Who Participates in Risk Transfer Markets? The Role of Transaction Costs and Counterparty Risk

Eric Stephens                  James R. Thompson
Carleton University            University of Waterloo
eric.stephens@carleton.ca      james@uwaterloo.ca

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Abstract

We analyze the role of transaction costs in risk transfer markets. For example, when these markets are in their infancy, they are characterized by few contracts and high costs. In such a scenario, we show, only highly risk-averse buyers (e.g., those with significant exposure to the underlying risk) exist in the market alongside high quality counterparties, and no asymmetric information can be present on the quality of the counterparty to which the risk is ceded. With lower transaction costs, we show, less risk-averse buyers (e.g., those with little or no exposure to the underlying risk) will enter the market, thereby increasing risk transfer; however, these buyers will choose to contract with less stable counterparties. Further, we find that when transaction costs are low, asymmetric information can exist in equilibrium. Finally, we analyze the effect of a transaction tax, which is viewed simply as an increase in transaction costs. Under perfect information, such a tax reduces the relative number of unstable counterparties. Under asymmetric information, information rents decrease, and under simple conditions, the relative number of unstable counterparties also decreases.

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1 Introduction

The rate of development of risk transfer markets in the past 40 years has been extraordinary. Seemingly overnight, new instruments to disperse many forms, of risk such as interest rate, credit and foreign exchange, have arisen. A characterizing feature of the evolution of these markets is that they begin small, with high transaction costs. As more players join the market, transaction costs decrease as contracts are standardized and more efficient means to find counterparties are developed. For example, consider the formation of the International Swaps and Derivative Association (ISDA) in 1985. Prior to this, each deal required a customized contract and great care needed to be taken to mitigate legal risk, a cost which ISDA standardization decreased considerably. Over-the-counter (OTC) markets, which are the motivating example of this paper due to their bilateral nature and potential exposure to counterparty risk, had a notional size of approximately 614 trillion dollars as of 2007, accounting for almost 84 percent of the total derivatives market. The massive growth in OTC contracts can be seen by looking back as recently as 2001, when the estimated notional size was just under 100 trillion dollars. Further, OTC contracts such as credit default swaps, which were in their infancy only a decade before, played an important role in the credit crisis of 2007-2009. In addition, the resurgence of interest in financial transaction taxes within the European Union and elsewhere makes the study of transaction costs of further relevance.

In this paper we model the effect of transaction costs in a simple market for risk transfer. The extant literature focuses largely on the effect of transaction costs on security demand, prices and volatility. In contrast, we abstract from these issues and study transaction costs from a different point of view, namely that decreased costs can bring new players into the market, and those new players may contract with very different counterparties. We find that when transaction costs are sufficiently high, only buyers that are very averse to risk (i.e., have strong hedging motives) can exist in the market and contract with relatively safe sellers (the party to which the risk is ceded). Furthermore, we show that with high transaction costs, the market cannot sustain asymmetric information regarding either the quality of the seller or the underlying risk. In other words, full information is a prerequisite for contracting to take place when transaction costs are high (e.g., when markets are in their infancy).

When transaction costs are low, the market will include buyers that are relatively less risk averse. We show that less risk-averse buyers may contract with less stable counterparties, so decreasing transaction costs tends to result in an increase in the relative number of unstable sellers.

Asymmetric information on the quality of the seller (degree of counterparty risk) can exist in equilibrium when transaction costs are sufficiently low, and has two noteworthy effects on the

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1For example, it was recently estimated that in Europe the average transaction cost in the OTC market has declined to as little as 55 Euros per 1 million Euro transaction, and this amount is expected to decrease further as electronic platforms become more prevalent (Deutsche Börse Group, “The Global Derivatives Market: An Introduction”, White Paper (2008)).

2For a discussion of this and related issues see Ludwig (1993).

relative number of unstable sellers in the market: a reduction, as less risk-averse buyers drop out of
the market, and an increase, as buyers who otherwise would have contracted with stable sellers
under perfect information contract with unstable sellers instead of with a seller of unknown quality.
Which effect dominates depends on competition amongst unstable sellers.

Finally, we analyze a transaction tax, which we view simply as an increase in transaction costs.
We show that such a tax pushes buyers with relatively low risk aversion out of the market, which
consequently reduces the relative number of unstable counterparties (this requires some assumptions
on competition when there is asymmetric information regarding seller type). Furthermore, we show
that a tax will decrease the information rents that can be extracted when there is asymmetric
information. However, it also decreases risk transfer generally, and in extreme cases it can cause
the market to collapse.

The intuition behind our results is as follows. A contingent contract is written on an underlying
risk in which buyer insure themselves from a potential loss, much like an insurance contract. There
are both good and bad sellers, which have different probabilities of default (counterparty risk). We
focus on the most interesting case, in which the bad sellers offer lower quality protection for a lower
price, while the good sellers must charge more for superior protection. On the demand side of the
market, we first model a population of buyers who differ solely in their aversion to risk. Buyers
that are highly risk averse are willing to pay higher prices for better protection from good sellers,
while buyers that are less risk averse tolerate more counterparty risk from bad sellers in exchange
for lower prices. When transaction costs are low, the market is partitioned between those who
contract with bad sellers and those who contract with good ones. The most responsive buyers to
transaction costs are those who are the least risk averse. Consequently, when transaction costs
increase, buyers that would have contracted with bad sellers are the first to drop out of the market.
Thus, the relative number of bad sellers in the market is decreasing in transaction costs.

The impact of asymmetric information is more subtle. To analyze the case of asymmetric
information over seller type, we assume that sellers have private information over their type and
thus bad sellers can pool with the good ones. Bad sellers that hide their type can extract rents,
since they can charge higher prices than they otherwise could have; however, it is also possible that
bad sellers who reveal their type earn rents as well. This occurs because when transaction costs
are low, revealed bad sellers contract with buyers that strictly prefer them over contracting with
the pool (i.e., those buyers that have little aversion to risk). This eases the competition among
bad sellers since the pooling bad sellers compete with good sellers in the pool instead of with the
revealed bad sellers. Thus, revealed bad sellers can charge higher prices than if there was perfect
information.

We find that when transaction costs are low, there are two noteworthy effects of asymmetric
information on the relative number of good and bad sellers in the market. First, since the revealed
bad sellers charge higher prices, this drives the least risk-averse buyers out of the market. Since these
parties would have otherwise contracted with bad sellers under perfect information, this decreases
the relative number of bad sellers. Conversely, contracting with the pool of good and bad sellers
is less attractive than contracting with the good seller with certainty (which can be done in the perfect information case). As such, we show that some buyers that would otherwise contract with a good seller with perfect information opt instead for a bad seller over the pool with asymmetric information, thus increasing the relative number of bad sellers. Which effect dominates depends on the degree of competition among the revealed bad sellers. When there are sufficiently many, competition drives the price down to that which would have resulted under perfection information, and the latter effect dominates. Furthermore, when transaction costs are sufficiently high, the market will cease to exist with asymmetric information of this type where it otherwise would have in its absence. This is the case in which with perfect information, only highly risk-averse buyers are in the market contracting with good sellers. With asymmetric information, the value of the contract goes down since the pool of sellers offers an inferior contract to that of the good seller (under perfect information). With sufficiently high transaction costs, even the most risk-averse buyers leave the market when they otherwise would have remained under perfect information.

The implications of a transaction tax depend on the informational environment. A tax causes the least risk-averse buyers to drop out of the market, thereby reducing the relative number of bad sellers. Further, with asymmetric information regarding seller quality, a tax reduces the information rents. This is because revealed bad sellers must lower their price, since the transaction tax means that some of the buyers who would have otherwise contracted with them over the pool instead drop out of the market. This causes increased competition among the revealed sellers and so they decrease the price to attract more buyers, thereby lowering their information rents. By lowering the price, some buyers from the pool switch to bad sellers, making it less likely that a pooling bad seller will obtain a contract and thus decreasing the rents of the pooling bad sellers as well.

