The Demand and Supply of Favours in Dynamic Relationships *

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Abstract

We characterise the optimal demand and supply of favours in a dynamic principal-agent model of joint production, in which heterogenous project opportunities arrive stochastically and are publicly observed upon arrival, utility from these projects is non-transferable and commitment to future production is limited. Our results characterise the optimal dynamic contract, and we establish that the principal's supply of favours (the production of projects that benefit the agent but not the principal) is backloaded, that the principal's demand for favours (the production of projects that benefit the principal but not the agent) is frontloaded, and that the production of projects is ordered by their comparative advantage, that is, by their associated efficiency in extracting (for demanded projects) and providing (for supplied projects) utility to the agent. Furthermore, we provide an exact construction of the optimal contract when project opportunities follow a Markov process.

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1 Introduction

Many long-running contractual relationships feature activities that can only be undertaken through mutual consent and effort. In these cases, the currency which supports current activities is the promise of collaboration in future activities, so that such relationships depend on a web of mutual obligations generated by the exchange of favours. In this paper, we study a dynamic relationship between a principal and an agent in which (a) heterogenous joint production opportunities, henceforth called projects, arrive according to an arbitrary stochastic process, (b) utility from these projects is non-transferable, and (c) commitment to future production is limited. Our results characterise optimal dynamic contracts in this environment.

Many economic environments share the key features of our setting. In large firms, the terms of an employee's formal contract are often (at least partially) outside the purview of the employee's manager. The authority of the manager, instead, involves some discretion in the choice of tasks to allocate to the employee. The net benefits to the manager and employee from a specific task might differ, and the manager's task selection at any point in their relationship must balance these possibly conflicting incentives. In fact, informal contracts, which are agreements regarding actions that are hard to describe ex ante and thus enforce ex post, constitute an important element of manager-employee relationships and can explain observed differences in firm performance (Gibbons and Henderson, 2013).[1] Our model is also general enough to encompass environments outside the standard applications considered by the contracting literature. For example, it can capture the trading of votes in legislatures: a party leader can recruit the vote of a party member for her preferred bills in return for a commitment to support the member's preferred bills in the future. In this application, our results uncover the properties of those bills that can be traded in such intertemporal deals.

Because projects are heterogeneous, the critical decision in these relationships is the selection of those projects that are actually produced. An optimal dynamic contract represents a production plan that maximises the principal's ex ante utility from the relationship but respects the agent's ability to walk away, at any point, from the relationship. Because optimal contracts are Pareto-undominated in any period, mutually beneficial projects are produced and mutually disagreeable projects are not produced. Therefore, optimal contracts are completely characterised by their production of projects that benefit the principal but not the agent (i.e., their *demand* for favours) and by their production of projects that benefit the agent but no the

[1]Informal contracts have been found to play role in transportation (Blader, Gartenberg, Henderson, and Prat, 2015), bankruptcy of General Motors (Helper and Henderson, 2014), international trade (Antràs and Foley, 2015; Macchiavello and Morjaria, 2015), drug discovery (Henderson, 1994; Henderson and Cockburn, 1994) and primary care quality (Gittell, 2002).

principal (i.e., their *supply* of favours).

Our main results establish that, in an optimal contract, (a) the demand for favours is *frontloaded*, (b) the supply of favours is *backloaded* and (c) the production of projects is ordered by their rank in *comparative advantage*. More specifically, an optimal contract is associated with the terms it promises to the agent at any given time, and these terms are represented by history-contingent time thresholds that build in the frontloading of demand and backloading of supply properties: for a demanded project, this threshold specifies time at which the principal commits to stop the production of this project, while for a supplied project, this threshold specifies the time at which the principal commits to start the production of this project. While backloading and frontloading describe the production dynamics of any one project, the selection of projects is driven by their rank in comparative advantage: the absolute value of the ratio of the payoffs to the principal and to the agent from a project. A demanded project that ranks high in comparative advantage is an efficient tool for the principal when she extracts utility from the agent: it provides her with large benefit per util cost to the agent. Conversely, a supplied project that ranks low in comparative advantage is an efficient tool for the principal when she provides utility to the agent: it costs her little per util benefit to the agent. The terms of an optimal contract specify a threshold project in the comparative advantage ranking such that all more efficient projects are produced (i.e., demanded projects above the threshold and supplied projects below the threshold) and all less efficient projects are not produced (i.e., demanded projects below the threshold and supplied projects above the threshold).

All our main results are driven by the fact that the production decisions specified by an optimal contract must respect the relative efficiency criterion embedded in projects' comparative advantage. More specifically, the principal will not promise to supply a favour to the agent through a less efficient project if some future opportunity with a more efficient project is passed over. Similarly, if the principal ever passes over demanding a favour that has high efficiency in terms of extracting utility from the agent, then any future opportunity at less efficient projects must also be passed over.

Optimal contracts give rise to simple production dynamics. The terms of an optimal contract are updated whenever the principal demands a favour from the agent and when this happens they become more generous towards the agent: the time thresholds decrease, which implies that the principal stops demanding and starts supplying favours sooner, and the threshold project moves up in comparative advantage, which increases the scope of supplied projects and correspondingly decreases the scope of demanded projects. Because any demand for a favour by the principal is associated with her commitment to supply a favour in the future, the dynamics of the relationship are driven by the principal's accumulation of increasingly less efficient commitments, which forces her to ration her demand for favours and concentrate it only on the most efficient demanded projects. Moreover, while the history of the agent's individual rationality constraints drives the updating of the terms of the optimal contract, all contracts eventually converge to an ex ante Pareto-efficient contract: a contract that maximises the principal's ex ante utility from the relationship subject only to a constraint on the agent's ex ante utility. An ex ante Pareto-efficient contract specifies stationary production decisions and splits the set of projects into those that are always produced and those that are never produced (i.e., such a contract identifies a time-independent thresholds contract in the comparative advantage ranking).

These results imply that the terms of an optimal contract favour the agent as the relationship progresses, and hence are in line with well-known backloading results for dynamic relationships (e.g., Ray, 2002). However, our results are not driven by the standard calculus through which promising high future rewards to the agent optimally provides incentives for his current actions. Rather, it is the principal's accumulation of supply commitments, driven by her past demands and leading to the rationing of future demands, that favours the agent. Moreover, our optimal contracts display several features similar to dynamic risk-sharing contracts with limited commitment (Thomas and Worrall, 1988; Ljungqvist and Sargent, 2004, Chapter 19): an optimal contract transitions to more generous terms following a demand for a favour, it stays constant between revisions and converges to an efficient contract. The first property is driven by the principal minimising the costs of providing incentives for project production. Between revisions, an optimal contract stays constant but the players' utility varies because some projects are produced and some not depending on their ranking in comparative advantage. In risk-sharing contracts it is the utility that is kept constant, due to risk aversion.

Remarkably, our main results depend neither on the players' discounting, nor on the size of the projects, nor on the stochastic process that drives arrival of the project opportunities. The reason is that these determine the amount of production, not how production is organised in a dynamic relationship. We show this in Section 4 when studying dynamic contracts under the assumption that the project process is Markov. While this is a significant restriction, the upside is that in this environment we provide an explicit construction of the optimal contract. We also recover the *amnesia* property (Kocherlakota, 1996): an optimal contract disposes of history dependence whenever its terms are revised. Optimal contracts do not satisfy the amnesia property in general because non-Markov project processes can display arbitrary history dependence.

We study two further variants of the model. First, in Section 5 we show that a model with transfers constitutes a special case of our general model. Our results then imply that the availability of monetary instruments does not crowd out production and that money in dynamic relationships flows from the agent to the principal in the early stages and from the principal to the agent in the later stages. Second, in Section 6 we relax the assumption that the principal can commit to the production decisions specified by the contract. We provide a necessary and sufficient condition for the optimal contract with commitment to be optimal without commitment. We also explore, through examples, how the absence of commitment affects the properties of optimal contracts highlighted by our main results.

Our work is closely related to the literature on informal risk-sharing in the presence of stochastic endowment shocks (Thomas and Worrall, 1988; Kocherlakota, 1996; Dixit, Grossman, and Gul, 2000).[2] This literature works with endowment processes that are either *iid* or Markov, while we allow for an arbitrary stochastic process generating project opportunities, and studies consumption smoothing driven by risk aversion, while we assume that players are risk-neutral, although their marginal utility from production varies stochastically, and focus on the selection of those projects that are produced.

The second literature we relate to combines informal risk-sharing with hidden information, typically about endowment shocks, and hence about the ability to provide a favour, or about the players' utility from production. This literature analyses simple counting *chips mechanisms* (Möbius, 2001), their generalisations (Hauser and Hopenhayn, 2008) and dynamic contracts with and without commitment (Guo and Hörner, 2015; Lipnowski and Ramos, 2016).[3] In this work truth-telling constraints ration players' demands for favours, an effect absent in our model. Instead, by abstracting from informational asymmetries between the principal and the agent, we allow for a rich space of possible project opportunities, and we obtain detailed results on the dynamics of project selection and production.

The work most closely related to ours is by Bird and Frug (2017) who study the production of projects in a dynamic relationship with hidden information. In their model projects arrive according to an independent Poisson processes, and their arrival is agent's private information. The hidden information environment makes their work different and complementary to ours. Like us, Bird and Frug (2017) show that comparative advantage organises project production.

[2]Subsequent work extends the informal risk-sharing model and incorporates multiple groups (Ligon, Thomas, and Worrall, 2002), group deviations (Genicot and Ray, 2003), storage (Ábrahám and Laczó, 2016), risk-aversion heterogeneity (Laczó, 2014a,b), or social networks (Bloch, Genicot, and Ray, 2008; Jackson, Rodriguez-Barraquer, and Tan, 2012; Ambrus, Möbius, and Szeidl, 2014). Within macroeconomics, the model has been used to study consumption smoothing (Kocherlakota, 2004; Krueger and Perri, 2006; Broer, Kapička, and Klein, 2017) and endogenously incomplete markets (Kehoe and Levine, 1993; Kehoe and Perri, 2002; Ábrahám and Cárceles-Poveda, 2006). Several contributions test the model empirically (Ligon et al., 2002; Mazzocco, 2007; Laczó, 2014b; Bold and Broer, 2016).

[3]See Abdulkadiroğlu and Bagwell (2012, 2013); Thomas and Worrall (1990); Kováč, Krähmer, and Tatur (2013) and Li, Matouschek, and Powell (2017) for further work.

Differently from us, their optimal mechanism compensates the agent for disclosing a demand project immediately after the disclosure, so as to free-up principal's capacity to motivate future disclosures. That is, their optimal mechanism frontloads the agent's compensation.

The third literature we relate to combines informal risk-sharing with sequential actions. The standard tension in this literature arises when a first-moving player invests and thus can be expropriated by a second-moving player, giving rise to a *hold-up* situation (Thomas and Worrall, 1994; Board, 2011).[4] This literature has close connections to the literature on *relational contracts* (Levin, 2003) that typically exploits transferability of utility to focus on stationary equilibria, while optimal relationships are not stationary in our environment, even if we assume that project opportunities are generated by an *iid* process.[5]

As a technical aside, we note that characterisations of dynamic contracts typically rely on recursive formulations and thus on the Markov structure of the model's stochastic processes.[6] Our most general results apply to an arbitrary stochastic process generating project opportunities, so that our proofs need to rely on 'direct', not recursive, arguments.

2 Model

A principal and an agent participate in a long-lived relationship in which a joint project opportunity arises in each period $t = 1, 2, \ldots$. Specifically, let $\mathcal{U} \subset \mathbb{R}^2$ be a finite set and let $u = \{u_t\}_{t\geq 1}$ be a $\mathcal{U}$-valued stochastic process that describes the arrival of projects over time. Let $u^t = (u_1, \ldots, u_t)$ denote a project history at t, and let $\mathcal{H}$ denote the set of all such histories for all times t. Because optimal contracts are indeterminate at histories that occur with zero probability, we assume that $\mathbb{P}_0(u^t) > 0$ for all project histories u^t. This is the only assumption that we impose on the project process u for our main results, and we do so mainly to ease the exposition.[7] In Section 4, we assume further that u is Markov to study the properties of optimal

[4]Further applications include Albuquerque and Hopenhayn (2004); Opp (2012); Kovrijnykh (2013) and Thomas and Worrall (2014). Several political economy contributions study self-enforcing voting (Maggi and Morelli, 2006) or the effects of possible expropriation by self-interested politicians (Acemoglu, Golosov, and Tsyvinski, 2008, 2011a,b; Aguiar, Amador, and Gopinath, 2009; Aguiar and Amador, 2011; Ales, Maziero, and Yared, 2014; Yared, 2010).

