V\left(\left(\widetilde{W}(q'), y_h\right)\right) = V\left(\left(\widetilde{W}(q'), y_l\right)\right)\end{aligned}$$

and $\widetilde{M}_0(q')$ denotes the *ex ante* surplus associated with the match:

$$\widetilde{M}_0(q') \equiv \alpha\widetilde{M}\left(\left(\widetilde{W}(q'), y_h\right)\right) + (1-\alpha)\widetilde{M}\left(\left(\widetilde{W}(q'), y_l\right)\right),$$

where $\widetilde{M}\left(\left(\widetilde{W}(q'), y_l\right)\right)$ is the *ex post* surplus associated with a low productivity match:

$$\frac{\widetilde{M}\left(\left(\widetilde{W}(q'), y_l\right)\right)}{1+r} = \frac{V\left(\left(\widetilde{W}(q'), y_l\right)\right) - V(s_u)}{1+r} + \left(\frac{r+\delta}{r+\delta+(1-\delta)\alpha f(q')}\right)\frac{y_l - \widetilde{W}(q')}{r+\delta}$$

and $\widetilde{M}\left(\left(\widetilde{W}(q'), y_h\right)\right)$ is the *ex post* surplus associated with a high productivity match:

$$\begin{aligned}\frac{\widetilde{M}\left(\left(\widetilde{W}(q'), y_h\right)\right)}{1+r} &= \frac{V\left(\left(\widetilde{W}(q'), y_h\right)\right) - V(s_u)}{1+r} \\ &\quad + \left(1 - \frac{r+\delta}{r+\delta+(1-\delta)\alpha f(q')}\right)\frac{y_h - w_e\left(\left(\widetilde{W}(q'), y_h\right)\right)}{r+\delta} \\ &\quad + \left(\frac{r+\delta}{r+\delta+(1-\delta)\alpha f(q')}\right)\frac{y_h - \widetilde{W}(q')}{r+\delta},\end{aligned}$$

which reflects the fact that *ex post* well matched workers will search for outside offers solely to elicit a retention offer from their current employer.

Noting that

$$\frac{\widetilde{V}_0(q') - V(s_u)}{1+r} = \frac{y_h}{r+\delta} - \left(\frac{r}{r+\delta}\right)\frac{V(s_u)}{1+r} - \frac{k_e}{\eta(q')\,q' f(q')(1-\alpha)\alpha} \tag{45}$$

and using (41)–(42), one can verify that

$$\begin{aligned}\frac{\widetilde{M}_0(q')}{1+r} &= \frac{y_h}{r+\delta} - \left(\frac{r}{r+\delta}\right)\frac{V(s_u)}{1+r} \\ &\quad - (1-\alpha)\left(\frac{r+\delta}{r+\delta+(1-\delta)\alpha f(q')}\right)\left(\frac{y_h - y_l}{r+\delta}\right) \\ &\quad - (1-\alpha)\left(1 - \frac{r+\delta}{r+\delta+(1-\delta)\alpha f(q')}\right)\left(\frac{k_e}{q' f(q')(1-\alpha)\alpha}\right).\end{aligned}$$

Lemma 6 *(i) $\widetilde{W}(q')$ and $\widetilde{V}_0(q')$ are strictly increasing and concave functions of q' on $(\widehat{q}_a, \widehat{q}_b]$. (ii) $\widetilde{M}_0(q')$ is a strictly concave function of q' on $(\widehat{q}_a, \widehat{q}_b] \subset (0,\infty)$ and it is maximized at $q' = \widehat{q}_b$; $\widetilde{M}_0(q') - \widetilde{V}_0(q')$ is a strictly decreasing and convex function of q' on $[\widehat{q}_a, \widehat{q}_b]$.*