As an extension, in Section 6, we consider the more standard problem of asymmetric information regarding the underlying risk on which the contract is written. We assume that there are two types of buyers: one with lower risk, the other with higher risk. With high transaction costs, we demonstrate a lemons result in which low-risk buyers are driven out of the market, so that asymmetric information cannot exist in equilibrium. This occurs because the low-risk buyers would be in the market if they could be identified as such; however, when forced to pool with high-risk types, they drop out. Thus, asymmetric information cannot be sustained in equilibrium when transaction costs are high. When transaction costs decrease, low-risk buyers are willing to pool with high-risk ones, since the lower transaction costs make the pooled contract more attractive. Thus, asymmetric information can be a feature of the equilibrium when transaction costs are sufficiently low. In this environment, a transaction tax causes proportionally more low-risk buyers to leave the market than high-risk buyers. To understand this result, consider a low-risk and high-risk buyer with the same level of risk aversion. Since they pay the same amount in the pool, the

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4 When there is asymmetric information over seller quality, the analysis is more complicated in that in addition to the least risk-averse buyers dropping out (who otherwise would have contracted with bad sellers), some buyers may wish to contract with unstable sellers when they otherwise would have contracted with an unknown pool in the absence of a transaction tax. This is discussed in further detail in Section 5.
low-risk buyers benefit less from the contract since they are less likely to receive a payment from
the seller and so are more likely to leave the market. Therefore, the equilibrium beliefs of the sellers
will put more weight on a buyer being a high-risk type when transaction costs increase, and so set
a higher price. Since the price increases as transaction costs increase, the information rents of the
high-risk types are decreasing in transaction costs.

This paper contributes to the literatures on transaction costs and counterparty risk. The trans-
action cost literature has largely focused on the effects that these costs have on demand, prices and
market volatility. Lo et al. (2005), Vayanos and Vila (1999), Vayanos (1998) and Allen and Gale
(1994) develop GE models to determine the effect of transaction costs on these variables. Lo et
al. (2005) demonstrate the intuitive illiquidity discount in asset markets and find that even small
transaction costs can have significant effects. Vayanos (1998) finds that asset prices can increase
or decrease, depending on demand. Vayanos and Vila (1999) show how one can get similar coun-
terintuitive results based on the interplay between an illiquid asset that is affected by transaction
costs and a liquid asset that is not. Similar to our work, Allen and Gale (1994) study limited
market participation; however, the focus in that paper is on how limited participation interacts
with market volatility. In contrast to these papers, we determine the type of players that enter and
exit the market, and we study the effects on counterparty risk in multiple informational environ-
ments. As such, we do not consider the GE effects of security demand on each individual player,
focusing instead on the incentive aspect of transaction costs. While this limits our ability to make
broad welfare statements, it greatly benefits the analysis by elucidating the mechanisms behind
our results as simply as possible. In a model of an intermediated market, Stephens and Thompson
(2013) show that some risky counterparties may wish to freely reveal themselves, and that a trans-
action tax can lead to more revelation. Since in that model all buyers and sellers contract with
a risk-neutral market maker, the authors cannot consider how transaction costs interact with the
choice of counterparty. This is where our paper makes its key contribution.

In the literature on counterparty risk and incentives, Thompson (2010), Acharya and Bisin
(2014) and Biais, Heider and Hoerova (2012) demonstrate moral hazards that may be present on the
sell side of the market, wherein the seller (or insurer) may take positions which increase counterparty
risk and are not in the best interest of the buyer (or insured party). In our paper, we model
counterparty risk exogenously, focusing on differences between sellers when there are heterogenous
buyers and transaction costs, which these papers do not study. Stephens and Thompson (2014a)
analyze counterparty risk with multiple sellers and, similar to the current paper, show that a risk-
averse buyer will tend to contract with a good seller. They also show that asymmetric information
on the seller can endogenously increase the counterparty risk of good sellers, as they are forced to
compete with bad ones. In contrast, we model many buyers and sellers so that we can show how
changes in transaction costs affect the composition of buyers and sellers in the market. In addition,
we consider asymmetric information on both sides of the market.

The rest of the paper is organized as follows. Section 2 outlines the model, and Section 3
analyzes the equilibrium with full information. In Section 4, we introduce asymmetric information
on seller quality, while in Section 5 we analyze the effect of a transaction tax. Section 6 considers an extension of the model to analyze asymmetric information on buyer risk type, and Section 7 concludes. Non-trivial proofs can be found in the Appendix.

2 Model

An individual (or institution) possesses a risky income stream that can be contracted on by a derivative product. For convenience, we assume that the buyer makes an upfront payment, which entitles them to a payout in the event that an observable loss occurs. An upfront payment is not crucial, and the analysis pertains to any state-contingent contract between two parties.

2.1 Sellers

The focus in this paper is on the buyer side of the market, thus we model relatively simplistic sellers. Specifically, we consider a reduced form representation of sellers from that found in Thompson (2010) and Stephens and Thompson (2014a), in which a formal portfolio choice is presented for the sellers (referred to as insurers in those papers). There, the seller is endowed with a portfolio and has a choice on how to invest the upfront payment (premium). This allows for endogenous counterparty risk, namely, counterparty risk that is affected by the risk transfer contract itself. In this paper we present a simplified version in which counterparty risk is exogenous.

There is a measure \( N \) of each of two types of risk-neutral sellers, which are referred to as “good” and “bad”, denoted \( j = \{G, B\} \). Both seller types possess a risky portfolio and is endowed with initial wealth \( W_j \), which we consider to be the size of the portfolio. We assume that good and bad types are exogenously different in that they have potentially different portfolios. The expected return of the portfolio for bad types is assumed to be \( r_B \), while the expected return for good types is \( r_G \). We assume that either portfolio can produce a return sufficiently low that the seller fails. With probability \( 1 - q_B \) a bad seller fails, while with probability \( 1 - q_G \) a good seller fails. In both cases of failure, the seller receives nothing (i.e., we assume limited liability). To distinguish between good and bad sellers, we let \( q_G > q_B \).

In the risk transfer contract we analyze, a seller collects \( P_j < 1 \) from the buyer in return for a contingent payment of size 1, and we assume that a seller only contracts with one buyer.\(^5\) The seller invests the premium in its portfolio, and thus it earns an expected return (conditional on not failing) of \( r_j \) from the premium. The probability of the buyer making a claim (i.e., a payment of 1 from the seller to the buyer) is \( 1 - p \). We now give a seller’s return, assuming that a contract is written.

\[
E(\pi_j) = q_j[p(1 + r_j)(W_j + P_j) + (1 - p)((1 + r_j)(W_j + P_j) - 1)], \quad j \in \{B, G\}. \tag{1}
\]

\(^5\)Our focus in this paper is not on the size of contract, but rather on determining which types of agents will purchase contracts, and with which types of counterparties. Adding endogenous contract size and/or non-exclusive contracts would needlessly complicate our analysis. For a generalization of the model that allows a contract size choice and the ability of an individual to contract with multiple sellers, see Stephens and Thompson (2014b).
We denote $P^0_j$ as the lowest price that a seller can charge and still break even on the contract, which is characterized by:

$$P^0_j = \frac{(1 - p)}{(1 + r_j)}.$$  \hspace{1cm} (2)

Clearly $r_B$ and $r_G$ determine which seller can charge a cheaper price in the presence of competition. This is discussed further in Section 3.