[5]Notable exceptions are Fuchs (2007); Halac (2012, 2014); Li and Matouschek (2013); Barron and Li (2015) and Fong and Li (2017a,b) who consider non-stationary dynamics in relational contracts.

[6]Use of 'promised utilities' as state variables has been pioneered by Spear and Srivastava (1987); Thomas and Worrall (1988); Abreu, Pearce, and Stacchetti (1990). Marcet and Marimon (2016) use past binding multipliers as history-dependent Pareto weights. Related but distinct is the dual recursive approach developed by Messner, Pavoni, and Sleet (2012) and Pavoni, Sleet, and Messner (2016). Golosov, Tsyvinski, and Werquin (2016) and Ljungqvist and Sargent (2004, Chapters 19 and 20) include comprehensive surveys of the recursive contracts techniques.

[7]Any process with zero-probability events can be expressed as the limit of a sequence of processes without such events. The limit of the corresponding sequence of optimal contracts, as characterised by our results, is an

contracts in more detail.

Given a project u_t at time t, the principal and the agent simultaneously decide whether or not to participate in the production of the project. Because we wish to capture environments in which projects are collaborative ventures that require the active engagement of both parties, we assume that project u_t is produced if and only if both players choose to participate. We let $u_t = (u_{P,t}, u_{A,t})$ denoted the payoffs to the principal and the agent if project u_t is produced, and we normalise each player's payoff from no-production to 0. For simplicity, we assume that the players' stage preferences over the production of projects are strict, that is, that $u_{A,t} \neq 0$ and $u_{P,t} \neq 0$ for all projects u_t. Therefore, player i (myopically) prefers to participate in the production of project u_t if $u_{i,t} > 0$ and prefers not to participate if $u_{i,t} < 0$. We model projects parsimoniously, but we can easily accommodate projects which are more complicated ventures with uncertain outcomes: in this case, u_t is interpreted as the expected utilities that the principal and the agent derive from these richer lotteries. Finally, the players discount future payoffs with common factor $\delta \in (0, 1)$.

Both project histories and production decisions, and hence all players' payoffs, are publicly observable and verifiable. A *contract* $\kappa : \mathcal{H} \to [0, 1]$ maps project histories into production probabilities. Given a project history u^t at time t, $\kappa(u^t)$, henceforth κ_t for short with history u^t understood, is the probability with which contract κ specifies that the project at t is produced. That is, a contract specifies a complete plan for what projects should be produced by the principal and the agent in all contingencies that can arise during their relationship. Furthermore, contracts allow for the use of a public randomisation device which determines whether or not production occurs following any given history.[8] Let $\mathcal{K}$ denote the set of all contracts.

Given a contract κ and a history u^t at time t, let

$$U_{i,t} = \mathbb{E}_t \sum_{t'=t}^{\infty} \delta^{t'-t} \kappa_{t'} u_{i,t'},$$

denote the associated discounted sum of payoffs to player i starting from t, where the expectation is taken conditional on the information available at t, which resides in project histories u^t. Notice that the linearity of the stage utilities in production probabilities implies that intertemporal smoothing of production decisions due to risk-aversion plays no role in our results. For future

optimal contract for the limiting process. In this sense, our assumption generates a selection of optimal contracts for project processes with zero-probability events.

[8]Interior production probabilities are useful to resolve rounding issues associated with the fact that production choices are discrete, but they are not essential (e.g., they would not be needed in the continuous time version of our model). Furthermore, as we show below, optimal contracts are essentially bang-bang.

reference, note that we can rewrite

$$U_{i,t} = \kappa_t u_{i,t} + \delta \mathbb{E}_t U_{i,t+1}.$$

We assume that production decisions within the relationship are contractible, but that the agent has the option to irreversibly quit the relationship at the beginning of every period t, after the realisation of project u_t but before the realisation of the production decision (determined by κ_t). Quitting yields a payoff of 0 to both players, which is the payoff they receive when no project is ever produced. It follows that an *optimal contract* κ^* is a solution to the problem

$$\max_{\kappa \in \mathcal{K}} \ \mathbb{E}_0 U_{P,1}$$
$$\text{subject to } U_{A,t} \geq 0 \text{ for all project histories } u^t. \qquad (IR_{A,t})$$

In words, an optimal contract maximises principal's ex-ante utility from the relationship subject to being *individually rational* for the agent following all project histories.[9] Our most general results maintain the assumption that the principal can be contractually obligated not to quit the relationship. In Section 6 we study the case of two-sided lack of commitment and require contracts to be individually rational for the principal also (i.e., satisfy $U_{P,t} \geq 0$ for all project histories u^t).

In all periods, an optimal contract must specify production decisions that are (stage) Pareto-undominated. In particular, this implies that if the preferences of the principal and the agent over the project at t are aligned, then an optimal contract implements jointly optimal production decisions.[10]

Lemma 1. *If contract κ^* is optimal, then*

1. *if $u_{P,t}, u_{A,t} > 0$, then $\kappa^*_t = 1$, and*

2. *if $u_{P,t}, u_{A,t} < 0$, then $\kappa^*_t = 0$.*

This simple observation has the important implication that an optimal contract can be identified with the production decisions it prescribes for those projects on which the principal and the agent disagree. To this end, define the sets $\mathcal{D} = \{v \in \mathcal{U} : v_P > 0 > v_A\}$ and $\mathcal{S} = \{w \in \mathcal{U} : w_A > 0 > w_P\}$. Given a contract κ, we say that the principal *demands a favour with probability κ_t at t* whenever $v_t \in \mathcal{D}$, and conversely that the principal *supplies a favour with*

[9] By the countability of project histories, standard arguments establish that an optimal contract always exists (e.g., Dixit et al., 2000).

[10] The proofs of all results are in Appendix A.

probability κ_t *at* t whenever $w_t \in \mathcal{S}$. Note that the production of projects $u \in \mathcal{D} \cup \mathcal{S}$ must rely on dynamic incentives: finite relationships and static incentives cannot support the demand or supply of favours.

Useful benchmarks to understand the effect of limited commitment on optimal contracts are *ex ante Pareto-efficient contracts*, which maximise the principal's expected period-1 utility subject to a lower bound $\overline{u}$ on the agent's expected period-1 utility. Formally, an efficient contract κ^e is a solution to

$$\max_{\kappa \in \mathcal{K}} \mathbb{E}_0 U_{P,1} \text{ subject to } \mathbb{E}_0 U_{A,1} \geq \overline{u}.$$

As we show below, optimal contracts and ex ante Pareto-efficient contracts have a similar structure, although that of the former is significantly richer. In particular, our characterisation of optimal contracts can be applied directly to characterise ex ante Pareto-efficient contracts, so that we will return to the latter benchmark after having stated our main results.

3 Optimal Contracts

The decomposition of an optimal contract into the demand and supply of favours turns out to be a fruitful way to describe project selection dynamics. In this section, we provide a characterisation of optimal contracts and show that the demand for favours is frontloaded, the supply of favours is backloaded and the production of projects is ordered by their comparative advantage. For ease of exposition, we break down our characterisation of the optimal contract into two parts. In Proposition 1, we introduce the form of the optimal contract along with its broad properties. Here we exploit front/backloading to describe the optimal contract through (history dependent) time thresholds. In Proposition 2, we state our main results on project selection in optimal contracts.

Proposition 1. *Fix any project history* u^t*. Without loss of generality for optimal payoffs, the optimal contract takes the following cutoff form: for all* $v \in \mathcal{D}$*, there exists a time threshold* T_t^v *such that*

$$\kappa_t^* = \begin{cases} 1 & \text{if } T_t^{v_t} \geq t+1, \\ T_t^{v_t} - t & \text{if } t < T_t^{v_t} < t+1, \\ 0 & \text{if } T_t^{v_t} \leq t; \end{cases} \tag{1}$$

and for all for all $w \in \mathcal{S}$, there exists a time threshold T_t^w such that

$$\kappa_t^* = \begin{cases} 1 & \text{if } T_t^{w_t} \leq t, \\ t+1-T^{w_t} & \text{if } t < T_t^{w_t} < t+1, \\ 0 & \text{if } T_t^{w_t} \geq t+1. \end{cases} \tag{2}$$

Given any projects v and w, the thresholds T_t^v and T_t^w have the following properties.

1. *$T_{t'}^v = T_t^v$ and $T_{t'}^w = T_t^w$ for all $t' > t$ such that $\sum_{s=t}^{t'} \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} = 0$.*

2. *T_t^v and T_t^w are non-increasing in t.*

3. *If $T_{t-1}^v > T_t^v$ or $T_{t-1}^w > T_t^w$, then $U_{A,t}^* = 0$.*

4. *$T_t^w = \infty$ if $\sum_{s=1}^{t} \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} = 0$.*

Optimal contracts can be characterised by simple time-threshold rules. At any time t the collection of history-dependent thresholds $\{T_t^v\}_{v \in \mathcal{D}}$ identifies those projects that are used by the principal to demand favours: any project with $T_t^{v_t} \geq t+1$ is produced; no project with $T_t^{v_t} \leq t$ is produced; and any project with $t < T_t^{v_t} < t+1$ is produced with interior probability. Therefore, $\lceil T_t^v \rceil$ is the earliest time at which the principal plans to demand favour v with zero probability, conditional on the relationship's status at time t. Similarly, time thresholds $\{T_t^w\}_{w \in \mathcal{S}}$ characterise the optimal supply of favours: no project with $T_t^{w_t} \geq t+1$ is produced; any project with $T_t^{w_t} \leq t$ is produced; and any project with $t < T_t^{w_t} < t+1$ is produced with interior probability. Again, $\lfloor T_t^w \rfloor$ is the earliest time at which the principal plans to supply favour w with positive probability, conditional the relationship's status at time t. Note that optimal contracts are typically bang-bang, which is due to the linearity of stage payoffs in production probabilities.

Optimal contracts are constant when the relationship is in between two favours demanded by the principal (Part 1). That is, the principal adjusts her plan for extracting utility from, and returning utility to, the agent only after she has asked for a new favour, for which she incurs a new utility debt. Also, asking for a new favour never leads the principal to become less generous towards the agent: both the time T_t^v at which she stops demanding project v and the time T_t^w at which she starts supplying project w can only move forward (Part 2). Notice the associated backloading property in the optimal supply favours: by agreeing to use w to supply a favour to the agent following some history, the principal also commits to supplying a favour to the agent in all future occurrences of w. There is also an associated frontloading property in the optimal demand for favours: if the principal ever passes on the opportunity to demand a favour v following some history, then the principal also commits to never demanding a favour in all future occurrences of v. These properties are derived through arguments that rely on the intertemporal reallocation of production, and the intuition behind them is most effectively explained in combination with the results on project selection contained in Proposition 2 below.

While the principal can adjust her future demand and supply for favours whenever she demands a new favour, she only does this if the failure to do so would violate the agent's individual rationality constraint. In particular, if the optimal contract becomes more generous when the principal demands a new favour, then the agent must be indifferent between enacting the project and quitting the relationship (Part 3). This does not imply that the agent's individual rationality constraint always binds in an optimal contract. In fact, the agent's constraint must be slack for some histories following a demand for a favour: if not, then the agent's continuation payoff following the favour would be 0, and because providing a favour is costly, the agent's feasibility constraint would fail. Therefore, the principal's incentives to hold the terms of the contract fixed while no intervening favours are demanded does benefit the agent, who obtains positive utility when not strictly necessitated by incentives. However, the principal delays adjusting thresholds until doing so is necessary. Finally, and not surprisingly, it is never optimal for the principal to supply any favours to the agent before she has demanded any favours (Part 4).

We turn to the paper's main result, which determines which projects are undertaken in the relationship between the principal and the agent and how this selection evolves over time. Given our characterisation of optimal contracts in Proposition 1, this task is reduced to describing the relationships between the various thresholds $\{T_t^v\}_{v\in\mathcal{D}}$ and $\{T_t^w\}_{w\in\mathcal{S}}$. For all these results, a first task is to determine those projects that the principal would prefer to use to demand favours from the agent, or to supply favours to the agent, irrespective of any dynamic incentive considerations. To this end, we define a *comparative advantage* ordering of projects such that $u \succ u'$ if and only $|{}^{u_P}/_{u_A}| > |{}^{u'_P}/_{u'_A}|$. In words, if $v, v' \in \mathcal{D}$ and $v \succ v'$, then project v has a comparative advantage over project v' when it is used by the principal to demand favours from the agent: in this case the ratio ${}^{v_P}/_{|v_A|}$ measures the efficiency of project v, from the principal's perspective, as a tool for extracting utility from the agent. Conversely, if $w, w' \in \mathcal{S}$ and $w' \succ w$, then project w has a comparative advantage over project w' when it is used by the principal to supply favours to the agent: in this case the ratio ${}^{w_A}/_{|w_P|}$ measures the efficiency of project w, from the principal's perspective, as a tool for providing utility to the agent. Put differently, comparative advantage ranks demanded projects according to the principal's benefit per util cost to the agent, and supplied projects according to principal's cost per util benefit to the agent. For simplicity, we assume that the ordering $\succ$ is complete on $\mathcal{D} \cup \mathcal{S}$, i.e., that all project pairs are ranked strictly by comparative advantage.