Proof: It replicates the arguments in Proposition 1 with minor changes. **QED**

The first-order conditions for an interior solution of Problem (P4) are given by

$$\frac{\widetilde{V}_0(q') - V(s_u)}{1+r} = \lambda q \left(\frac{1-\eta(q)}{\eta(q)}\right)\frac{k_u}{q f(q)}, \tag{46}$$

$$\lambda = \frac{f(q)\,\partial\widetilde{V}_0/\partial q'}{q f(q)\left(\partial\widetilde{V}_0/\partial q' - \partial\widetilde{M}_0/\partial q'\right)} \tag{47}$$

and

$$q f(q)\left(\frac{\widetilde{M}_0(q') - \left(\widetilde{V}_0(q') - V(s_u)\right)}{1+r}\right) = k_u, \tag{48}$$

where λ is the multiplier associated with the employer's zero-profit constraint, given by (48). Equation (46) coincides with the standard matching efficiency condition if and only if the multiplier equals $1/q$. Consider equation (47). The multiplier is the expected value of surplus to the worker associated with a higher labor market queue at the margin ($f(q)\,\partial\widetilde{V}_0/\partial q'$) evaluated in terms of the employer's surplus ($q f(q)\left(\partial\widetilde{V}_0/\partial q' - \partial\widetilde{M}_0/\partial q'\right)$). The expected surplus of a match is maximized at $\partial\widetilde{M}_0/\partial q' = 0$, which implies that $\lambda = 1/q$. Lemma 6 implies that this happens exactly at the corner when $\widetilde{W}(q') = y_l$.

Following similar steps as in the proof of Proposition 1 one can verify that an interior solution to Problem (P4) satisfies

$$\frac{y_h - b}{r+\delta} - \frac{k_e}{\eta(q')\,q' f(q')(1-\alpha)\alpha} = \lambda q\left(\frac{1-\eta(q)}{\eta(q)}\right)\frac{k_u}{q}\left(\frac{1}{f(q)} + \frac{1}{r+\delta}\right), \tag{49}$$

where λ is given by (47), and

$$\frac{k_u}{qf(q)} = -(1-\alpha)\left(\frac{r+\delta}{r+\delta+(1-\delta)\alpha f(q')}\right)\left(\frac{y_h-y_l}{r+\delta}\right) + \left(\frac{k_e}{q'f(q')(1-\alpha)\alpha}\right)\left(\frac{(1-\alpha)(r+\delta)}{r+\delta+(1-\delta)\alpha f(q')}+\frac{1-\eta(q')}{\eta(q')}+\alpha\right). \tag{50}$$

Lemma 7 *There is a number* $\kappa_0 \in \left(0,(1-\alpha)\alpha\frac{y_h-y_l}{r+\delta}\right)$ *such that equations (47), (49) and (50) have a unique solution* $(\lambda,q,q') = (\lambda^*,q_u^*,q_e^*)$, *with* $q_u^* \in (0,\infty)$, $q_e^* \in (\widehat{q}_a,\widehat{q}_b)$, *and* $\lambda^* q_u^* > 1$, *for all* (k_u,k_e) *such that* $k_e \in (0,\kappa_0)$ *and* $k_u \in \left[\alpha\frac{y_h-y_l}{r+\delta},\alpha\frac{y_h-b}{r+\delta}\right)$, *where* $\widehat{q}_a$ *is given by (43) and* $\widehat{q}_b$ *is given by (44).*

Proof: Differentiating equation (47) one can verify that the following inequality is necessary and sufficient for $\partial\lambda_i q/\partial q' < 0$:

$$\frac{-\partial^2\widetilde{M}_0/\partial q'^2}{-\partial^2\widetilde{V}_0/\partial q'^2} > \frac{\partial\widetilde{M}_0/\partial q'}{\partial\widetilde{V}_0/\partial q'}.$$

The left side of the inequality is greater than one, since $\widetilde{M}_0 - \widetilde{V}_0$ *is a strictly convex function of* q'. The right side is smaller than one, since $\widetilde{M}_0 - \widetilde{V}_0$ *is a strictly decreasing function of* q'. Hence, $\partial(\lambda_i q)/\partial q' < 0$. Moreover, note that $\lambda_i q \geq 1$ if and only if $\partial\tilde{M}_0/\partial q' \geq 0$.