### 2.2 Buyers and Transaction Costs

There are many buyers that all share the same underlying asset risk, but which have different preferences over the desire to shed that risk. The asset can take one of two state contingent values: with probability $p$ it returns $R_B > 1$ (the high state), and with probability $1 - p$ it returns nothing (the low state). As described above, the buyer can contract with a seller to transfer the risk of the low state. If buyer $k$ incurs the loss from the low state and has not transferred this risk, it suffers the cost $Z_k \geq 0$.\(^6\) We assume that there is a measure $\mathbb{Z}$ of buyers such that $Z_k \in [0, \mathbb{Z}]$.\(^7\) This cost could represent an endogenous reaction caused by a shock to the buyer’s portfolio; however, we will not model this here. It is this cost that makes the buyer averse to holding risk.\(^8\) With only one asset type, the question of whether $Z$ is known to sellers is unimportant for the majority of the results in the paper. In Section 6, we allow for two asset types and make the natural assumption that $Z$ is not known to sellers.

When the buyer experiences a loss and is under contract with seller $j$, the buyer receives 1 if the seller is solvent and suffers the cost $Z_k$ if the seller is insolvent.\(^9\) We assume that entering into a contract comes with an exogenous cost $\Delta$. As discussed above, this cost can be interpreted in a number of ways, including legal fees, search costs or, as will be discussed below, as a transaction tax. The following expression characterizes the expected payoff of buyer type $k$ that contracts with seller type $j$:

$$E(\pi_{kj}) = pR_B + (1 - p)q_j - (1 - p)(1 - q_j)Z_k - P_j - \Delta \quad \forall Z_k \in [0, \mathbb{Z}].$$  \hspace{1cm} (3)

\(^6\)In an earlier version of the paper, we referred to those with high $Z_k$ as “hedgers” and those with low $Z_k$ (e.g., $Z_k = 0$) as “speculators”. This approach is in line with Keynes (1930) and Hicks (1946), who present early work that differentiates hedgers from speculators according to their risk aversion. Subsequent literature such as Hirshleifer (1975, 1977) has also emphasized informational asymmetries and beliefs in addition to risk aversion when modeling speculation. To analyze beliefs would require a full market microstructure model, which is beyond the scope of this paper. One could model beliefs of the buyer through the $Z$ parameter in addition to capturing risk aversion. Those buyers that have a more favorable view of the contract would have a higher $Z$. This would, of course, complicate the model; however, we note that the results below will obtain when there is divergent beliefs, provided that the high $Z_k$ types have a higher willingness to pay, irrespective of beliefs, due to risk aversion. Given the potential for confusion that may arise from these labels, we instead simply refer to buyers as having low- and high-risk aversion. Given the potential for confusion that may arise from these labels, we instead simply refer to buyers as having low- and high-risk aversion.

\(^7\)We suppose that buyers are uniformly distributed in $[0, \mathbb{Z}]$, that is, they are described by Lebesgue measure.

\(^8\)The variable $Z$ is used as a means to obtain risk transfer in a straightforward manner and was first introduced in Thompson (2010). A more formal discussion of this approach can be found in Stephens and Thompson (2014a).

\(^9\)The cost of counterparty risk and the cost of not entering the market are set equal for simplicity and can be allowed to differ without changing the qualitative results.
3 Full Information

We first consider the case in which there is no asymmetric information. The definition of equilibrium is given by the following.

Definition 1 An equilibrium is a set of prices \((P_G, P_B)\) and a contract choice for each buyer such that:

i. Prices are set by Bertrand competition amongst sellers.

ii. Given prices, buyers maximize expected profit by choosing whether to purchase a contract, and if so, at which type of seller.

Each buyer chooses whether to participate, and if so, a seller type with whom to contract. Given that the underlying asset risk is the same for each buyer, their choice depends solely on their preference parameter \(Z_k\). Since those with high \(Z_k\) suffer a larger cost when they do not transfer the risk, and when they do transfer the risk but the seller is unable to fulfil the contract, they are the most willing to enter the market and the most willing to contract with a good seller. We assume that the measure of each type of seller is sufficiently large to ensure competition under perfect information drives prices down to that which earn zero profit. We define \(\hat{Z}\) as the buyer that is indifferent between contracting with a good and a bad seller at zero-profit prices:

\[
\hat{Z} = \frac{P^0_G - P^0_B}{(1-p)(q_G - q_B)} - 1. \tag{4}
\]

We assume that when a buyer is indifferent between a good and bad seller, the good is chosen. We wish to focus on the most interesting case in which both sellers types are potentially active in equilibrium, thus we let \(Z > \hat{Z}\). This assumption does not imply that both types of sellers must be active, since (4) does not consider the effect of transaction costs on market participation. Define \(Z_j\) as the buyer that is indifferent between contracting with seller \(j\) and not contracting at all.

\[
Z_j = \frac{P_j + \Delta}{q_j(1-p)} - 1. \tag{5}
\]

Given that \(Z_k \geq 0\), it is obvious from (4) that for any buyer to contract with a bad seller, it must be that \(P^0_G - P^0_B > 0\). Intuitively, the bad seller cannot obtain a contract by setting a higher price and exposing the buyer to more counterparty risk than the good seller. If \(P^0_G - P^0_B < 0\), the result is obvious: buyers will either contract with good sellers, or not at all. For this reason, we restrict our discussion to the more interesting case in which \(r_B > r_G\) so that \(P^0_G - P^0_B > 0\). This case is the most natural given our set-up. Since we made the bad sellers more risky, it makes sense that the portfolio of that seller can earn higher returns. For example, the bad seller could represent a hedge fund with a high-return, high-risk portfolio, for which it is profitable to take contracts at a lower price, whereas the good seller could represent a well-diversified (safer) institution whose
investments are more conservative and thus lower yielding. This is made explicit in both Thompson (2010) and Stephens and Thompson (2014a), and we do not pursue the details here.

The following proposition characterizes the effect of transaction costs on the composition of buyers and sellers in the market.

**Proposition 1**

i. When $\Delta > (1 + Z)(1 - p)q_G - P^0_G$, there is no market for risk transfer.

ii. When $\left(\frac{P^0_G - P^0_B}{q_G - q_B}\right) - P^0_G \leq \Delta \leq (1 + Z)(1 - p)q_G - P^0_G$, only good sellers exist in the market.

iii. When $\Delta < \left(\frac{P^0_G - P^0_B}{q_G - q_B}\right) - P^0_G$, both good and bad sellers exist in the market.

When $\Delta > (1 + Z)(1 - p)q_G - P^0_G$, transaction costs are prohibitively high so that no buyer wishes to contract with any seller. Thus, the risk transfer market does not exist. In the intermediate case, only those buyers for which $Z_k \geq \hat{Z}$ contract in the market. In other words, only relatively risk-averse buyers participate in the market and contract only with good sellers. When transaction costs are relatively low, buyers for which $Z_k < \hat{Z}$ participate in the market and contract with bad sellers. Since contracts are of fixed size, total risk transfer is simply the total measure of buyers that contract in the market. The following corollary to Proposition 1 summarizes the effect of transaction costs on risk transfer and counterparty risk.

**Corollary 1**

i. When $\Delta \leq (1 + Z)(1 - p)q_G - P^0_G$ (i.e., when a market exists), risk transfer increases (decreases) as transaction costs decrease (increase).

ii. When $\Delta < \left(\frac{P^0_G - P^0_B}{q_G - q_B}\right) - P^0_G$ (i.e., when both types of seller are active), the relative number of bad sellers increases (decreases) as transaction costs decrease (increase).

As transaction costs decrease, the participation constraint of all buyers is relaxed and so permits some who otherwise would not find it optimal to contract to enter the market. When transaction costs are sufficiently low, further decreases in such costs will bring in buyers who prefer to contract with bad sellers, and thus the relative number of bad sellers increases as transaction costs decrease.