Proposition 2. *The optimal time thresholds* (1) *and* (2) *have the following properties.*

1. *Fix projects* $\overline{v} \succ \underline{v}$. *If* $T_t^{\overline{v}} < t+1$, *then* $T_t^{\underline{v}} \leq t$.

2. *Fix projects* $\underline{w} \succ \overline{w}$. *If* $T_t^{\overline{w}} > t$, *then* $T_t^{\underline{w}} = \infty$.

3. *Let* $\overline{W}_{t-1} = \min_{\succ}\{w \in \mathcal{S} : T_{t-1}^{w} > t+1\}$. *If* $v \succ \overline{W}_{t-1}$, *then* $T_t^{v} > t$.
   Let $\underline{W}_{t-1} = \max_{\succ}\{w \in \mathcal{S} : T_{t-1}^{w} < t+2\}$. *If* $\underline{W}_{t-1} \succ v$, *then* $T_t^{v} \leq t$.

We first discuss the result that optimal contracts order the supply of favours according to comparative advantage (Part 2). Note that following any history at most one threshold project w^* has $t < T_t^{w^*} < \infty$: for all worse-ranked projects $w \succ w^*$ we have $T_t^{w} = \infty$ (and hence $\kappa_t^* = 0$), and for all better-ranked projects $w^* \succ w$ we have $T_t^{w} \leq t$ (and hence $\kappa_t^* = 1$). Furthermore, because the time thresholds $\{T_t^{w}\}_{w \in \mathcal{S}}$ are non-increasing in t, the threshold project w^* is increasing (with respect to $\succ$) over time. That is, the principal transitions to supplying favours via less advantageous projects as the relationship matures and the stock of past demands accumulates. The heterogeneity of project opportunities generates multiple currencies that the principal can use to reward the agent. The rank of supplied projects in comparative advantage determines the principal's preferences over these currencies as tools for returning utility to the agent. Optimal contracts then require that the principal first commits to supply better-ranked, and hence less costly, favours before committing to supply those favours that are worse ranked, and hence more costly.

There is a tight connection between the prioritising of favours through comparative advantage from Proposition 2 and the backloading of the supply projects from Proposition 1, which explains why they are proved together in the Appendix. Notably, by concentrating her supply of favours in the future, the principal ensures that she exhausts her stock of advantageous currencies before tapping into her remaining stocks. Establishing this requires considering simple intertemporal reallocations of production. Contrary to our results, suppose that at some time t the principal supplied favour $\underline{w}$ but that her future supply of better-ranked favours $\underline{w} \succ \overline{w}$ is not exhausted: i.e., there exists some continuation history at $t' > t$ such what $u_{t'} = \overline{w}$ and $\kappa_{t'}^* < 1$. We show that the principal can gain by decreasing her supply of favour $\underline{w}$ at t and increasing her supply of favour $\overline{w}$ at t'. This can be done while keeping the agent indifferent at t, so that no individual rationality constraint is violated at any time $r \leq t$, and the agent is clearly better off at t' (and hence at any time between t and t').

The demand for favours is also ordered by comparative advantage (Part 1): there exists at most one threshold project v^* that has $t < T_t^{v^*} < t+1$: for all worse-ranked projects

$v^* \succ v$ we have $T_t^v \leq t$ (and hence $\kappa_t^* = 0$), and for all better-ranked projects $v \succ v^*$ we have $T_t^v \geq t+1$ (and hence $\kappa_t^* = 1$). Again, because the time thresholds $\{T_t^v\}_{v \in \mathcal{D}}$ are non-increasing, the threshold project v^* is increasing (with respect to $\succ$) over time. That is, the principal stops demanding favours that are worse ranked in comparative advantage before stopping her demand for better-ranked favours. Because demanded projects that rank high in comparative advantage represent beneficial currencies for extracting utility from the agent, the principal uses these projects longer than the less beneficial ones. Again, this result is tied to the frontloading of the demand for favours established in Proposition 1: the principal abandons less advantageous currencies before ceasing the use of more advantageous ones. These results are established by arguments analogous to those for the supply of favours sketched above. Specifically, contrary to our results, suppose that at some time t the principal passed on an opportunity to demand project $\overline{v}$ but that she demands lower-ranked favour $\overline{v} \succ \underline{v}$ at some continuation history at $t' > t$. Again, we show that the principal can gain by shifting some production from t' to t, while meeting all of the agent's individual rationality constraints at all histories.

From parts 1 and 2, we know that project priorities in the demand and the supply of favours follows from comparative advantage. What remains is to describe the connection between the principal's selection of the projects that are used to demand favours and those that are used to supply favours, which involves a simple comparison of marginal costs and benefits (Part 3). The marginal benefit to the principal from demanding project v is determined by this project's rank in comparative advantage, which normalises the principal's gain by the agent's loss, because the latter indexes the utility debt incurred by this project's production. An important remark is that the marginal cost to demanding favour v is endogenous. Specifically, the principal's accumulated commitments to supplying favours can differ at the various histories at which project v can be demanded. Therefore, the marginal cost of asking for an additional favour is measured by the comparative advantage ranking of the highest-ranked project w available to the principal, which must be one of those projects that have not yet been committed to supplying favours. In other words, the principal's ability to demand an additional favour must depend on what remains of the stocks of currencies that she uses to repay the agent.

To say more, we need additional notation, namely the projects $\overline{W}_{t-1} = \min_{\succ}\{w \in \mathcal{S} : T_{t-1}^w > t+1\}$ and $\underline{W}_{t-1} = \max_{\succ}\{w \in \mathcal{S} : T_{t-1}^w < t+2\}$. To interpret these definitions, suppose that the principal is in a position to demand a favour from the agent on some project v at time t, and note that the collection of thresholds $\{T_{t-1}^w\}_{w \in \mathcal{S}}$ describe the supply commitments accumulated in the relationship's history up to t. The project $\overline{W}_{t-1}$ is the principal's preferred project among those that, if she does not demand v, would be used to supply favours to the agent with interior or zero probability at $t+1$. Similarly, the project $\underline{W}_{t-1}$ is the principal's least preferred project among those that, if she does not demand v, would be used to supply favours to the agent with positive or unit probability at $t+1$. In words, $\overline{W}_{t-1}$ is the best uncommitted supply project and $\underline{W}_{t-1}$ is the worst committed supply project. Note that we have $\overline{W}_{t-1} \succ \underline{W}_{t-1}$ whenever $\overline{W}_{t-1} \neq \underline{W}_{t-1}$. Because any marginal increase in supply commitments at t will be delivered in future occurrences of project $\overline{W}_{t-1}$, the principal must demand a favour with some probability at t if $v \succ \overline{W}_{t-1}$. Conversely, because any marginal reduction in supply commitments at t will reduce the production of future occurrences of projects that rank no better than $\underline{W}_{t-1}$ in comparative advantage, the principal cannot demand a favour with any probability if $\underline{W}_{t-1} \succ v$.

Note that this last result ties together the fact that the cutoff demanded and supplied projects v^* and w^* both increase over time, and it illustrates how the frontloading of the demand for favours is the natural complement to the backloading of their supply: as the principal accumulates supply commitments over time, her marginal cost for asking new favours increases, choking off her ability to demand additional favours. In other words, early in the relationship the principal has large stocks of advantageous currencies with which to reward the agent, so that she can demand payments from the agent in currencies that are not advantageous. Late in the relationship if the principal's stocks of advantageous currencies are exhausted, then she can only demand payment from the agent in those currencies that the agent is most willing to transfer to the principal.

Our characterisation of optimal contracts with limited commitment in Propositions 1 and 2 can be used to describe ex ante Pareto-efficient contracts, that is, those production plans that would be optimal for the principal if the agent could commit to future commitment decisions.

Corollary 1. *Lemma 1 applies to ex ante Pareto-efficient contracts as well, and, without loss of generality for optimal payoffs, any such contract has the cutoff form described by (1) and (2). Furthermore,*

1. *Thresholds $\{T_t^v\}_{v\in\mathcal{D}}$ and $\{T_t^w\}_{w\in\mathcal{S}}$ are history independent.*

2. *There exists at most one project $u^e \in \mathcal{D}\cup\mathcal{S}$ with $1 < T_1^{u^e} < \infty$. Furthermore, given any $u \in \mathcal{D}\cup\mathcal{S}$,*

$$\text{if } u \succ u^e \text{ then } T_1^u = \infty, \text{ and if } u^e \succ u \text{ then } T_1^u = 1.$$

In efficient contracts, as in optimal contracts, production decisions must agree with the players' preferences when these agree. Furthermore, efficient contracts partition the set of projects $\mathcal{D}\cup\mathcal{S}$ on which the players disagree into those projects that are produced and those projects that are not produced. For almost all projects $u \in \mathcal{D}\cup\mathcal{S}$ production decisions are stationary, in that they specify a fixed production decision following all histories with the same current project opportunity. At most one threshold project u^e has non-stationary production decisions: if $u^e \in \mathcal{D}$, then u^e is demanded for a finite number of periods, and if $u^e \in \mathcal{S}$, then u^e is not supplied until a finite number of periods have elapsed. Furthermore, the set of those projects that are produced and the set of those projects that are not produced are ordered by comparative advantage.

To compare efficient and optimal contracts, note that because the cutoff demanded and supplied projects v^* and w^* both increase over time and the number of potential projects is finite, the optimal contract must converge, along each sequence of realisations of the project process, to a stationary contract characterised by two sets of projects, those that are always produced and those that are never produced. In other words, by Corollary 1, optimal contracts converge to efficient contracts. The key difference is that efficient contracts allocate production decisions evenly and identically across histories (again, with the exception of threshold project u^e), while optimal contracts must tailor production decisions to the agents' individual rationality constraints, which track the history of demands made by the principal. Therefore, some favour v may be demanded by the principal following some histories but not others, which means that the principal can demand favours ranked lower than v in the former histories but can only demand favours ranked higher than v in the latter histories. The principal could gain by smoothing out her demands across these types of histories, but this is incompatible with the agent's incentives.

4 Markov Project Processes

From Section 3, we know that the principal's selection of projects in both the demand and supply of favours is driven by their rank in comparative advantage. Somewhat surprisingly, the absolute benefit to the principal of demanding favour v, measured by v_P, or the absolute cost to the principal of supplying favour w, measured by w_P, do not on their own determine how these projects are treated by the optimal contract. Instead, the value of project v to the principal must be measured relative to the cost it imposes on the agent, just as the cost of project w for the principal must be measures relative to the benefit it procures to the agent. More broadly, our characterisation of optimal contracts is also independent of the players' discounting and of the stochastic process governing u. While these factors are not inputs into the optimal project selection decision, they are important for determining the effect of a demand for a favour on the agent's individual rationality constraint. In other words, while project size, discounting and the availability of future opportunities for favours as embedded in the project process do not affect optimal project prioritisation, they will affect the realised scope and level of production.

In this section we impose additional structure on the process driving joint project opportunities: we assume that u is a Markov process. This allows us to sharpen our results considerably, and in fact we provide a complete characterisation of optimal contracts in this case. Furthermore, simple examples are very tractable in this setting, so that we can easily illustrate the properties and dynamics of optimal contracts. An important note is that a Markov project process u does not generate optimal contracts that are themselves stationary, that is, that specify the same production decisions following all histories with the same current project opportunity. In fact, optimal contracts are history-dependent even in the special case of *iid* project processes. There are two related explanations for this fact. First, as we established in Propositions 1 and 2, the backloading of the supply of favours and the frontloading of the demand for favours naturally induces non-stationarity in project production. Second, the history of binding individual rationality constraints matters for optimal contracts, and different occurrences of the same project can be treated differently if they are preceded by different histories of production decisions.

Recall that the agent's individual rationality constraint can only bind if the principal demands a favour. The key to Proposition 3 below, which is our characterisation of optimal contracts with Markov project processes, is to associate a contract that extends a minimal level of generosity towards the agent to all projects at which the principal can demand a favour (projects $v \in \mathcal{D}$). The critical implication of the process u being Markov is that these minimally generous contracts are history-independent. In turn, these minimally generous contracts are used to construct the time-thresholds $\{T_t^v\}_{v\in\mathcal{D}}$ and $\{T_t^w\}_{w\in\mathcal{S}}$ that characterise optimal demand and supply processes in Propositions 1 and 2 through an explicit, recursive procedure.