Note that (49) characterizes q as a decreasing function of q' on $(q_c,\widehat{q}_b)$, where q_c is given by

$$\frac{y_h-b}{r+\delta} - \frac{k_e}{\eta(q_c)q_c f(q_c)(1-\alpha)\alpha} = 0. \tag{51}$$

To verify that $q_c < \widehat{q}_b$, note that (44) implies that

$$\frac{k_e}{\eta(\widehat{q}_b)\widehat{q}_b f(\widehat{q}_b)(1-\alpha)\alpha} < \frac{y_h-y_l}{r+\delta}, \tag{52}$$

for all $k_e < (1-\alpha)\alpha\left(\frac{y_h-y_l}{r+\delta}\right)$, which, together with the fact that $y_l > b$, implies that $\widehat{q}_b > q_c$, as required. It is evident that $q_c > 0$. Furthermore, as illustrated in Figure 1, q approaches ∞ as q' approaches q_c, whereas q approaches some number in $(0,\infty)$ as q' approaches $\widehat{q}_b$.

Note that (50) characterizes q as an increasing function of q' on a subset of $(q_c,\widehat{q}_b)$. Specifically, note that q approaches zero as q' approaches zero. Hence, q approaches some number in $(0,\infty)$ as q' approaches q_c.

Next, writing (44) as

$$\frac{\alpha k_e}{\eta(\widehat{q}_b)\widehat{q}_b f(\widehat{q}_b)(1-\alpha)\alpha} = -(1-\alpha)\left(\frac{r+\delta}{r+\delta+(1-\delta)\alpha f(\widehat{q}_b)}\right)\left(\frac{y_h-y_l}{r+\delta}\right) + \left(\frac{k_e}{\widehat{q}_b f(\widehat{q}_b)(1-\alpha)\alpha}\right)\left(\frac{(1-\alpha)(r+\delta)}{r+\delta+(1-\delta)\alpha f(\widehat{q}_b)}+\frac{1-\eta(\widehat{q}_b)}{\eta(\widehat{q}_b)}+\alpha\right),$$

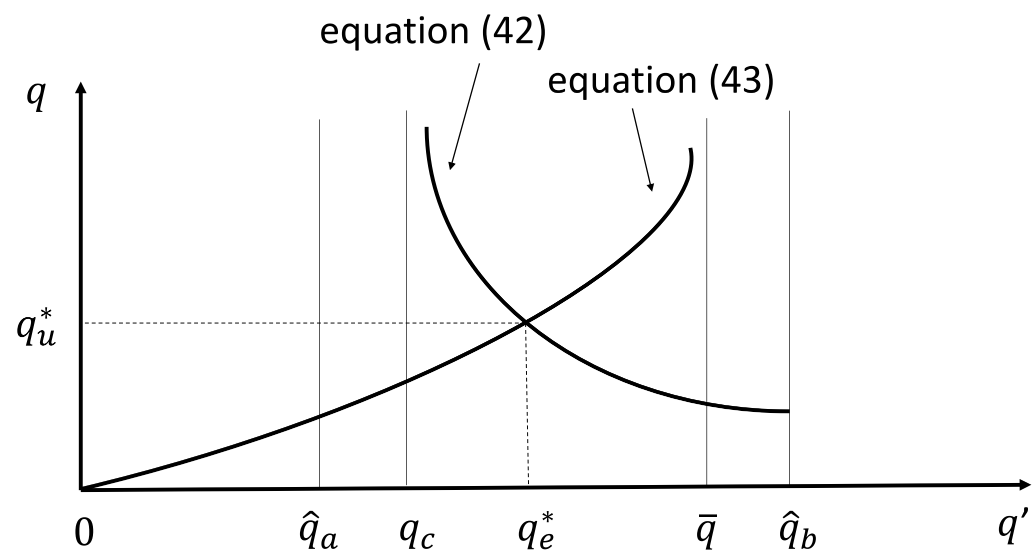

Figure 1: existence and uniqueness of an interior solution

and comparing this with (50), it follows that

$$\frac{\alpha k_e}{\eta\left(q_c\right) q_c f\left(q_c\right)(1-\alpha) \alpha} > k_u \geq \frac{\alpha k_e}{\eta\left(\widehat{q}_b\right) \widehat{q}_b f\left(\widehat{q}_b\right)(1-\alpha) \alpha}$$

is necessary and sufficient to ensure that there is a number $\bar{q} \in (q_c, \widehat{q}_b]$ such that q approaches ∞ as q' approaches $\bar{q}$. To see why, note that the first inequality is necessary and sufficient to ensure that the right side of (50) becomes larger than k_u as q' approaches q_c. Then note that the second inequality is necessary and sufficient to ensure that the right side of (50) becomes smaller than k_u as q' approaches $\widehat{q}_b$.