### 4 Asymmetric Information on Seller Quality

An important feature of many markets for financial risk transfer is that the buyer may not have perfect information as to the counterparty risk of the seller, whereas the seller is better informed about its own risk.\(^\text{10}\) To introduce asymmetric information, we assume that bad sellers have the...
ability to conceal their type from buyers.\textsuperscript{11} Denote the fraction of bad sellers who choose to conceal their type by \( \phi \) (thus the fraction who reveal their type is \( 1 - \phi \)). As will be described in detail below, the buyer may choose to contract with a revealing bad seller, or with a pool which consists of both good and bad sellers. Since prices are observable but seller type is not, the buyer is uncertain of the likelihood that a pooled seller will fulfill their obligations. The probability that a seller chosen from the pool succeeds is given by 
\[
q_{pl} = \frac{(Nq_G + \phi Nq_B) / N (1 + \phi)}{1 + \phi} = \frac{q_G + \phi q_B}{1 + \phi}.
\]
Relative to the full information setting described above (in which \( \phi = 0 \) by definition), the value of \( \phi \) is a feature of the equilibrium (to be defined below).

To analyze asymmetric information on seller quality, we restrict ourselves to the most interesting case in which there exists a market under full information. This requires that transaction costs are not so high that even the most risk-averse buyer does not participate. This is ensured by the following assumption (see Proposition 1).

**Assumption 1** \( \Delta < (1 + Z)(1 - p)q_G - P_{0G}^G \).

Furthermore, in this section we also assume that the number of sellers of each type is sufficiently large to ensure that prices would be that which earn zero profit for both types of sellers under perfect information. This is a simplifying assumption that provides a sufficient condition for a unique solution in Lemma 3 below, and it can be relaxed while still obtaining our results.

**Assumption 2** \( N \geq Z + 1 \).

We must also consider the beliefs of the buyers. We assume that the buyer only learns the quality of the seller if the price is below that to which a good seller could charge and still earn non-zero profit.\textsuperscript{12} We now define the equilibrium when there is asymmetric information over seller quality. Define \( E(\pi_{pl}^B(\phi)) \) as the expected profit of a bad seller when pooling, which depends on the total fraction of bad sellers that pool. Likewise, define \( E(\pi_{rev}^B(\phi)) \) as the expected profit of a bad seller that reveals. Contrary to the perfect information case, the definition of equilibrium must take into account the choice now available to bad sellers: whether to reveal or to pool. In equilibrium, the payoff to revealing must not exceed that to pooling, otherwise more bad sellers will reveal.

**Definition 2** An equilibrium is a set of prices \((P_G, P_B)\), a contract choice for each buyer and a value \( \phi \) such that:

i. Prices are set by Bertrand competition amongst sellers.

ii. Upon observing a premium \( P \) less than \( P_{0G}^G \), a buyer’s belief is that the seller is bad. For prices \( P \geq P_{0G}^G \), beliefs reflect population averages.

\textsuperscript{11}In a previous version of the paper, we considered a more general case in which an exogenous fraction of bad sellers are revealed to the buyers by nature, while the rest are able to conceal their type. All the main results to be discussed below follow through in this more general environment.

\textsuperscript{12}We could use more general beliefs; however, this will only serve to complicate the analysis without offering additional insights. The approach taken here corresponds to Stephens and Thompson (2014a).
iii. Given prices and beliefs, buyers maximize expected profit by choosing whether to purchase a contract, and if so at which type of seller.

iv. $\phi = 1$ if $E(\pi_{pl}^B(\phi = 1)) > E(\pi_{rev}^B(\phi = 1))$, otherwise $\phi \in [0, 1)$ such that $E(\pi_{pl}^B(\phi)) = E(\pi_{rev}^B(\phi))$.

The analysis is broken down into two cases: the first, in which no bad seller obtains a contract under perfect information, and the second, in which they do.

4.1 Asymmetric Information on Seller Quality: High Transaction Costs

We first consider the relatively straightforward case in which no buyer would knowingly contract with a bad seller, as given by condition ii of Proposition 1. Given competition among good sellers, each charges $P^G_0$ in equilibrium. Because setting a price below $P^G_0$ results in a bad seller being revealed, and therefore unable to participate in the market, competition among good and bad sellers implies the pooling price must be $P^G_0$ and the equilibrium distribution of pooling bad sellers is $\phi^* = 1$. To ensure that the market exists under asymmetric information, we must put an upper bound on transaction costs. The following condition on $\Delta$ ensures that there are buyers in the market who contract with the pool of sellers.

$$(P^0_G - P^0_B)(q_B + q_G) - P^0_G \leq \Delta < (1 + Z)(1 - p)\frac{q_B + q_G}{2} - P^0_G,$$

The left hand side of this expression is condition ii of Proposition 1 and ensures that no buyer would contract with a bad seller under perfect information. The right hand side of this expression ensures that the transaction cost is not so high that no buyer would contract with the pool. This leads to the following proposition:

**Proposition 2** When $(1 + Z)(1 - p)\frac{q_B + q_G}{2} - P^0_G < \Delta < (1 + Z)(1 - p)q_G - P^0_G$ the market collapses due to the presence of asymmetric information over seller quality.

Thus, when transaction costs are sufficiently high, the market collapses (the left hand side of the expression in the proposition) where it otherwise would have existed with perfect information (the right hand side of the expression in the proposition). The intuition behind this result is that with perfect information, buyers can contract with a good seller with certainty. With asymmetric information, the expected quality of the pool is less than that of a good seller while price remains unchanged.

4.2 Asymmetric Information on Seller Quality: Low Transaction Costs

As detailed in Proposition 1, for sufficiently low transaction costs, some buyers prefer to contract with bad sellers rather than with good ones. Since a pooling bad seller only obtains the contract with some probability, it follows that there may be an incentive for a subset of bad sellers to reveal their type and obtain a contract with certainty. In other words, bad sellers can either charge the
price $P_B^G$ and pool with the good sellers (as above) or reveal themselves and set a price $P_B$ given competition from other revealing bad sellers. The following lemma characterizes properties of the revealed bad sellers’ price, which are consistent with competition for a given value of $\phi$.

**Lemma 1** The price at the bad sellers, $P_B(\phi)$, satisfies:

1. $N(1 - \phi) = \hat{Z}_{pl}(P_B(\phi)) - Z_B(P_B(\phi))$ if $N(1 - \phi) < \hat{Z}_{pl}(P_B^0) - Z_B(P_B^0)$
2. $P_B(\phi) = P_B^0$ if $N(1 - \phi) \geq \hat{Z}_{pl}(P_B^0) - Z_B(P_B^0)$.