Proposition 3. *Suppose that the project process u is Markov. For all $v, v' \in \mathcal{D}$ and all $w \in \mathcal{S}$, there exist $\tau^{v'v}, \tau^{v'w} \geq 0$ that recursively define the optimal time thresholds T_t^v from (1) and T_t^w from (2) as follows.*

1. *$T_0^v = T_0^w = \infty$ for all $v \in \mathcal{D}$ and $w \in \mathcal{S}$.*

2. *Given any $t \geq 1$,*

$$T_t^v = \begin{cases} t + \tau^{v_t v} & \text{if } t + \tau^{v_t v} < T_{t-1}^v, \\ T_{t-1}^v & \text{otherwise,} \end{cases} \quad \text{for all } v \in \mathcal{D}, \text{ and}$$

$$T_t^w = \begin{cases} t + \tau^{v_t w} & \text{if } t + \tau^{v_t w} < T_{t-1}^w, \\ T_{t-1}^w & \text{otherwise,} \end{cases} \quad \text{for all } w \in \mathcal{S}.$$

Given an opportunity for the principal to demand favour v_t at time t, $\lceil \tau^{v_t v} \rceil$ is interpreted as the number of periods from t during which the principal would demand favour $v \in \mathcal{D}$, and $\lfloor \tau^{v_t w} \rfloor$ is interpreted as the number of periods from t before the principal starts to supply favour $w \in \mathcal{S}$. Whether or not the principal actually implements the contract described by $\{\tau^{v_t v}\}_{v \in \mathcal{D}}$ and $\{\tau^{v_t w}\}_{w \in \mathcal{S}}$ following some demand for favour v_t depends on the project history. If v_t is the principal's first opportunity to demand a favour, then she will temporarily commit to time thresholds $\{t+\tau^{v_t v}\}_{v \in \mathcal{D}}$ and $\{t+\tau^{v_t w}\}_{w \in \mathcal{S}}$ from t on. These commitments are revisited whenever an opportunity for a new favour $v_{t'}$ arises at $t' > t$, in which case there are two possibilities. First, if the inherited contract is sufficiently generous to satisfy constraint $(IR_{A,t'})$, then the principals' commitments from t are extended from t' on, i.e., the terms of the contract remain unchanged. Note that commitments determined by $\{\tau^{v_t v}\}_{v \in \mathcal{D}}$ and $\{\tau^{v_t w}\}_{w \in \mathcal{S}}$ may have initially been less generous than those determined by $\{\tau^{v_{t'} v}\}_{v \in \mathcal{D}}$ and $\{\tau^{v_{t'} w}\}_{w \in \mathcal{S}}$. However, at t' the time remaining before the principal stops demanding favour v is $t + \tau^{v_t v} - t'$ and the time remaining before she starts to supply favour w is $t + \tau^{v_t w} - t'$, so that inherited commitments can be more generous than new commitments if enough time has elapsed since an original demand for a favour. Second, if inherited commitments fall short of those determined by $\{\tau^{v_{t'} v}\}_{v \in \mathcal{D}}$ and $\{\tau^{v_{t'} w}\}_{w \in \mathcal{S}}$, then the contract is updated to these more generous commitments, which are themselves revisited the next time a project arrives at which a favour can be demanded by the principal.

Proposition 3 reproduces the properties of optimal contracts for general project processes derived in Propositions 1 and 2 and adds an exact description of how the history of binding individual rationality constraints shapes the current state of the relationship between the principal and the agent. We can go further: a key step in the proof of Proposition 3 is the construction of a ranking of projects $v \in \mathcal{D}$ in terms of the stringency of their corresponding individual rationality constraints for the agent under the optimal contract.

Corollary 2. *Given any projects $\overline{v}, \underline{v} \in \mathcal{D}$, if either*

$$\tau^{\overline{v}v} \le \tau^{\underline{v}v} \text{ for some } v \in \mathcal{D} \text{ or } \tau^{\overline{v}w} \le \tau^{\underline{v}w} \text{ for some } w \in \mathcal{S},$$

then both

$$\tau^{\overline{v}v} \le \tau^{\underline{v}v} \text{ for all } v \in \mathcal{D} \text{ and } \tau^{\overline{v}w} \le \tau^{\underline{v}w} \text{ for all } w \in \mathcal{S}.$$

Given two projects $\overline{v}$ and $\underline{v}$ at which the principal can demand a favour, the resulting time thresholds $(\{\tau^{\overline{v}v}\}_{v \in \mathcal{D}}, \{\tau^{\overline{v}w}\}_{w \in \mathcal{S}})$ and $(\{\tau^{\underline{v}v}\}_{v \in \mathcal{D}}, \{\tau^{\underline{v}w}\}_{w \in \mathcal{S}})$ can be ranked uniformly in terms of their generosity to the agent. This yields a concrete sense in which project $\overline{v}$ is more costly to demand for the principal than project $\underline{v}$: in return, the principal must commit (at least provisionally) to demand less, and supply more, future favours to the agent. Also, note that Corollary 2 allows us to recover the relevant stationarity property of optimal contracts with Markov project processes: while an optimal contract will not specify similar production decisions following all histories with the same current project opportunity, it will specify similar production decisions following histories with the same current project opportunities that have met the same most costly demand for a favour, irrespective of the other properties of these histories.

Intuitively, this ranking of projects $v \in \mathcal{D}$ by the stringency of their individual rationality constraints depends on two factors: the absolute cost to the agent associated with the demand for favour v (indexed by $|v_A|$), and the value to the agent of future project opportunities conditional on having reached project v. This last factor depends on the project process u. However, if the project process is *iid*, then the value to the agent of future project opportunities is history-independent. In that case, the stringency of agent-feasibility constraints for favours that the principal can demand are ranked solely by their stage costs to the agent.

Corollary 3. *If the project process u is iid, then, given any projects $\overline{v}, \underline{v} \in \mathcal{D}$,*

$$\tau^{\overline{v}v} \leq \tau^{\underline{v}v} \text{ for all } v \in \mathcal{D} \text{ and } \tau^{\overline{v}w} \leq \tau^{\underline{v}w} \text{ for all } w \in \mathcal{S} \text{ if and only if } |\overline{v}_A| \geq |\underline{v}_A|.$$

This provides a comparative statics result of sorts, which shows how optimal contracts vary with the properties of the project process u. As noted above, even if the process u is *iid*, the optimal contract is not stationary and depends on the relationship's history of transitions to favours that are more costly to demand for the principal. However, in this case the ranking of favours in terms of their cost to the principal is independent of the process u. If instead u is Markov but not *iid*, then the ranking of projects $v \in \mathcal{D}$ by the stringency of their individual rationality constraints, while stationary, depends on the details of the process u; having reached $\overline{v}$ instead of $\underline{v}$ implies not only different costs to the agent but also different distributions over future project opportunities. We illustrate this in Example 1 below.

The proof of Proposition 3 constructs an optimal contract through an inductive sequence of reduced problems. To this end, fix any project $v' \in \mathcal{D}$ and suppose that $u_1 = v'$. We define the reduced problem

$$\max_{\kappa \in \mathcal{K}} U_{P,1} \quad \text{subject to} \quad U_{A,1} \geq 0, \tag{3}$$

which corresponds to the problem of finding an optimal contract conditional on $u_1 = v'$, but in which only constraint $(IR_{A,1})$ is required to hold. This problem has a solution determined by fixed time thresholds $\left(\{\tau^{v'v}\}_{v \in \mathcal{D}}, \{\tau^{v'w}\}_{w \in \mathcal{S}}\right)$: contrary to the corresponding history-dependent thresholds of Propositions 1 and 2, these do not need to be adjusted at times $t > 1$ because no future individual rationality constraints need to be accommodated. Nevertheless, this solution has properties that are expected given our results for general project processes. First, if $\tau^{v'\underline{v}} > 0$, then $\tau^{v'\overline{v}} = \infty$ for all $\overline{v} \succ \underline{v}$. In words, an optimal contract for problem (3) has the principal select a threshold demanded project, with all projects ranked higher in comparative advantage always demanded, and all projects ranked lower in comparative advantage never demanded. Second, if $\tau^{v'\underline{w}} < \infty$, then $\tau^{v'\overline{w}} = 0$ for all $\underline{w} \succ \overline{w}$. In words, an optimal contract for problem (3) also has the principal select a threshold supplied project, with all projects ranked higher in comparative advantage never supplied, and all projects ranked lower in comparative advantage always supplied. Third, if $v \succ w$ and $\tau^{v'v} < \infty$, then $\tau^{v'w} = 0$. In words, the lowest-ranked project among those that are demanded by the principal must be succeeded (in comparative advantage) by the highest-ranked project among those that are supplied by the principal.

The last point implies the following comparison of solutions to the reduced problem (3) for different initial u_1: if the principal demands less favours following $u_1 = \overline{v}$ than following $u_1 = \underline{v}$, then she must also supply more favours following $u_1 = \overline{v}$ than following $u_1 = \underline{v}$. In words, solutions to (3) following $u_1 = \overline{v}$ and $u_1 = \underline{v}$ are ordered by the stringency of their initial individual rationality constraints, or equivalently by their generosity to the agent.

Now consider the project v^1 with the most generous solution to (3), which we denote by κ^{1*}. Because the process u is Markov and this contract becomes more generous over time, then it must satisfy constraint $(IR_{A,t})$ if $u_t = v^1$ at all times $t > 1$. Furthermore, contract κ^{1*} must also satisfy $(IR_{A,t})$ if $u_t = v \neq v^1$ at all times $t > 1$. This follows because, again, u is Markov, and also because κ^{1*} is the most generous solution to (3) among all initial projects u_1. Finally, because the solution to the reduced problem (3) satisfies $(IR_{A,t})$ for all t, it follows that no contract can simultaneously yield higher payoff to the principal following u^t with $u_t = v^1$ and respect agent's individual rationality at all $t' \geq t$.

With project v^1 assigned as the costliest project to demand for the principal in our ordering of projects $v \in \mathcal{D}$, we proceed inductively to define the second project in this ordering. Given any $u_1 = v' \neq v^1$, we define the reduced problem

$$\begin{aligned} \max_{\kappa \in \mathcal{K}} U_{P,1} \;\text{ subject to }\; & U_{A,1} \geq 0, \\ & U_{A,t} \geq 0 \text{ at each history with } u_t = v^1, \\ & \kappa = \kappa^{1*} \text{ at each history with } u_t = v^1 \text{ and } U_{A,t} = 0, \end{aligned} \tag{4}$$

which corresponds to the problem of finding an optimal contract conditional on (i) $u_1 = v'$, (ii) on constraint $(IR_{A,1})$ being required to hold, and (iii) on constraint $(IR_{A,t})$ being required to hold at all histories with $u_t = v^1$, with the contract κ^{1*} being specified whenever this constraint binds at such histories. For the same reasons as above, the solution to problem (4) is such that (i) either the relationship has transitioned to κ^{1*} following some occurrence of v^1, or otherwise (ii) this solution is determined by fixed time thresholds $\left(\{\tau^{v'v}\}_{v\in\mathcal{D}}, \{\tau^{v'w}\}_{w\in\mathcal{S}}\right)$. Furthermore, these time thresholds are ranked by their generosity to the agent. The second project v^2 in our ordering of projects $v \in \mathcal{D}$, interpreted as the second-costliest favour for the principal to demand, is therefore the project for which the solution to (4) is the most generous to the agent, and we can define the corresponding contract κ^{2*}. Finally, for the same reasons as above, this contract is agent-feasible at all $t > 1$, so that no contract can yield higher payoffs to the principal and respect agent's individual rationality following any history at which project v^2 occurs. This inductive process can be repeated to rank all projects $v \in \mathcal{D}$ in terms of how costly they are for the principal to demand and to complete the construction of the optimal contract.

Note the critical feature of this construction: the project v^2 is determined by anticipating transitions to the more generous contract that follows a demand for the costliest favour v^1 at some future time. Demands of less costly favours are also anticipated, but in these cases the adjustments of κ^{2*} are not necessary to satisfy the agent's individual rationality constraint. Furthermore, anticipating costlier favours in the future allows the principal to demand more of less costly favours than she could do otherwise. To see this, note that from above we know that the solution to problem (3) given $u_1 = v^2$ does not, in general, satisfy $(IR_{A,t})$ if $u_t = v^1$ at time $t > 1$. Also, the contract κ^{1*} is not optimal for the principal if $u_1 = v \neq v^1$, because it is too generous towards the agent. Therefore, by anticipating that the relationship may transition to the more generous κ^{1*} following some histories, the solution to problem (4) for $u_1 = v^2$ can be less generous to the agent before such a transition than the solution to problem (3) for $u_1 = v^2$ would be. We illustrate this in Example 1 below.