It follows from (51) and (52) that

$$\alpha \frac{y_h - b}{r+\delta} > k_u \geq \alpha \frac{y_h - y_l}{r+\delta} \tag{53}$$

is sufficient to ensure that there is a number $\bar{q} \in (q_c, \widehat{q}_b)$ such that q approaches ∞ as q' approaches $\bar{q}$. Hence, (53) is sufficient to ensure that equations (47), (49) and (50) have a unique solution $(\lambda, q, q') = (\lambda^*, q_u^*, q_e^*)$, with $q_u^* \in (0, \infty)$, $q_e^* \in (q_c, \widehat{q}_b)$ and $\lambda^* q_u^* > 1$. Figure 1 illustrates this.

It remains to provide conditions under which $q_e^* > \widehat{q}_a$. Comparing the definition of $\widehat{q}_a$ in (43) and that of q_c in (51), it follows that there is a number $\kappa_0 \in \left(0, (1-\alpha)\alpha \frac{y_h - y_l}{r+\delta}\right)$ such that $\widehat{q}_a \leq q_c$, for all $k_e \in (0, \kappa_0)$. Hence, $q_e^* \in (\widehat{q}_a, \widehat{q}_b)$ for all $k_e \in (0, \kappa_0)$, as required. **QED**

Letting $w_u^* \equiv \widetilde{W}\left(q_e^*\right)$ and $w_e^* \equiv w_e\left((w_u^*, y_h)\right) = w_e\left((w_u^*, y_h)\right)$, it follows that $w_e^*(s) > y_l$ and $w_u^* < y_l$, since $q_e^* > \widehat{q}_a$ and $q_e^* < \widehat{q}_b$.

It is straightforward to characterize ψ. The unemployment rate is given by

$$\psi(s_u) = \frac{\delta}{\delta + f(q_u^*)}.$$

The wage distribution has two mass points: $\left(w_u^*, w_e^*\right)$. The mass of workers earning the wage w_u^* is

$$\psi((w_u^*, y_l)) + \psi((w_u^*, y_h)) = \left(\frac{f(q_u^*)}{\delta + (1-\delta)\alpha f(q_e^*)}\right)\psi(s_u),$$

where $q_e^* \equiv q_e((w_u^*, y_l)) = q_e((w_u^*, y_h))$. The mass of workers earning the wage w_e^* is

$$\psi((w_e^*, y_h)) = \left(\frac{(1-\delta)\alpha f(q_e^*)}{\delta}\right)\left(\psi((w_u^*, y_l)) + \psi((w_u^*, y_h))\right).$$

Next, we show that the allocation we have characterized can be supported by a refined equilibrium. To that end, consider first a deviation in the market for non-entry jobs. Suppose the beliefs of a deviating firm are consistent with the fact that workers hired under the same pooling contract not only are observationally equivalent, but they also have identical incentives and so they cannot be separated. Conditional on these off-equilibrium beliefs, the allocation that solves Problem (P1) with $\rho = 1-\alpha$ is the only one that is consistent with a refined equilibrium that exhibits pooling contracts in the market for entry jobs. This is because it is the allocation that maximizes the value of search on the job subject to potential employers making non-negative profits. Hence, the relevant issue here is whether a pooling contract in the market for entry jobs can ever arise along the equilibrium path that is consistent with a refined equilibrium. To address this issue, we now consider a deviation in the market for entry jobs.

The deviating contract in the market for entry jobs may be pooling or separating. If it is a pooling contract, then a worker who seeks the deviating contract must believe that future employers will not be able to separate different types of workers who earn the same wage and have the same incentives. Otherwise a sub-optimal pooling contract may in fact dominate the best pooling contract along the equilibrium path, simply because both workers and future employers believe that the wage associated with an otherwise inferior pooling contract will in fact be revealing. Accordingly, the pooling contract that solves Problem (P2) with $\rho = 1-\alpha$ cannot be dominated by any other pooling contract. Note that these off-equilibrium beliefs are reasonable in the sense of our refinement.