Where, analogous to $\hat{Z}$ described in (4), we define

$$\hat{Z}_{pl}(P_B(\phi)) = \frac{(P_B^0 - P_B(\phi))(1 + \phi)}{(1 - p)(q_G - q_B)} - 1,$$

which represents the buyer that is indifferent between contracting with the pool of sellers or with a revealed bad seller, and

$$Z_B(P_B(\phi)) = \frac{P_B(\phi) + \Delta}{q_B(1 - p)} - 1$$

represents the buyer that is indifferent between contracting with a bad seller and dropping out of the market. Lemma 1 determines the equilibrium price that revealing bad sellers will charge. Consider condition $i$, wherein demand is such that the bad sellers earn positive profit. The left hand side of the equality represents the number of revealing bad sellers, while the right hand side represents the number of buyers who choose to contract with bad sellers. In this case, the revealed bad sellers raise their price above that which would earn zero profit until excess demand is eliminated. This case occurs if, at the zero profit price of the bad sellers, there would be more buyers than sellers (this is the inequality of condition $i$). Condition $ii$ is the case in which there are more revealed bad sellers than buyers with whom contract. In this case, the price must be that of zero profit due to competition for contracts. Given Lemma 1, we define $\hat{\phi}$ such that $\phi \leq \hat{\phi}$ implies $P_B(\phi) = P_B^0$ and $\phi > \hat{\phi}$ implies $P_B(\phi) > P_B^0$. Using (7) and (8), rewrite item $i$ of Lemma 1 as follows:

$$(1 - \phi)N = \frac{(P_B^0 - P_B)(1 + \phi)}{(1 - p)(q_G - q_B)} - \frac{P_B + \Delta}{q_B(1 - p)}.$$  

Rearranging for $P_B$ yields:

$$P_B = \frac{\frac{P_B^0(1+\phi)}{(1-p)(q_G-q_B)} - (1 - \phi)N}{q_B(1-p)} + \frac{\Delta}{(1-p)(q_G-q_B)}.$$  

The following lemma describes some useful properties of $P_B(\phi)$:

**Lemma 2**

1. $P_B(\phi) < P_B^G$
\[ \frac{dP_B(\phi)}{d\phi} \geq 0, \text{ where the inequality is strict for } P_B(\phi) > P_B^0. \]

**Proof.** See Appendix.

The first item is relatively straightforward. The price that a revealing bad seller can charge must be below that which the pool is charging, otherwise no buyer would ever contract with a revealed bad seller. The second item says that the more bad sellers there are pooling with the good sellers, the higher the price that the revealed bad sellers can charge. This occurs because the competition among revealed bad sellers decreases when there are less of them. In equilibrium, \( \phi \) and \( P_B \) must be such that the expected profit of a pooling bad seller is at least that of a revealing bad seller. This is characterized by the following condition.

\[
(1 + r_B)P_B(\phi) - (1 - p) \leq \frac{Z - \hat{Z}_{pl}(P_B(\phi))}{N(1 + \phi)} - [(1 + r_B)P_B^0 - (1 - p)] \tag{11}
\]

Where \( (Z - \hat{Z}_{pl}(P_B(\phi)))/(N(1 + \phi)) \) is the probability that a pooling bad seller obtains a contract (recall that there are \( N \) good sellers and \( \phi N \) potential pooling bad sellers), so that the right hand side of (11) is the expected profit of a pooling bad seller, \( E(\pi_{pl}^B(\phi)) \). The left hand side is the expected profit of a revealing bad seller, \( E(\pi_{rev}^B(\phi)) \). The following result determines the equilibrium value of \( \phi \), which we denote \( \phi^* \).

**Lemma 3** There exists a unique equilibrium \( \phi^* \in (\hat{\phi}, 1] \) that satisfies (11).

**Proof.** See Appendix.

The equilibrium \( \phi^* \) defined in the lemma is set such that a bad seller is indifferent between revealing and pooling or strictly prefers the latter. In the case where the preference is strict, i.e., (11) is a strict inequality, \( \phi^* = 1 \). Note that all the information rents of the bad sellers disappear and the profits approach zero for both revealing and pooling sellers as \( N \) becomes arbitrarily large. To understand this, consider a pooling seller. As \( N \) becomes large, the probability of obtaining the contract becomes small. As bad sellers exit the pool (i.e., reveal), the revealing price is driven down to that which earns zero profit.

When considering the effect of asymmetric information on counterparty risk, we wish to look beyond the obvious effect, namely that buyers can no longer contract with good sellers with certainty. The following proposition highlights two less obvious effects on the relative number of good and bad sellers.

**Proposition 3**

\[ (i. \quad \text{There are } (P_G^0 \cdot \phi^* - P_B(\phi^*) \cdot (1 + \phi^*) + P_B^0)/(1 - p)(q_G - q_B) \text{ buyers that contract with good sellers with perfect information but choose bad sellers over the pool with asymmetric information.}) \]

\[ (ii. \quad \text{There are } (P_B(\phi^*) - P_B^0)/q_B(1 - p) \text{ buyers that contract with bad sellers with perfect information but drop out of the market with asymmetric information.}) \]
For $N$ sufficiently large, the first effect dominates the second.

Proof. See Appendix.

Given the information friction, buyers who would contract with good sellers under perfect information may contract with bad sellers instead of the pool of good and bad under asymmetric information. The number of buyers who do this is given by $\hat{Z}_{pl}(P_B(\phi^*)) - \hat{Z} > 0$, which is equivalent to the condition in the proposition. These buyers switch to bad sellers to avoid the pooled price (i.e., the lower quality coverage for the same price as would have been paid at good sellers with perfect information), thereby increasing the relative number of bad sellers in the market. Given that competition between revealed bad sellers decreases under asymmetric information, those that reveal can also extract rents by charging higher prices. In this case, risk transfer decreases as those with low $Z$ drop out of the market. The number of buyers that drop out is given by $Z_B(P_B(\phi^*)) - Z_B(P^0_B)$, which is equivalent to the condition in the proposition. Thus, some contracts which would have been with bad sellers under perfect information no longer trade with asymmetric information, thereby decreasing the relative number of bad sellers. As $N$ increases, the expected return in the pool decreases for a fixed $\phi$. Thus, in equilibrium, more bad sellers reveal, thereby driving down the price of revealed bad sellers, i.e., $P_B(\phi^*)$ decreases. Since $P_B(\phi^*)$ decreases, it follows that $\hat{Z}_{pl}(P_B(\phi^*))$ increases. Thus, it is straightforward to see why when $N$ is sufficiently large, the first effect of Proposition 3 dominates the second.

5 A Transaction Tax

Transaction taxes have received much attention since the onset of the credit crisis in 2007. The consideration of such a tax in our model is relatively straightforward and allows for insights not currently found in the literature.\textsuperscript{13} A transaction tax can be viewed as an increase in transaction costs, which as we have seen has important implications for market outcomes. It is important to emphasize that the intention of this section is to add a new element to the debate over transaction taxes. Since our framework is not constructed for a welfare analysis, we do not make unconditional statements as to whether such a tax is desirable. The following proposition summarizes the effect of a transaction tax (i.e., an increase in transaction costs) on both the composition of buyers and sellers in the market, as well as on the rents that arise under asymmetric information.

Proposition 4 When both seller types are active under full information, the introduction of a transaction tax (i.e., an increase in transaction costs):

i. Causes the lowest $Z_k$ buyers to drop out of the market, thereby reducing risk transfer under both full and asymmetric information.

\textsuperscript{13}For an in-depth discussion of transaction taxes see, for example, Matheson (2011).
ii. Reduces the relative number of bad sellers in the market under full information. With asymmetric information on seller quality, the relative number of bad sellers falls when there is a sufficient quality difference between good and bad sellers or when \(N\) is large.

iii. Reduces the information rent that can be extracted when there is asymmetric information on seller quality.

**Proof.** See Appendix.

A tax pushes the lowest \(Z_k\) buyers out of the market because the effective cost of protection increases, and buyers that are the least risk averse find it too expensive to contract. When transaction costs are sufficiently small to begin with, under perfect information, those that drop out are low \(Z_k\) buyers, which are precisely those that choose to contract with bad sellers. Thus, the proportion of bad to good sellers falls with the tax. In the unknown seller case, there are two key effects. That which is described in the perfect information case, and that in which a transaction tax induces some buyers that would otherwise contract with the pool to contract with revealed bad sellers. This is because revealed bad sellers charge a cheaper price, since there are now fewer buyers willing to contract with them. For this effect to dominate depends crucially on the form of competition, namely that pooling sellers cannot lower the price, while revealing bad sellers can. If, for example, \(N\) is large so that the revealed bad seller price is pushed down to that which earns zero profit, both of the prices could not decrease. In this case, the second effect is not present since the revealed bad sellers cannot price the contract any lower. In an enriched model of competition, both the price in the pool and that of revealed bad sellers would be flexible, thereby muting the second effect. In our model, in which the revealing price can decrease while the pooling price cannot, it stands that there would be little advantage to contracting with the pool over revealed bad sellers if the quality of good and bad sellers is sufficiently close. As such, we focus on the most interesting case, in which there is sufficient quality difference so that the main effect on the relative number of bad sellers is that of low \(Z_k\) types dropping out of the market, as in the perfect information case.