For all the examples in the paper, we adopt the following simple setting, with a Markov project process over three projects, two that the principal can demand as favours and one that the principal can supply as a favour.

Example. Suppose that $\mathcal{D} = \{\overline{v}, \underline{v}\}$, that $\mathcal{S} = \{w\}$ and that $\mathcal{U} = \mathcal{D} \cup \mathcal{S}$ (i.e., there are no projects on which the preferences of the principal and the agent are aligned). Given any time t and projects $u, u' \in \mathcal{U}$, let $\mathcal{P}_t(u_{t+1} = u' | u_t = u) = p_{uu'}$. Let projects be ordered in comparative advantage such that $\overline{v} \succ \underline{v} \succ w$.

Example 1. Suppose that $|\overline{v}_A| > |\underline{v}_A|$, so that the agent finds project $\overline{v}$ more costly to produce than project $\underline{v}$. Given any $0 \leq q \leq 1/3$, suppose that $p_{\overline{v}u} = 1/3$ for all $u \in \mathcal{U}$, that $p_{\underline{v}\overline{v}} = 1/3$, $p_{\underline{v}\underline{v}} = 1/3 + q$ and $p_{\underline{v}w} = 1/3 - q$, and that $p_{w\overline{v}} = 1/3$, $p_{w\underline{v}} = 1/3 - q$ and $p_{ww} = 1/3 + q$. Notice that for any value of q, the unique stationary distribution of the project process assigns probability

$^1\!/_3$ to projects $\overline{v}$, $\underline{v}$ and w. Therefore, higher values of q increase the persistence of states $\underline{v}$ and w, while leaving fixed the unconditional expectation of the occurrence of any given project.

If $q = 0$, then the project process is *iid* and from Corollary 3, we have that under the optimal contract the most stringent individual rationality constraint for the agent is associated to project $\overline{v}$. If we assume that

$$-|\overline{v}_A| + \frac{\delta/3}{1-\delta}\left[w_A - |\overline{v}_A|\right] > 0 \tag{5}$$

and that

$$-|\overline{v}_A| + \frac{\delta/3}{1-\delta}\left[w_A - \left[|\overline{v}_A| + |\underline{v}_A|\right]\right] < 0, \tag{6}$$

then it follows that the solution to (3) for $u_1 = \overline{v}$ has $\tau^{\overline{v}\overline{v}} = \infty$ (the principal always demands favour $\overline{v}$), $0 < \tau^{\overline{v}\underline{v}} < \infty$ (the principal demands favour $\underline{v}$ for a finite number of periods), and $\tau^{\overline{v}w} = 0$ (the principal always supplies favour w).

The thresholds $\{\tau^{\underline{v}u}\}_{u\in\mathcal{U}}$ are given by solution to (4) for $u_1 = \underline{v}$. Because the thresholds $\{\tau^{\overline{v}u}\}_{u\in\mathcal{U}}$ that solve (3) given $u_1 = \overline{v}$ are admissible in (4) given $u_1 = \underline{v}$ but are such that $U_{A,1} > 0$, we must have $\tau^{\underline{v}\underline{v}} > \tau^{\overline{v}\underline{v}}$. That is, the principal demands favour $\underline{v}$ for more periods following an initial demand for $\underline{v}$ than for $\overline{v}$.

Under the optimal contract, no production occurs until the first realisation of $\overline{v}$ or $\underline{v}$. If $\overline{v}$ is the first opportunity for the principal to demand a favour, then the thresholds $\{\tau^{\overline{v}u}\}_{u\in\mathcal{U}}$ describe future production decisions. Furthermore, these are never adjusted in the future, as the continuation contracts are individually rational for the agent following future realisations of both $\overline{v}$ and $\underline{v}$. If instead $\underline{v}$ is the first opportunity for the principal to demand a favour, then the principal temporarily implements the less generous thresholds $\{\tau^{\underline{v}u}\}_{u\in\mathcal{U}}$, anticipating that they may be adjusted to $\{\tau^{\overline{v}u}\}_{u\in\mathcal{U}}$, if needed, following the first arrival of project $\overline{v}$. Notice that even as $\tau^{\overline{v}\underline{v}} \to 0$, we must nevertheless have that $\tau^{\underline{v}\underline{v}} > 0$. This illustrates the history-dependence of the set of projects that are produced in the relationship, in the sense that whether or not project $\underline{v}$ is ever produced depends on the order in which the projects at which the principal can demand a favour are realised: the least costly favour $\underline{v}$ can precede, but not follow, the most costly favour $\overline{v}$.

Now suppose that $q = {}^1\!/_3$. First, note that, given any $u \in \mathcal{U}$ and any $t > 1$, $\mathbb{P}_1(u_t = u | u_1 = \overline{v})$ is independent of q. This implies that the solution to the reduced problem (3) given $u_1 = \overline{v}$ is also independent of q. Therefore, under assumptions (5) and (6) as above, this solution has $\tau^{\overline{v}\overline{v}} = \infty$, $0 < \tau^{\overline{v}\underline{v}} < \infty$, and $\tau^{\overline{v}w} = 0$. For the remainder of the example, consider parameters for this example such that $\tau^{\overline{v}\underline{v}} \to \infty$. Second, let $\underline{U}^{\overline{\tau}}_{A,1}$ denote the payoff to the agent from the contract specified by thresholds $\{\tau^{\overline{v}u}\}_{u\in\mathcal{U}}$, but evaluated conditional on $u_1 = \underline{v}$. Because the thresholds $\{\tau^{\overline{v}u}\}_{u\in\mathcal{U}}$ solve (3) given $u_1 = \overline{v}$, we have that $\overline{U}^{\overline{\tau}}_{A,1} = 0$. Moreover, the thresholds (in the limit) specify a stationary contract with full production of all projects and thus $\underline{U}^{\overline{\tau}}_{A,1} \to -|\underline{v}_A| + \delta\left[{}^{2}\!/_{3}\underline{U}^{\overline{\tau}}_{A,1} + {}^{1}\!/_{3}\overline{U}^{\overline{\tau}}_{A,1}\right]$, or, equivalently

$$\underline{U}^{\overline{\tau}}_{A,1} \to \frac{-|\underline{v}_A|}{1 - {}^{2\delta}\!/_{3}} < 0.$$

It follows that the thresholds that solve (3), given $u_1 = \overline{v}$ and $u_1 = \underline{v}$, must satisfy $\tau^{\underline{v}\underline{v}} < \tau^{\overline{v}\underline{v}}$. In words, the optimal contract's most stringent individual rationality constraint for the agent is associated to project $\underline{v}$, so that even though the stage cost to the agent of producing this project is lower than that of project $\overline{v}$, the optimal contract following this project is more generous to the agent. This is driven by the differences in the continuation project opportunities following $\overline{v}$ and $\underline{v}$; when $q = {}^{1}\!/_{3}$, the project process transitions from $\overline{v}$ to w with probability ${}^{1}\!/_{3}$, but never transitions from $\underline{v}$ to w. That is, conditional on $\underline{v}$, the principal has few opportunities to supply favour w to the agent, which limits the demands for the less costly favour $\underline{v}$ that the agent can accept.

5 Transfers

So far, we have not explicitly allowed for monetary transfers between the principal and the agent. This streamlines the presentation of our model and it tightens its relationship to the applications we find most relevant, in which transfers are either illegal, considered unethical, or are dictated by contracts not under the control of the principal. However, given the flexibility allowed by our general specification of the process driving project arrivals, we can show that a model with transfers is included as a special case of our model, so that all our results from above are valid without change.

To see this, assume that the set $\mathcal{U}$ includes projects $m^D \in \mathcal{D}$ and $m^S \in \mathcal{S}$, and that these satisfy $(m^D_P, m^D_A) = (\bar{k}, -\bar{k})$ and $(m^S_P, m^S_A) = (-\bar{k}, \bar{k})$. In particular, note that this implies that $m^D_P/|m^D_A| = |m^S_P|/m^S_A = 1$. Suppose that, fixing $0 < \epsilon < 1$, (i) given any history with $u_t \in \mathcal{U} \setminus \{m^D, m^S\}$, we have that $\mathbb{P}_t(u_{t+1} = m^D) = 1 - \epsilon$, and (ii) given any history with $u_t = m^D$, we have that $\mathbb{P}_t(u_{t+1} = m^S) = 1 - \epsilon$. The limiting model as $\epsilon \to 0$ is essentially equivalent to one with a stage game in which every project opportunity is followed by the possibility of transfers between the principal and the agent, and furthermore these payments become unlimited as $\overline{k} \to \infty$ as well.

While our characterisation of the optimal contract also describes the optimal contract in the model with transfers, our results also provide specific implications regarding the use of money in the dynamic relationships captured by our environment. First, the ability for the principal to use transfers to reward the agent does not crowd out the use of production to supply favours: e.g., projects $w \in \mathcal{S}$ with $m^S \succ w$ are always used by the principal to supply favours to the agent whenever the principal also uses transfers to reward the agent. The fact that transfers are available does not imply that the principal will reimburse the agent for a demand for a favour using money, as some future project opportunities may be more efficient for returning utility to the agent. Furthermore, in an optimal contract the principal may even supply favours using projects $w \succ m^S$ ranked lower than money in comparative advantage, if the constraint on the ability to transfer money is tight. However, as $\overline{k} \to \infty$, the principal will always use money instead of lower-ranked projects. Second, the direction of the flow of money between the principal and the agent varies over the relationship's lifetime: the principal demands transfers from the agent early in the relationship, and supplies transfers to the agent later in the relationship.

6 No Commitment

A key feature of optimal contracts in this dynamic environment is that the principal's generosity towards the agent increases over time, through a decrease in her demand for favours and an increase in her supply of favours. An intuitive conjecture is that an optimal contract should be individually rational for the principal as long as it is individually rational in the later stages of the relationship. This is not correct in general because given an arbitrary project process, the monotonic worsening in the contract's terms for the principal need not generate a monotonic decrease in her payoffs. However, if the project process is Markov, then with probability 1 the terms of the contract stop evolving in finite time and furthermore, given any project u, the principal's payoff decreases over all successive realisations of this project. Therefore, we have a simple necessary condition for an optimal contract to be robust to a lack of commitment by the principal: after the contract has stabilised to its most generous terms towards the agent, the principal must have the incentive to supply the favour she finds the most expensive to supply.

Example 2. Return to the *iid* setting from Example 1 with $q = 0$, in which project $\overline{v}$ has the most stringent individual rationality constraint for the agent. Under conditions (5) and (6), the principal's lowest payoff under the optimal contract occurs when she supplies favour w after having stopped demanding favour $\underline{v}$. Therefore, the optimal contract is individually rational for the principal if and only if

$$-|w_P| + \frac{\delta/3}{1-\delta}\left[\overline{v}_P - |w_P|\right] \geq 0.$$

It is easy to see that in some cases, the optimal contract with commitment cannot be individually rational for the principal. If we assume instead that

$$-|\overline{v}_A| + \frac{\delta/3}{1-\delta}\left[w_A - |\overline{v}_A|\right] < 0, \tag{7}$$

then it follows that the solution to (3) for $u_1 = \overline{v}$ has $\tau^{\overline{v}\overline{v}} < \infty$, $\tau^{\overline{v}\underline{v}} = 0$ and $\tau^{\overline{v}w} = 0$. That is, under the optimal contract the principal eventually ceases to demand any favours and always supplies favour w, and her individual rationality constraint must fail at any such history.

With Markov project processes, we can select from the set of optimal contracts to limit the backloading of the agent's payoffs and hence minimise the principal's commitment problem. To do this, we must first provide an alternative characterisation of optimal contracts in this case. We do this through contracts with piecewise constant probabilities, that is, contracts that specify constant, and typically interior, production probabilities for all projects, with these probabilities revised when the principal demands a new favour.

Lemma 2. *Suppose that the project process u is Markov. For all $v, v' \in \mathcal{D}$ and all $w \in \mathcal{S}$, there exist $0 \leq \kappa^{v'v}, \kappa^{v'w} \leq 1$ that recursively define an optimal contract κ^* as follows.*

1. *$\kappa_0^v = 1$ for all $v \in \mathcal{D}$ and $\kappa_0^w = 0$ for all $w \in \mathcal{S}$.*
2. *Given any $t \geq 1$,*

$$\kappa_t^v = \begin{cases} \kappa^{v_t v} & \text{if } \kappa^{v_t v} < \kappa_{t-1}^v, \\ \kappa_{t-1}^v & \text{otherwise,} \end{cases} \quad \text{for all } v \in \mathcal{D},$$

$$\kappa_t^w = \begin{cases} \kappa^{v_t w} & \text{if } \kappa^{v_t w} > \kappa_t^w, \\ \kappa_{t-1}^w & \text{otherwise,} \end{cases} \quad \text{for all } w \in \mathcal{S}.$$

Under a contract with piecewise constant probabilities, the payoffs to both the principal and the agent are constant for successive realisations of the same project in the absence of further demands by the principal. From our previous results, it also follows that under such contracts production probabilities are decreasing over time for projects that can be demanded as favours and increasing over time for projects that can be supplied as favours. The result of

Lemma 2 is analogous to Proposition 3, and in fact it consists in selecting constant probability solutions to the sequence of reduced problems starting with (3).[11] Furthermore, among the class of solutions to this sequence of problems, a solution with piecewise constant probabilities maximises the principal's minimal payoff over all histories. From this remark follows a simple necessary and sufficient condition for the optimal contract to be robust to limited commitment by the principal.