Now suppose that the deviating contract $\left(w_l^d, w_h^d\right)$ is such that $w_h^d \neq w_l^d = y_l$. Then, the profits of the deviating firm must be such that

$$q^d f\left(q^d\right)\alpha\left(\frac{y_h - w_h^d}{r+\delta}\right) \geq k_u,$$

where q^d is the corresponding queue. Accordingly, it must be that $w_h^d < y_l$ for all $k_u \geq \alpha\frac{y_h - y_l}{r+\delta}$.

Next note that

$$\frac{V\left(\left(w^d, y_h\right)\right)}{1+r} - \frac{V(s_u)}{1+r} = \frac{w^d}{r+\delta} - \left(\frac{r}{r+\delta}\right)\frac{V(s_u)}{1+r},$$

where $V(s_u)$ is the value of unemployment in the candidate non-revealing equilibrium. Since $\frac{rV(s_u)}{1+r} > b$, for all $b \in (0, y_l)$, it follows that for each $\alpha \in (0,1)$ there is a number $\beta_0 \in (0, y_l)$ such that $V\left(\left(w_h^d, y_h\right)\right) - V(s_u) < 0$ for all $b \in (\beta_0, y_l)$ and all $k_u \geq \alpha\frac{y_h - y_l}{r+\delta}$. When this is the case, unemployed workers who meet a deviating firm are better off rejecting any job offer associated with a high productivity match. But then a deviating contract such that $w_h^d \neq w_l^d = y_l$ cannot be profitable for the firm.

Next suppose that the deviating contract $\left(w_l^d, w_h^d\right)$ is such that $w_h^d \neq w_l^d \neq y_l$. There are two cases to consider. If the deviating contract is such that $V\left(\left(w_h^d, y_h\right)\right) - V(s_u) < 0$, then the contract attracts unemployed workers if and only if

$$f\left(q^d\right)\left[(1-\alpha)V\left(\left(w_l^d, y_l\right)\right) - V(s_u)\right] > f(q_u^*)\left(\widetilde{V}_0(q_e^*) - V(s_u)\right),$$

and if $V\left(\left(w_h^d, y_h\right)\right) - V(s_u) = 0$, then the contract attracts unemployed workers if and only if

$$f\left(q^d\right)(1-\alpha)\left[V\left(\left(w_l^d, y_l\right)\right) - V(s_u)\right] > f(q_u^*)\left(\widetilde{V}_0(q_e^*) - V(s_u)\right).$$

In either case one can verify that there is a number $\alpha_0 \in (0,1)$ such that for each $\alpha \in (\alpha_0, 1)$ no deviating job will attract unemployed workers, for any (b, k_u, k_e) such that $b \in (\beta_0, y_l)$ — for β_0 as defined above — $k_u \in \left[\alpha\frac{y_h - y_l}{r+\delta}, \alpha\frac{y_h - b}{r+\delta}\right)$ and $k_e \in \left(0, (1-\alpha)\,\alpha\frac{y_h - y_l}{r+\delta}\right)$. To see why, for given values of α and k_e satisfying the assumptions of the proposition, let $k_e = m(1-\alpha)\alpha$, with

$$m = q_e^* f(q_e^*)\left(\frac{y_h - w_e^*}{r+\delta}\right).$$

Maintaining $k_e = m(1-\alpha)\alpha$, for the fixed value of m, we have

$$\lim_{(\alpha,k_e)\to(1,0)} f(q_u^*)\left(\widetilde{V}_0(q_e^*) - V(s_u)\right) > 0,$$

which is well defined for some feasible values of k_u. In particular, note that there is a value of k_u satisfying (53) such that there is an interior solution with all the properties specified in Lemma 7, for all $\alpha \in (0,1)$ and $k_e < (1-\alpha)\,\alpha\frac{y_h - y_l}{r+\delta}$. One such example is $k_u = \alpha\frac{y_h - y_l}{r+\delta}$.

It follows that there is a number $\alpha_0 \in (0,1)$ such that for each $\alpha \in (\alpha_0, 1)$ there is a refined equilibrium that is non-revealing, for all (b, k_u, k_e) such that $b \in (\beta_0, y_l)$, $k_u \in \left[\alpha\frac{y_h - y_l}{r+\delta}, \alpha\frac{y_h - b}{r+\delta}\right)$ and $k_e \in (0, \kappa_0)$, where β_0 is given above and κ_0 is given in Lemma 7. The construction of Q is now standard. **QED**

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