The final result, that information rents decrease, follows easily from the above arguments and holds regardless of what happens to the relative number of bad sellers. A tax pushes the lowest \(Z_k\) buyers out of the market, thereby pushing down the price that revealed bad sellers can charge, thus lowering their information rent. The pooling bad sellers must also have a lower information rent in equilibrium, since they must be indifferent between pooling and revealing. In this case, when low \(Z_k\) buyers leave the market, a portion of revealing sellers choose instead to pool, thereby lowering the expected profit in the pool due to the decreased probability that a bad seller obtains a contract.

6 Extension: Asymmetric Information on the Underlying Asset

We turn to the more traditional problem of asymmetric information regarding the quality of the underlying asset. Acharya and Johnson (2007), for example, provide evidence in the context of
credit derivatives that those who purchase these contracts (banks in their case) can have superior information as to the quality of the underlying asset. To model this situation, we consider the case in which seller quality is known and add one new feature to the model of Section 2. Assume now that a buyer possesses one of two types of assets with equal probability: (R)isky and (S)afe, indexed by \( i \in \{ R, S \} \). Each asset yields return \( R_B \) with probability \( p_i \); otherwise it defaults with probability \( 1 - p_i \) and returns nothing. It is assumed that the risky asset is less likely to return \( R_B \), so that \( p_R < p_S \). There is a measure \( Z \) of each buyer type that differ by their preference parameter \( Z \), again distributed over \([0, \overline{Z}]\). We assume that \( Z \) is not known to the sellers.\(^{14}\)

Whereas in Section 2.1 each seller knew the probability that the asset would return zero, they now form a belief \( b_j \) as to the probability of the low state. Our concept of equilibrium is analogous to that above. We assume sufficient competition such that a seller charges their zero profit price given beliefs, which is denoted \( P_0^j(b_j) \). Further, in equilibrium, the sellers’ beliefs are assumed to be consistent.

When buyer type is unknown, a seller can only charge one pooling price.\(^{15}\) To characterize beliefs, let \( \theta_G (\theta_B) \) represent the probability that the buyer of the contract at a good (bad) seller is the safe type. We modify the zero profit prices as defined in (2) to explicitly account for beliefs (which are consistent in equilibrium):

\[
P_0^j(b_j) = \frac{b_j}{1 + r_j} = \frac{(1 - p_S)\theta_j + (1 - p_R)(1 - \theta_j)}{1 + r_j}.
\]

In what follows, we denote prices \( P_0^j(\theta_j) \) where convenient. Analogous to (4) under perfect information, we define the \( \hat{Z} \) for which a safe and risky type buyer is indifferent between contracting with a good and bad seller:

\[
\hat{Z}^S = \frac{P_0^G(\theta_G) - P_0^B(\theta_B)}{(1 - p_S)(q_G - q_B)} - 1
\]

\[
\hat{Z}^R = \frac{P_0^G(\theta_G) - P_0^B(\theta_B)}{(1 - p_R)(q_G - q_B)} - 1.
\]

Under full information, sellers offer prices \( P_0^j(1 - p_S) \) and \( P_0^j(1 - p_R) \) if they contract with a safe or risky type. As in Section 3, we analyze the most interesting case, in which some members of both buyer types would potentially contract with both seller types under perfect information.

\(^{14}\)For the sellers to observe \( Z \), they would have to know the motivation of the buyers to trade, which would be an extreme assumption. Even in such a case, we could obtain our qualitative results; however, the analysis would be more tedious. For a formal market microstructure model that allows for multiple unknown attributes of traders see, for example, Gervais (1997).

\(^{15}\)Note that we rule out non-linear pricing and thus the possibility of a separating equilibrium. This is reasonable given that financial risk transfer contracts are non-exclusive, i.e., the seller cannot preclude the buyer from purchasing the same contract from other sellers. Stephens and Thompson (2014b) show that if the buyer can split the contract over many sellers, separation cannot be achieved through a menu of contracts. However, separation may be possible if there is aggregate risk to which a bad seller is subjected. In the current paper, the fixed contract size rules out separation through menus of contracts, and since there is no aggregate risk, extending the model in the spirit of Stephens and Thompson (2014b) would justify focusing on only pooling in equilibrium.
(actual participation in the market with either seller type will depend on transaction costs). This requires $Z > \hat{Z}^S > \hat{Z}^R$ for any set of beliefs, where the first inequality is true for sufficient $Z$ and the second is true by definition, as can be seen by comparing (13) and (14). Note that because $Z > \hat{Z}^S > \hat{Z}^R$, the most risk-averse buyer $\bar{Z}$ of either underlying risk type will always choose the good seller when participating in the market. The following assumption, which is analogous to Assumption 1, ensures that at least some positive measure of both buyer types wish to participate under full information.

**Assumption 3** \( \Delta < (1 + \bar{Z})(1 - p_i)q_G - P^0_G(1 - p_i), \ i \in \{R, S\} \).

We now turn to market participation and define $Z^S_j$ ($Z^R_j$) as the $Z$ for which a safe (risky) type buyer is indifferent between contracting at seller $j$ and not entering in the market:

\[
Z^S_j = \min \left( Z, \frac{P^0_j(\theta_j) + \Delta}{(1 - p_S)q_j} - 1 \right) \quad (15)
\]

\[
Z^R_j = \frac{P^0_j(\theta_j) + \Delta}{(1 - p_R)q_j} - 1. \quad (16)
\]

Given $P^0_j((1 - p_S)\theta_j + (1 - p_R)(1 - \theta_j)) \leq P^0_j(1 - p_R)$, Assumption 3 implies that there will be risky types in the market, or equivalently $Z^R_j < \bar{Z}$. The safe types, however, may or may not be in the market when they are forced to pool with the risky types. Using the preceding definitions, the following expression provides implicit expressions for $\theta_G$ and $\theta_B$, which are the proportions (probabilities) of safe buyers at the good and bad sellers, respectively. Note that these expressions are only defined when there are buyers at the respective sellers.

\[
\theta_G = \frac{Z - \hat{Z}^S}{2\bar{Z} - (\hat{Z}^S + \hat{Z}^R)} \in [0, 1/2] \quad (17)
\]

\[
\theta_B = \frac{\hat{Z}^S - Z^S_B}{Z^S - Z^S_B + \hat{Z}^R - Z^R_B} \in [1/2, 1] \quad (18)
\]

That the proportion of safe buyers fall into these ranges can be shown simply by substituting expressions (13)-(16) into the definitions of $\theta_G$ and $\theta_B$.\(^{16}\) The following proposition represents the main result of this section:

**Proposition 5** When $(1 + \bar{Z})(1 - p_S)q_G - P^0_G(1 - p_S) \geq \Delta \geq (1 + \bar{Z})(1 - p_S)q_G - P^0_G(1 - p_R)$, the market contains both underlying risk types under full information, but only risky types under asymmetric information. When $\Delta < (1 + \bar{Z})(1 - p_S)q_G - P^0_G(1 - p_R)$, risk transfer can occur with asymmetric information in which both types of buyers participate in the market.

**Proof.** See Appendix.

\(^{16}\)Note that $\theta_G = \frac{1}{2}$ and $\theta_B = \frac{1}{2}$ in the case when $p_S = p_R$. 