Proposition 4. *Suppose that the project process u is Markov. Let $\overline{v} \in \mathcal{D}$ have the most generous solution to the reduced problem (3) with $u_1 = \overline{v}$, and consider its representation with piecewise constant probabilities. Then the principal's payoff from an optimal contract is the same with and without commitment if and only if*

$$\min\{\overline{U}_{P,2} : u_2 \in \mathcal{S}\} \geq 0.$$

We can discuss the key factors under which the condition from Proposition 4 is satisfied by continuing our example.

Example 3. Returning to the final part of Example 2, under condition (7) we have that the constant probability solution to (3) with $u_1 = \overline{v}$ has $\kappa^{\overline{v}\underline{v}} = 0$ and $\kappa^{\overline{v}w} = 1$. Furthermore, $0 < \kappa^{\overline{v}\overline{v}} < 1$ is given by the unique solution to the equation

$$-\kappa^{\overline{v}\overline{v}}|\overline{v}_A| + \frac{\delta/3}{1-\delta}\left[w_A - \kappa^{\overline{v}\overline{v}}|\overline{v}_A|\right] = 0. \tag{8}$$

Therefore, while the time-threshold representation of the optimal contract is not individually rational for the principal, its representation with piecewise constant probabilities is individually rational if

$$-|w_P| + \frac{\delta/3}{1-\delta}\left[\kappa^{\overline{v}\overline{v}}\overline{v}_P - |w_P|\right] \geq 0,$$

which, after substituting (8), is rewritten as

$$\frac{\overline{v}_P}{|\overline{v}_A|} - \left[\frac{1 - {}^2\!/_3\delta}{\delta/3}\right]^2 \frac{|w_P|}{w_A} \geq 0. \tag{9}$$

Because $\overline{v} \succ w$, we have that $\frac{\overline{v}_P}{|\overline{v}_A|} > \frac{|w_P|}{w_A}$ and it follows that condition (9) is always satisfied if δ is high. That is, the comparative advantage ranking indicates that there are gains from trade that the principal can capture by demanding favour $\overline{v}$ and supplying favour w, but these may

[11] The proof follows from almost identical argument and is omitted.

not be realised in the absence of commitment if the principal is shortsighted. Alternatively, the gains from trade from this relationship can be made bigger if the comparative advantage gap between $\overline{v}$ and w grows. A final factor which affects the principal's value from her relationship with the agent, but which is absent from this example, is the production of common-interest projects. An immediate observation is that the optimal contract with commitment must also be optimal without commitment if, following each history, the probability that common-interest projects will arise in the continuation game is sufficiently high.

A characterisation of optimal contracts with two-sided lack of commitment is much less tractable than our results to date for the case of one-sided commitment, even in the case of Markov project processes. In the remainder of this section, we illustrate through examples how the key features of optimal contracts that we have described so far are affected when the condition of Proposition 4 is violated.

Example 4. Return to the *iid* setting from Example 1 with $q = 0$, in which project $\overline{v}$ has the most stringent individual rationality constraint for the agent. Suppose that

$$-|\overline{v}_A| + \frac{\delta/3}{1-\delta}\left[w_A - [|\overline{v}_A| + |\underline{v}_A|]\right] > 0,$$

so that with commitment, the optimal contract with piecewise constant probabilities is such that $\kappa^{\overline{v}\overline{v}} = \kappa^{\overline{v}\underline{v}} = 1$ and $\kappa^{\overline{v}w} < 1$. Note that this also implies that $\kappa^{\underline{v}\overline{v}} = \kappa^{\underline{v}\underline{v}} = 1$ and $\kappa^{\underline{v}w} < \kappa^{\overline{v}w} < 1$. Finally, we have that $\kappa^{\overline{v}w}$ solves the equation

$$-|\overline{v}_A| + \frac{\delta/3}{1-\delta}\left[\kappa^{\overline{v}w} w_A - |\overline{v}_A| - |\underline{v}_A|\right] = 0,$$

so that

$$\kappa^{\overline{v}w} = \frac{\frac{1-2\delta/3}{\delta/3}|\overline{v}_A| + |\underline{v}_A|}{w_A}.$$

By Proposition 4, the optimal contract with commitment is not individually rational for the principal if and only if

$$-\kappa^{\overline{v}w}|w_P| + \frac{\delta/3}{1-\delta}\left[\overline{v}_P + \underline{v}_P - \kappa^{\overline{v}w}|w_P|\right] < 0,$$

rewritten as

$$\kappa^{\overline{v}w} > \frac{\delta/3}{1-2\delta/3}\frac{\overline{v}_P + \underline{v}_P}{|w_P|}.$$

The optimal contract without commitment will feature piecewise constant probabilities. Furthermore, if the project process is *iid*, it can be shown that these probabilities will have the following property: for any $u \in \mathcal{U}$, if $\kappa^{u\overline{v}} < 1$, then $\kappa^{u\underline{v}} = 0$. Recall that with commitment, the fact that $\overline{v} \succ \underline{v} \succ w$ implies that for any $v \in \{\overline{v}, \underline{v}\}$, if $\kappa^{vv'} < 1$, then $\kappa^{vw} = 1$. As we will show, this last property is not robust to the absence of commitment: there can be a gap in terms of comparative advantage between those projects that the principal demands and those that she supplies as favours. This gap increases the marginal return of supplying additional favours (which, again, is measured in comparative advantage against those favours that are demanded) and mitigates the principal's commitment problem. Note that this makes precise how the scale and scope of production is affected by the principal's inability to commit.

We will provide a heuristic construction of the optimal contract without commitment. The first question is: starting from the optimal contract with commitment, can the principal's individual rationality constraint conditional on $u_t = w$ be slackened by reducing the production probability of supplied favour w? However, to satisfy the agent's most stringent individual rationality constraint, which follows a demand for favour $\overline{v}$, any such change must to be accompanied by a decrease in the demand for favour $\underline{v}$. Specifically, given any reduced probability $k^{\overline{v}w} \leq \kappa^{\overline{v}w}$ in the supply of favour w, the reduced probability $k^{\overline{v}\underline{v}} \leq 1$ in the demand for favour $\underline{v}$ must solve

$$-|\overline{v}_A| + \frac{\delta/3}{1-\delta}\left[k^{\overline{v}w} w_A - |\overline{v}_A| - k^{\overline{v}\underline{v}}|\underline{v}_A|\right] = 0,$$

so that

$$k^{\overline{v}\underline{v}} = \frac{-\frac{1-2\delta/3}{\delta/3}|\overline{v}_A| + k^{\overline{v}w} w_A}{|\underline{v}_A|}. \tag{10}$$

Given any history with $u_t = w$, the principal's payoff is

$$U_{P,t} = -k^{\overline{v}w}|w_P| + \frac{\delta/3}{1-\delta}\left[\overline{v}_P + k^{\overline{v}\underline{v}}\underline{v}_P - k^{\overline{v}w}|w_P|\right],$$

which, after substituting (10), can be rewritten as

$$U_{P,t} = \frac{\delta/3 w_A}{1-\delta} k^{\overline{v}w}\left[\frac{\underline{v}_P}{|\underline{v}_A|} - \frac{1-2\delta/3}{\delta/3}\frac{|w_P|}{w_A}\right] + \frac{\delta/3|\overline{v}_A|}{1-\delta}\left[\frac{\overline{v}_P}{|\overline{v}_A|} - \frac{1-2\delta/3}{\delta/3}\frac{\underline{v}_P}{|\underline{v}_A|}\right] \tag{11}$$

Because the optimal contract without commitment is not individually rational for the principal, we know that $U_{P,t} < 0$ if $k^{\overline{v}w} = \kappa^{\overline{v}w}$ and $k^{\overline{v}\underline{v}} = 1$. Furthermore, we have that $\partial/\partial k^{\overline{v}w} U_{P,t} < 0$ if and only if

$$\frac{\underline{v}_P}{|\underline{v}_A|} < \frac{1 - {}^{2\delta}/_{3}}{{}^{\delta}/_{3}} \frac{|w_P|}{w_A}. \tag{12}$$

Recall that because $\underline{v} \succ w$, we have that $\frac{\underline{v}_P}{|\underline{v}_A|} > \frac{|w_P|}{w_A}$. Therefore, conditional on $u_t = w$, the principal is made better off by reducing her supply of favours even if she must also decrease her demand for $\underline{v}$ as long as the comparative advantage the $\underline{v}$ has over w is not too great. It is important to note that even if the operation just described increases the principal's payoff conditional on $u_t = w$, it nevertheless reduces her initial (time 1) expected payoff. The reason is that her initial expected payoff depends on her payoff conditional on $u_t = \overline{v}$ and $u_t = \underline{v}$, because the principal supplies favours only after having demanded some favour.

When (12) fails, it follows that absence of commitment leads to a complete breakdown of production: the optimal contract without commitment specifies that the principal never demands or supplies any favours. This breakdown is particularly striking because in this case the optimal contract with commitment maximises the principal's payoff conditional on $u_t = w$. However, even this most beneficial contract cannot compensate her for the supply of favour w. This case occurs if δ is sufficiently high.

When (12) is satisfied, there is a tension between the principal's ex ante and ex post incentives to trade the demand of favour $\underline{v}$ against the supply of favour w: she benefits from such trades when demanding favours, but suffers losses from them when it comes time to supply the favours. Let $\underline{k}^{\overline{v}w}$ be such that $k^{\overline{v}\overline{v}} = 1$ and $k^{\overline{v}\underline{v}} = 0$, and note that (10) implies that

$$\underline{k}^{\overline{v}w} = \frac{1 - {}^{2\delta}/_{3}}{{}^{\delta}/_{3}} \frac{|\overline{v}_A|}{w_A}.$$

Let $\underline{U}_{P,t}$ denote the principal's payoff from a history with $u_t = w$ given $k^{\overline{v}\overline{v}} = 1$, $k^{\overline{v}\underline{v}} = 0$ and $k^{\overline{v}w} = \underline{k}^{\overline{v}w}$. If we have that $\underline{U}_{P,t} \geq 0$, then the optimal contract without commitment must have $k^{\overline{v}\overline{v}} = 1$, $k^{\overline{v}\underline{v}} \geq 0$ and $k^{\overline{v}w} \geq \underline{k}^{\overline{v}w}$, with $k^{\overline{v}\underline{v}}$ and $k^{\overline{v}w}$ chosen such that $U_{P,t} = 0$. In words, starting from the optimal contract with commitment, the principal reduces the probability with which she supplies favour w and demands favour $\underline{v}$ until her individual rationality constraint binds. In this case, the absence of commitment does not affect the set of projects that are produced, but the scale of production is reduced: whereas the optimal contract with commitment traded the demand for favour $\underline{v}$ against the supply of favour w until all gains from trade were exhausted, the optimal contract without commitment must sacrifice some of these gains to counter the principal's ex post resistance to supplying favour w with high probabilities. The condition that

$\underline{U}_{P,t} \geq 0$ can be computed from (11) to yield

$$\frac{\overline{v}_P}{|\overline{v}_A|} \geq \left[\frac{1 - {}^{2\delta}\!/_3}{{}^{\delta}\!/_3}\right]^2 \frac{|w_P|}{w_A}. \tag{13}$$

For the optimal contract without commitment to involve some production of project $\underline{v}$, it must be the case that demanding favour $\overline{v}$ in the future compensates the principal for the supply of favour w. Therefore, condition (13) requires that project $\overline{v}$ has a sufficiently large comparative advantage over project w. Note that (13) is always satisfied for large δ, while it fails for low δ.