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Without asymmetric information, Assumption 3 ensures that both safe and risky buyers will be in the market. However, with asymmetric information, if the transaction costs are too high \((1 + Z)(1 - ps)q_G - P^B_G(1 - ps) \geq \Delta \geq (1 + Z)(1 - ps)q_G - P(1 - p_R))\), the safe buyers do not wish to pool with the risky buyers, and so drop out of the market. Therefore, in equilibrium only risky buyers are left, and thus the sellers know with certainty with whom they are contracting so that asymmetric information ceases to exist. In this case, the amount of risk transfer decreases relative to the full information case. The mechanism behind this result is that of a lemons problem, first explored in Akerlof (1970).

6.1 Transaction Tax with Asymmetric Information on the Underlying Asset

We can also consider the implications of a transactions tax in this environment. It is straightforward to show that part \(i\) of Proposition 4 continues to hold with asymmetric information on the underlying asset. With respect to part \(ii\) of Proposition 4, we can show that the relative number of bad sellers in the market is unambiguously reduced by a transaction tax with asymmetric information on the underlying asset. To show this, we define \(\theta_{B,G}\) as the proportion of bad sellers in the market given by:

\[
\begin{align*}
\theta_{B,G} &= \frac{(\hat{Z}^S - Z^S_j) + (\hat{Z}^R - Z^R_j)}{(Z - \hat{Z}^S + \hat{Z} - \hat{Z}^R) + (\hat{Z}^S - Z_j^S) + (\hat{Z}^R - Z_j^R)} \\
&= 1 - \left[\frac{2Z - (\hat{Z}^S + \hat{Z}^R)}{2Z - (Z^S_j + Z^R_j)}\right]
\end{align*}
\]

From (13) and (14) it follows that \(\hat{Z}^S\) and \(\hat{Z}^R\) are independent of \(\Delta\). From (15) and (16), it can be seen that \(Z^S_j\) and \(Z^R_j\) are increasing in \(\Delta\). It follows that \(\frac{d\theta_{B,G}}{d\Delta} < 0\), so that the proportion of good (bad) sellers is increasing (decreasing) in transaction costs.

Finally, we note that information rents in this environment are reduced by such a tax as well. From (15) and (16), it is clear that with asymmetric information on the underlying asset, an increase in \(\Delta\) causes a greater reduction of safe types in the market than risky, which in turn reduces \(\theta_G\) and \(\theta_B\). It is straightforward to show that the pooling price is strictly decreasing in \(\theta_G\) and \(\theta_B\), and so the information rents of risky buyers are decreasing in \(\Delta\) (ignoring the trivial case where \(\theta_G = 0\) and \(\theta_B = 0\), in which there are no information rents).

In summary, when the underlying asset quality is unknown to sellers, transaction costs have a heterogenous effect on high- and low-risk buyers. More of the low risk buyers will drop out of the market as they find protection less necessary/beneficial than the high-risk buyers. As such, the quality of the pool decreases, and so the equilibrium price gets closer to that which the high-risk buyers would pay under perfect information, and thus their information rents decrease.
7 Conclusion

In this paper we analyze transaction costs in a simple market for risk transfer. We show that when transaction costs are high (which would be typical for a risk transfer market in its infancy), the market is used only by the most risk-averse buyers, and only high-quality counterparties can be present. We show that a prerequisite for a market to exist with high transaction costs is that there be no asymmetric information on the quality of the counterparty. As transaction costs decrease, we show, less risk-averse buyers (e.g., those that might have less exposure to the underlying risk) will join the market, leading to an increase in the relative number of lower quality counterparties that are active. A transaction tax is then analyzed as simply an increase in transaction costs. We show that such an increase can reduce the relative number of unstable sellers and the information rents that can be extracted when there is asymmetric information.

Going forward, it would be fruitful to embed transaction costs into a financial fragility/contagion framework by, for example, connecting the sellers through a market, in the spirit of Allen and Gale (2000), and Allen and Carletti (2006).
8 Appendix

Proof of Lemma 2

Part i): we re-write our expression for $P_B(\phi)$ (10) as follows:

$$P_B(\phi) = P_G^0 \left[ \frac{(1+\phi)}{(1-p)(q_G-q_B)} \right] - \left[ \frac{(1-\phi)N + \frac{\Delta}{q_B(1-p)}}{q_B(1-p) + (1-p)(q_G-q_B)} \right]$$

since

$$\frac{(1+\phi)}{(1-p)(q_G-q_B)} < 1. \quad (21)$$

Part ii): Since $P_B^0$ is constant, $dP_B^0/d\phi = 0$. Consider now the case in which $P_B(\phi) > P_B^0$. Define the following two variables:

$$\eta_1 = \frac{P_G^0 (1+\phi)}{(1-p)(q_G-q_B)} - (1-\phi)N - \frac{\Delta}{q_B(1-p)}, \quad (22)$$

$$\eta_2 = \frac{1}{q_B(1-p) + (1-p)(q_G-q_B)} \quad (23)$$

Thus,

$$\frac{dP_B(\phi)}{d\phi} = \frac{P_G^0 (1+\phi)}{(1-p)(q_G-q_B)} + \frac{N}{\eta_2} \left( \frac{m}{(1-p)(q_G-q_B)} \right)$$

$$> 0, \quad (24)$$

using $P_B(\phi) = \eta_1/\eta_2$ and $P_G^0 > P_B(\phi)$, as shown in part i).

Proof of Lemma 3

Define the return for revealing and pooling bad sellers as follows:

$$F(\phi) = (1+r_B)P_B(\phi) - (1-p), \quad G(\phi) = \frac{Z - \hat{Z}_{pl}}{N(1+\phi)} ((1+r_B)P_G^0 - (1-p)), \quad (27)$$
and define \( H(\phi) = G(\phi) - F(\phi) \). We now show that \( H'(\cdot) < 0 \).

\[
H'(\phi) = \frac{dP_B(\phi)}{d\phi} \left( \frac{(1 + r_B)P_G^0 - (1 - p)}{N(1 - p)(q_G - q_B)} - (1 + r_B) \right) - \frac{(1 + \bar{Z})[(1 + r_B)P_G^0 - (1 - p)]}{N(1 + \bar{\phi})^2}.
\] (28)

From Lemma 2, \( dP_B(\phi)/d\phi \geq 0 \), thus a sufficient condition for \( H'(\cdot) < 0 \) is

\[
(1 + r_B)P_G^0 - (1 - p) \leq 0 \Leftrightarrow \frac{(1 + r_B)P_G^0 - (1 - p)}{(1 + r_B)(1 - p)(q_G - q_B)} \leq N.
\] (29)

Inserting the zero profit prices and simplifying we have

\[
\frac{r_B - r_G}{(1 + r_B)(1 + r_G)(q_G - q_B)} = \hat{Z} + 1 < Z + 1 \leq N,
\] (30)

where the last inequality holds by Assumption 2. Now consider \( \phi = \hat{\phi} \). From Lemma 1 and equation (2), we have \( P_B(\phi) = P_B^0 = (1 - p)/(1 + r_B) \), so that \( F(\hat{\phi}) = 0 \) and thus, \( H(\hat{\phi}) > 0 \) whenever the probability of obtaining a contract in the pool is non-zero. If \( H(1) \geq 0 \), \( \phi^* = 1 \), since \( H(\cdot) \) is non-increasing. If \( H(1) < 0 \), then there is a unique interior value \( \phi^* < 1 \), which satisfies \( H(\phi^*) = 0 \).

\[ \blacksquare \]

**Proof of Proposition 3**

With perfect information, the \( Z_k \) that is indifferent between good and bad insurance differs from that of the \( Z_k \) that is indifferent between pooled and bad insurance under asymmetric information. The difference between the two is given by \( \hat{Z}_{pl}(P_B(\phi^*)) - \hat{Z} > 0 \). Substituting \( \hat{Z}_{pl}(P_B(\phi^*)) \) and \( \hat{Z} \) yields the condition in the proposition.