Finally, if (12) is satisfied but (13) fails, then the principal's individual rationality constraint can only be met if she gains by trading a reduction of her supply of project w against a reduction of her demand for project $\overline{v}$. That is, if favours are traded in the optimal contract without commitment, then this contract must be such that $k^{\overline{v}\underline{v}} = 0$ and $k^{\overline{v}\overline{v}}, k^{\overline{v},w} > 0$. In this case, the agent's payoff conditional on $u_t = \overline{v}$ must satisfy

$$-k^{\overline{v}\overline{v}}|\overline{v}_A| + \frac{{}^{\delta}\!/_3}{1-\delta}\left[k^{\overline{v}w}w_A - k^{\overline{v}\overline{v}}|\overline{v}_A|\right] = 0,$$

so that

$$k^{\overline{v}\overline{v}} = k^{\overline{v}w}\left[\frac{{}^{\delta}\!/_3}{1 - {}^{2\delta}\!/_3}\right]\frac{w_A}{|\overline{v}_A|}. \tag{14}$$

For any history with $u_t = w$, the principal's payoff is

$$U_{P,t} = -k^{\overline{v}w}|w_P| + \frac{{}^{\delta}\!/_3}{1-\delta}\left[k^{\overline{v}\overline{v}}\overline{v}_P - k^{\overline{v}w}|w_P|\right],$$

which, after substituting (14), can be rewritten as

$$U_{P,t} = k^{\overline{v}w}w_A\frac{\left[{}^{\delta}\!/_3\right]^2}{1 - {}^{2\delta}\!/_3}\left[\frac{\overline{v}_P}{|\overline{v}_A|} - \left[\frac{1 - {}^{2\delta}\!/_3}{{}^{\delta}\!/_3}\right]^2\frac{|w_P|}{w_A}\right].$$

Because (13) is assumed to fail, there is no value of $k^{\overline{v}w} > 0$ for which $U_{P,t} \geq 0$. It follows that the optimal contract without commitment in this case also leads to a breakdown in production, as it must specify that the principal never demands or supplies any favours. In other words, if the principal cannot be incentivised to supply favour w through future demands for the highest-ranked favour $\overline{v}$ (which is what the failure of (13) indicates), then reducing her supply of favours starting from the contract with $k^{\overline{v}\overline{v}} = 1$ and $k^{\overline{v}\underline{v}} = 0$, with the corresponding decrease in her demands for $\overline{v}$, cannot increase her payoffs conditional on $u_t = w$.

To recap, if the principal's individual rationality constraint fails under the optimal contract with commitment, then whether or not production can be supported without commitment depends on (*i*) project $\underline{v}$ not being too highly ranked in comparative advantage relative to w and (*ii*) project $\overline{v}$ being highly enough ranked relative to w. When these conditions hold, the optimal contract without commitment induces a wedge (in terms of comparative advantage) between those projects that are demanded and supplied as favours: this contract prioritises the trades that make the principal better off ex post ($\overline{v}$ against w) and compresses the trades that make the principal worse off ($\underline{v}$ against w). All the output loss due to the principal' inability to commit is concentrated on those projects involved in the latter trades.

Example 5. In Example 4, the principal's inability to commit drove a wedge between those projects that she demanded as favours and those projects that she supplied as favours. However, project selection decisions were still determined by their ranking in comparative advantage. In particular, project $\underline{v}$ was never demanded as a favour if opportunities to demand higher-ranked project $\overline{v}$ were still available. In this example we show that this property only arises if the project process is *iid*. If project opportunities are persistent, then the principal's inability to commit can also lead to lower-ranked projects being either demanded or supplied as favours ahead of higher-ranked projects.

Fix $0 < \epsilon < 1$ and suppose that $p_{\overline{v}\overline{v}} = 1 - \epsilon$, $p_{\overline{v}w} = \epsilon$, and $p_{uu'} = 1/2$ for all $u, u' \in \{\underline{v}, w\}$. Suppose further that $u_1 = \overline{v}$. In words, the initial project $\overline{v}$ is persistent, but after the first transition to project w, the project process alternates independently between $\underline{v}$ and w thereafter. It can be computed that if

$$\frac{1}{1-\delta(1-\epsilon)}\left[-|\overline{v}_A| + \delta\epsilon\left[\frac{1-\delta/2}{1-\delta}\right]w_A\right] < 0, \tag{15}$$

then it follows that with commitment, the optimal contract with piecewise constant probabilities is such that $\kappa^{\overline{v}\overline{v}} < 1$, $\kappa^{\overline{v}w} = 1$ and $\kappa^{\overline{v}\underline{v}} = 0$. That is, favour $\underline{v}$ is never demanded while favour w is always supplied. Clearly, this contract is not individually rational for the principal at any history with $u_t = w$.

In what follows, we provide a heuristic derivation of the optimal contract without commitment for a specific region of the parameter space. Namely, we assume that

$$\frac{\underline{v}_P}{|\underline{v}_A|} > \left[\frac{1-\delta/2}{\delta/2}\right]^2 \frac{|w_P|}{w_A}. \tag{16}$$

First, note that in any piecewise constant contract without commitment in which some production occurs, it must be that $k^{\overline{v}w} > 0$, which in turn implies that $k^{\overline{v}\underline{v}} > 0$. Second, under condition (16), it must be that any optimal contract without commitment must have either $k^{\overline{v}w} = 1$ or $k^{\overline{v}\underline{v}} = 1$. That is, we claim, given any contract with $k^{\overline{v}w}, k^{\overline{v}\underline{v}} < 1$, that there exists an alternative contract with $k'^{\overline{v}\underline{w}} > k^{\overline{v}\underline{w}}$ and $k'^{\overline{v}\underline{v}} > k^{\overline{v}\underline{v}}$ that Pareto-dominates it. To see this, the agent's payoff from $k^{\overline{v}\underline{w}}$ and $k^{\overline{v}\underline{v}}$ conditional on $u_t = \underline{v}$ is given by

$$U_{A,t} = -k^{\overline{v}\underline{v}}|\underline{v}_A| + \frac{\delta/2}{1-\delta}\left[-k^{\overline{v}\underline{v}}|\underline{v}_A| + k^{\overline{v}w}w_A\right],$$

which can be rearranged to yield

$$k^{\overline{v}\underline{v}} = k^{\overline{v}w}\frac{\delta/2}{1-\delta/2}\frac{w_A}{|\underline{v}_A|} - \frac{1-\delta}{1-\delta/2}\frac{U_{A,t}}{|\underline{v}_A|}. \tag{17}$$

The principal's payoff conditional on $u_t = w$ is given by

$$U_{P,t} = -k^{\overline{v}w}|w_P| + \frac{\delta/2}{1-\delta}\left[-k^{\overline{v}w}|w_P| + k^{\overline{v}\underline{v}}\underline{v}_P\right],$$

which, after substituting (17), can be rearranged to yield

$$U_{P,t} = \frac{[\delta/2]^2 w_A}{(1-\delta)(1-\delta/2)}k^{\overline{v}w}\left[\frac{\underline{v}_P}{|\underline{v}_A|} - \left[\frac{1-\delta/2}{\delta/2}\right]^2\frac{|w_P|}{w_A}\right] - \frac{\delta/2}{1-\delta/2}\frac{\underline{v}_P}{|\underline{v}_A|}U_{A,t}. \tag{18}$$

Therefore, given (16), it can be seen from (17) and (18) that there exist $k'^{\overline{v}\underline{w}}$ and $k'^{\overline{v}\underline{v}}$ that yield payoff $U_{A,t}$ to the agent following histories with $u_t = \underline{v}$ and payoff $U'_{P,t} > U_{P,t}$ to the principal following histories with $u_t = w$, as desired. In words, condition (16) ensures that projects $\underline{v}$ has a sufficient edge in comparative advantage over project w that exchanging the production of $\underline{v}$ against the production of w generates gains from trade for both the principal and the agent. Note also that this claim ensures that under condition (16) the optimal contract without commitment involves positive levels of production of both projects $\underline{v}$ and w, that is, $k^{\overline{v}w} > 0$ and $k^{\overline{v}\underline{v}} > 0$. The remaining question is whether this contract involves the production of project $\overline{v}$. Third, it must be the case that in any optimal contract without commitment, we have that $U_{P,t} = 0$ given any history with $u_t = w$. To see this, suppose that $U_{P,t} > 0$. Since $k^{\overline{v}\underline{v}} > 0$, we must have that $k^{\overline{v}\overline{v}} = 1$, as otherwise the principal would gain by reducing the demand for favour $\underline{v}$ and increasing the demand for favour $\overline{v}$, keeping the agent indifferent conditional on $u_t = \overline{v}$ (and hence increasing the agent's payoff conditional on $u_t = \underline{v}$). However, such a contract cannot satisfy the agent's individual rationality constraint conditional on $u_t = \overline{v}$: by (15), even if $k^{\overline{v}w} = 1$, the agent's payoff conditional on $u_t = \overline{v}$ would be such that $U_{A,t} < 0$.

Our claims above leave two cases for the optimal contract without commitment. In the first case, the optimal contract is such that $k^{\overline{v}w} < 1$, and then we must have that $k^{\overline{v}\underline{v}} = 1$ and $U_{P,t} = 0$ conditional on $u_t = w$. Because $k^{\overline{v}w} > 0$ and $U_{P,t} = 0$, from (18) we have that $U_{A,t} > 0$ conditional on $u_t = \underline{v}$, which, because we must have that $U_{A,t} = 0$ conditional on $u_t = \overline{v}$, implies that $k^{\overline{v}\overline{v}} > 0$. In the second case, the optimal contract without commitment has $k^{\overline{v}w} = 1$, and then we must have that $0 < k^{\overline{v}\underline{v}} \leq 1$ and $U_{P,t} = 0$ conditional on $u_t = w$. It also follows from (18) that $U_{A,t} > 0$ conditional on $u_t = \underline{v}$, so that again we have that $k^{\overline{v}\overline{v}} > 0$.

We know from our claims above that the optimal contract without commitment will maximise the level of production conditional on $u_t = w$ subject to $U_{P,t} = 0$. The two possible cases for this contract differ only in the levels of production for projects $\underline{v}$ and w that achieve this balance between production and the principal's incentives: in the first case, the principal always demands favour $\underline{v}$ but supplies favour w with interior probability and in the second case she always supplies favour w but demands favour $\underline{v}$ with interior probability. In the first case, the principal supplies favour w with a lower probability than with commitment and cannot participate in any additional production of this project without violating her individual rationality constraint. In the second case, the principal demands favour $\underline{v}$ even if she never demands it with commitment, and she cannot reduce the production of this project without violating her individual rationality constraint. In both cases, these distortions imply that $0 < k^{\overline{v}\overline{v}} < 1$: while there is production of project $\overline{v}$, it occurs with a lower probability without commitment than in the optimal contract with commitment. To recap, in this example the absence of commitment not only leads to a decrease in production, but it also spells the end of the comparative advantage ranking as the sole guide to optimal project selection: at a cost to gains from trade, the production of $\underline{v}$, which is needed to provide the principal with the incentives to supply favours to the agent, crowds out the production of $\overline{v}$.

7 Conclusion

In this paper, we have that in a dynamic model of joint production the principal's incentives for project selection lead to contracts whose terms of trade favour the agent over time. Specifically, this involves frontloading the production of projects that benefit the principal and backloading the production of projects that benefit the agent. This result obtains irrespective of the process that drives joint project opportunities, and we have also shown it to be robust to the inclusion of monetary transfers between the parties. Finally, while our characterisation of optimal contracts is not robust to the absence of commitment by the principal, our examples make clear that the principal's incentives for concentrating production on projects that are highly ranked by comparative advantage is still the key driving force without commitment, although in this case the scale and scope of production is reduced and the principal's individual rationality constraints can lead to non-monotonicity in the terms of optimal contracts. In sum, the dynamic project selection decisions highlighted by our results are robust.

A Proofs

Proof of Lemma 1. Suppose, towards a contradiction, that κ^* is optimal and that, for some project history u^t such that $u_{P,t}, u_{A,t} > 0$, we have that $\kappa^*_t < 1$. Fix a contract $\tilde{\kappa}$ that is identical to κ^* except that $\tilde{\kappa}_t = 1$ at u^t. It follows that $\tilde{\kappa}$ is individually rational because κ^* is individually rational. Furthermore, $\tilde{U}_{P,t} > U^*_{P,t}$, yielding the desired contradiction. The proof for the case of u^t such that $u_{P,t}, u_{A,t} < 0$ is similar, and is omitted. □

Proof of Propositions 1 and 2. As noted in the text, we divided the exposition of our characterisation of optimal contracts into a first part, which states their back/frontloading properties (Proposition 1), and a second part, which states their rules for project selection (Proposition 2). However, it is more natural to prove these results jointly. We proceed in a number of steps.