With perfect information, the smallest \( Z_k \) in the market is given by \( Z_B(P_B^0) \). With asymmetric information, the smallest \( Z_k \) is given by \( Z_B(P_B(\phi^*)) \). Thus, substituting these into their difference yields the expression in the proposition.

Consider the case where \( N \) becomes arbitrarily large. In this case \( P_B(\phi^*) \to P_B^0 \) so that \( Z_B(P_B(\phi^*)) - Z_B(P_B^0) \to 0 \), while \( \hat{Z}_{pl}(P_B(\phi^*)) \to \hat{Z} \to \hat{Z}_{pl}(P_B^0) - \hat{Z} > 0 \) so that the first effect dominates the second. Consider a finite \( N \). Since the first effect is increasing in \( N \) while the second is decreasing, standard reasoning considering endpoints shows that there exists an \( N \) such that the first effect dominates the second.

\[ \blacksquare \]

**Proof of Proposition 4**

i. With perfect information, it is straightforward to see that \( dZ_B/d\Delta > 0 \), where \( Z_B \) is given by (5). Now consider the case with asymmetric information. Assume, to the contrary, that
\(Z_B(P_B^*)\) remains the same or decreases. For it to remain the same, the \(P_B^*\) must fall by \(\Delta\), as can be seen from (8). However, in this case, the payoff to revealing is lower than that in the equilibrium without the transaction tax, while the payoff to pooling is the same. Thus, this cannot be an equilibrium. The same argument applies to the case in which \(Z_B(P_B^*)\) decreases. It follows that \(Z_B(P_B^*)\) must increase so that risk transfer decreases.

\(\text{ii.} \) With perfect information, this follows from Corollary 1. Now consider the case with asymmetric information. Define \(\gamma\) as the fraction of contracts with revealing bad sellers to those with the pool, given by
\[
\gamma = \frac{\hat{Z}_{pl}(P_B^*) - Z_B(P_B^*)}{Z - Z_B(P_B^*)}.
\] (31)

We wish to show that \(d\gamma/d\Delta < 0\). Differentiating yields
\[
\frac{d\hat{Z}_{pl}(P_B^*)}{d\Delta} - \frac{dZ_B(P_B^*)}{d\Delta} - \frac{d\hat{Z}_{pl}(P_B^*)}{d\Delta} \left(\frac{\hat{Z}_{pl}(P_B^*) - Z_B(P_B^*)}{(Z - Z_B(P_B^*))^2}\right) < 0.
\] (32)

In the limiting case where \(N\) becomes arbitrarily large, \(P_B = P_B^0\), and so \(d\hat{Z}_{pl}(P_B^*)/d\Delta = 0\). Therefore, (32) reduces to
\[
- \frac{dZ_B(P_B^*)}{d\Delta} < 0,
\] (33)

which follows easily by differentiating (8) with respect to \(\Delta\). Now consider the case in which \(N\) is finite. Taking derivatives and simplifying (32) using the definition of \(\gamma\) yields the following:
\[
q_G(1 - \gamma) - q_B(2 - \gamma - \phi) > 0,
\] (34)

which is satisfied whenever \(q_G - q_B\) is sufficiently large.

\(\text{iii.} \) From Part i we know that \(dP_B(\phi)/d\Delta < 0\), thus information rents decrease for revealing bad sellers. The expected payoff of a pooling bad seller, denoted \(E(\pi_{pl}^*)\) is
\[
E(\pi_{pl}^*) = \frac{Z - \hat{Z}_{pl}(P_B(\phi))}{N(1 + \phi)} \left[(1 + r_B)P_G^0 - (1 - p)\right]
\] (35)
\[
= \frac{Z - \left((P_G^0 - P_B(\phi))(1 + \phi) - 1\right)}{N(1 + \phi)} \left((1 + r_B)P_G^0 - (1 - p)\right),
\] (36)

and it follows that \(dE(\pi_{pl}^*)/d\Delta < 0\) since \(dP_B(\phi)/d\Delta < 0\), so that rents of the pooling bad sellers also decrease.
Proof of Proposition 5

Consider when \((1 + Z)(1 - p_S)q_G - P_G^0(1 - p_S) \geq \Delta \geq (1 + Z)(1 - p_S)q_G - P_G^0(1 - p_R)\). The left side ensures that \(\Delta\) is low enough to ensure market participation of safe types under full information (this also implies participation of risky types). The right hand side is the threshold value for \(\Delta\) such that a higher transaction cost results in a complete exodus of safe buyers. This can be seen from (15) as follows:

\[
\bar{Z} \leq \frac{P_G^0(\theta_G) + \Delta}{(1 - p_S)q_G} - 1 \Rightarrow \Delta \geq (1 + \bar{Z})(1 - p_S)q_G - P_G^0(\theta_G),
\]

(37)

where \(\theta_G = 0\) and so \(b_G = 1 - p_R\) as indicated in Proposition 5. When \(\Delta < (1 + \bar{Z})(1 - p_S)q_G - P_G^0(\theta_G)\), safe types can exist in equilibrium. To show this, we rewrite the expression for the fraction of safe types at the good and bad sellers, initially given in (17) and (18) as follows:

\[
\theta_G = \mathcal{G}(\theta_G, \theta_B) = \frac{1 + \bar{Z} - \left(\frac{P_G^0(\theta_G) - P_B^0(\theta_B)}{(1 - p_S)q_G - q_B}\right)}{2(1 + \bar{Z})} - \frac{\left(\frac{P_G^0(\theta_G) - P_B^0(\theta_B)}{(1 - p_S)q_G - q_B}\right)}{\left(\frac{P_G^0(\theta_G) - P_B^0(\theta_B)}{(1 - p_S)q_G - q_B}\right) + \left(\frac{P_G^0(\theta_G) - P_B^0(\theta_B)}{(1 - p_R)q_G - q_B}\right)}
\]

(38)

\[
\theta_B = \mathcal{B}(\theta_G, \theta_B) = \frac{\frac{P_G^0(\theta_G) - P_B^0(\theta_B)}{(1 - p_S)q_G - q_B} - \frac{P_B^0(\theta_B) + \Delta}{q_B(1 - p_S)}}{\frac{P_G^0(\theta_G) - P_B^0(\theta_B)}{(1 - p_S)q_G - q_B} - \frac{P_B^0(\theta_B) + \Delta}{q_B(1 - p_R)}}.
\]

(39)

Define \(\mathcal{F}(\theta_G, \theta_B) = \mathcal{G}(\theta_G, \mathcal{B}(\theta_G, \theta_B)) - \theta_G = 0\). It follows that since prices \(P_j^0(\theta)\) are linear in \(\theta_j\), \(\mathcal{F}(\cdot)\) is a continuous function that maps \([0, 1/2] \times [1/2, 1]\) into itself. Thus an equilibrium exists by Brouwer’s fixed point theorem. Note that \(\theta_G\) and \(\theta_B\) are only defined when there are buyers at both types of sellers. For sufficiently high \(\bar{Z}\), \(\theta^*_G > 0\), and for sufficiently low \(\Delta\), \(\theta^*_B > 0\). The case in which there are no buyers at the bad seller and so \(\theta_B\) is undefined is possible. To characterize the equilibrium in this case, we must define off equilibrium beliefs for bad sellers to pin down \(P_B^0\) and thus \(\theta_G(\theta_B)\).

\[\blacksquare\]
9 References