Claims for the Optimal Supply of Favours

Step 1. Fix and optimal contract κ^*, project history u^t, its superhistories $u^{t'}$ and $u^{t''}$, and projects $\underline{w} \succ \overline{w}$. Suppose that (i) $u_{t'} = \overline{w}$ and (ii) $u_{t''} = \underline{w}$ and $\sum_{s=t}^{t''} \kappa^*_s \mathbb{I}_{u_s \in \mathcal{D}} = 0$.[12] We show that

$$\text{if } \kappa^*_{t'} < 1, \text{ then } \kappa^*_{t''} = 0.$$

To see this suppose, towards a contradiction, that $\kappa^*_{t'} < 1$ at $u^{t'}$ and that $\kappa^*_{t''} > 0$ at $u^{t''}$. Now consider an alternative contract $\tilde{\kappa}$, identical to κ^* except that (i) $\kappa^*_{t'} < \tilde{\kappa}_{t'} \leq 1$ at $u^{t'}$, (ii) $0 \leq \tilde{\kappa}_{t''} < \kappa^*_{t''}$ at $u^{t''}$ and (iii)

$$\tilde{U}_{A,t} - U^*_{A,t} = \delta^{t'-t} \mathbb{P}_t(u^{t'})[\tilde{\kappa}_{t'} - \kappa^*_{t'}]\overline{w}_A - \delta^{t''-t} \mathbb{P}_t(u^{t''})[\kappa^*_{t''} - \tilde{\kappa}_{t''}]\underline{w}_A = 0. \tag{19}$$

Such a contract always exists, and furthermore $\tilde{\kappa}$ is individually rational for the agent. To see this, first note that, because $\tilde{U}_{A,t} = U^*_{A,t} \geq 0$, we have that $\tilde{\kappa}$ satisfies $(IR_{A,r})$ for all times $r \leq t$. Second, because $\tilde{U}_{A,t'} > U^*_{A,t'} \geq 0$, it follows that given any time $r > t$ and history u^r that is not a subhistory of $u^{t''}$, we have that $\tilde{U}_{A,r} \geq U^*_{A,r} \geq 0$. Third, even though we have that

[12] Throughout, $\sum_{s=t}^{t''} \kappa^*_s \mathbb{I}_{u_s \in \mathcal{D}} = 0$ denotes that, given history u^t and its superhistory $u^{t''}$, $\kappa^*_s = 0$ for any history u^s with $u_s \in \mathcal{D}$ that is superhistory of u^t and a subhistory of $u^{t''}$.

$\tilde{U}_{A,t''} < U^*_{A,t''}$, because $\tilde{\kappa}_{t''} \geq 0$ it also follows that

$$\begin{aligned}\tilde{U}_{A,t''} &\geq \delta \mathbb{E}_{t''} U^*_{A,t''+1} \\ &\geq 0.\end{aligned}$$

Finally, given time $r > t$ and history u^r which is a subhistory of $u^{t''}$, the fact that $\sum_{s=t}^{t''} \kappa^*_s \mathbb{I}_{u_s \in \mathcal{D}} = 0$ implies that

$$\begin{aligned}\tilde{U}_{A,r} &\geq \delta^{t''-r} \mathbb{P}_r(u^{t''}) \tilde{U}_{A,t''} \\ &\geq 0.\end{aligned}$$

It remains only to note that, by (19), we have

$$\begin{aligned}\tilde{U}_{P,t} - U^*_{P,t} &= -\delta^{t'-t} \mathbb{P}_t(u^{t'})[\tilde{\kappa}_{t'} - \kappa^*_{t'}]|\overline{w}_P| + \delta^{t''-t} \mathbb{P}_t(u^{t''})[\kappa^*_{t''} - \tilde{\kappa}_{t''}]|\underline{w}_P| \\ &= \delta^{t''-t} \mathbb{P}_t(u^{t''})[\kappa^*_{t''} - \tilde{\kappa}_{t''}]|\underline{w}_P| \left[1 - \frac{|\overline{w}_P|/\overline{w}_A}{|\underline{w}_P|/\underline{w}_A}\right] \\ &> 0,\end{aligned}$$

where the inequality follows because $\underline{w} \succ \overline{w}$, contradicting the optimality of κ^*.

Step 2. Step 1 implies that to any optimal contract κ^* corresponds a history-dependent threshold project mapping $W : \mathcal{H} \to \mathcal{S}$ such that, for all times t and histories u^t, if $u_t \in \mathcal{S}$, then

$$\kappa^*_t = \begin{cases} 1 & \text{if } W_t \succ u_t, \\ 0 & \text{if } u_t \succ W_t, \end{cases}$$

where for simplicity we denote $W(u^t)$ by W_t, with the project history understood. Furthermore, W_t is non-decreasing (with respect to $\succ$), and is such that, given any history u^t and its superhistory $u^{t'}$, $W_{t'} = W_t$ if $\sum_{s=t+1}^{t'} \kappa^*_s \mathbb{I}_{u_s \in \mathcal{D}} = 0$. The threshold is given by

$$W(u^t) = \max_{\succ} \left\{ w \in \mathcal{S} : \mathbb{P}_t(\kappa^*_{t'} > 0, u_{t'} = w, \sum_{s=t+1}^{t'} \kappa^*_s \mathbb{I}_{u_s \in \mathcal{D}} = 0) > 0 \right\},$$

if this is well-defined, and by

$$W_t = \min_{\succ} \mathcal{S},$$

otherwise.[13]

Step 3. Step 2 does not determine the optimal contracts at times t when $u_t = W_t$. We now show that, without loss of generality for optimal payoffs, we can restrict attention to contracts with the property that, given any history u^t and its superhistory $u^{t'}$ at which $u_{t'} = W_t$ and $\sum_{s=t+1}^{t'} \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} = 0$, there exists time $\hat{T}$ such that $\kappa_{t'}^* = 1$ if and only if $t' \geq \hat{T}$. More precisely, fix an optimal contract κ^* and history u^t, and consider an alternative contract $\hat{\kappa}$, identical to κ^* except that, at all superhistories $u^{t'}$ of u^t with $\sum_{s=t+1}^{t'} \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} = 0$ and $u_{t'} = W_t$,

$$\hat{\kappa}_{t'} = \begin{cases} 1 & \text{if } \hat{T} \leq t', \\ t' + 1 - \hat{T} & \text{if } t < \hat{T} < t + 1, \\ 0 & \text{if } \hat{T} \geq t' + 1. \end{cases}$$

Note that $\hat{U}_{A,t} \geq U_{A,t}^*$ if $\hat{T} = t$. Also, $\lim_{\hat{T} \to \infty} \hat{U}_{A,t} \leq U_{A,t}^*$. By continuity of $\hat{U}_{A,t}$ in $\hat{T}$, there exists some $\tilde{T} \geq t$ such that $\tilde{U}_{A,t} = U_{A,t}^*$. Also, note that

$$\begin{aligned} \tilde{U}_{P,t} - U_{P,t}^* &= |W_{P,t}| \, \mathbb{E} \left[\sum_{t' \geq t} \delta^{t'-t} [\kappa_{t'}^* - \tilde{\kappa}_{t'}] \right] \\ &= \frac{|W_{P,t}|}{W_{A,t}} \left[U_{A,t}^* - \tilde{U}_{A,t} \right] \\ &= 0. \end{aligned}$$

To verify that $\tilde{\kappa}$ is individually rational for the agent, first note that $\tilde{U}_{A,r} = U_{A,r}^* \geq 0$ either if (i) $r \leq t$ or if (ii) $r > t$ and history u^r is not a superhistory of u^t with $\sum_{s=t+1}^{r} \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} = 0$. Second, given any superhistory u^r of u^t with $\sum_{s=t+1}^{r} \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} = 0$ and $r > \hat{T}$, then, because $\tilde{\kappa}_s u_{A,s} \geq \kappa_s^* u_{A,s}$ for all $s \geq r$, it follows that $\tilde{U}_{A,r} \geq U_{A,r}^* \geq 0$. Third, given any superhistory u^r of u^t with $\sum_{s=t+1}^{r} \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} = 0$ and $r \leq \hat{T}$, then, by the previous point and because $\tilde{\kappa}_r u_{A,r} \geq 0$, it follows recursively that

$$\begin{aligned} \tilde{U}_{A,r} &\geq \delta \mathbb{E}_r \tilde{U}_{A,r+1} \\ &\geq 0. \end{aligned}$$

The last point is to establish that the procedure above, which modifies κ^* at a single history in a payoff-invariant way, can be extended simultaneously to all histories. We do this in Step 7

[13]Throughout, $\mathbb{P}_t(\kappa_{t'}^* > 0, u_{t'} = w, \sum_{s=t+1}^{t'} \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} = 0) > 0$ denotes that, given history u^t, the set of its superhistories $u^{t'}$ such that $\kappa_{t'}^* > 0$, $u_{t'} = w$ and $\sum_{s=t+1}^{t'} \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} = 0$ has positive probability.

below. Therefore, this step defines a history-dependent time threshold $T_t^{W_t}$ for each history u^t. By construction, for any history u^t and its superhistory $u^{t'}$ with $u_{t'} = W_t$ and $\sum_{s=t+1}^{t'} \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} = 0$ (and hence $W_{t'} = W_t$), we have that $T_t^{W_t} = T_{t'}^{W_{t'}}$.

Step 4. Step 3 defines a history-dependent time threshold $T_t^{W_t}$ for each history u^t. We now show that, without loss of generality for optimal payoffs, we can restrict attention to contracts with the property that, given any history u^t and any superhistory $u^{t'}$ such that $W_t = W_{t'}$, we have that $T_t^{W_t} \geq T_{t'}^{W_{t'}}$. By Step 3, given u^t, if $\mathbb{P}_t(\kappa_{t'}^* > 0, u_{t'} = W_t, \sum_{s=t+1}^{t'} \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} = 0) = 0$, then $T_t^{W_t} = \infty$, and the claim is true. Therefore, in what follows we assume that, given u^t, $\mathbb{P}_t(\kappa_{t'}^* > 0, u_{t'} = W_t, \sum_{s=t+1}^{t'} \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} = 0) > 0$.

Fix time t and history u^t, and let $\overline{T}_t = \sup\{T_{t'}^{W_{t'}} : t' \geq t, u_{t'} = W_t = W_{t'}\}$. It is possible, without loss of generality for optimal payoffs, to assume that $\overline{T}_t < \infty$. To see this, consider an alternative contract $\hat{\kappa}$, identical to κ^* except that, given any superhistory $u^{t'}$ of u^t with $W_t = W_{t'}$, (i) $\hat{T}_{t'}^{W_{t'}} = \hat{T}$ if $\sum_{s=t+1}^{t'} \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} = 0$, and (ii) $\hat{T}_{t'}^{W_{t'}} = \min\{T_{t'}^{W_{t'}}, \hat{T}\}$ if $\sum_{s=t+1}^{t'} \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} > 0$. We have that $\hat{U}_{A,t} \geq U_{A,t}^*$ if $\hat{T} = t$, and $\lim_{\hat{T} \to \infty} \hat{U}_{A,t} < U_{A,t}^*$ because $\mathbb{P}_t(\kappa_{t'}^* > 0, u_{t'} = W_t, \sum_{s=t+1}^{t'} \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} = 0) > 0$. By continuity, there exists $\tilde{T} < \infty$ such that $\tilde{U}_{A,t} = U_{A,t}^*$, as well as $\tilde{U}_{P,t} = U_{P,t}^*$. To verify that contract $\tilde{\kappa}$ is individually rational for the agent, first note that $\tilde{U}_{A,r} = U_{A,r}^* \geq 0$ unless history u^r is a superhistory of u^t with $W_r = W_t$. Second, by construction of $\tilde{\kappa}$, $\tilde{U}_{A,r} \geq U_{A,r}^* \geq 0$ for any superhistory u^r of u^t with $r \geq \tilde{T}$ and $W_r = W_t$ as well as for any superhistory u^r of u^t with $t < r < \tilde{T}$, $W_r = W_t$ and $\sum_{s=t+1}^{r} \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} > 0$. Third, given any superhistory u^r of u^t with $t < r < \tilde{T}$, $W_r = W_t$ and $\sum_{s=t+1}^{r} \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} = 0$, then, by the previous points and because $\tilde{\kappa}_r u_{A,r} \geq 0$, it follows recursively that

$$
\begin{aligned}
\tilde{U}_{A,r} &\geq \delta \mathbb{E}_r \tilde{U}_{A,r+1} \\
&\geq 0.
\end{aligned}
$$

Now consider an alternative contract κ^α, identical to κ^* except that, given any superhistory $u^{t'}$ of u^t with $W_t = W_{t'}$, (i) $T_{t'}^{\alpha, W_{t'}} = (1-\alpha)\overline{T}_t + \alpha t$ if $\sum_{s=t+1}^{t'} \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} = 0$, and (ii) $T_{t'}^{\alpha, W_{t'}} = (1-\alpha) T_{t'}^{W_{t'}} + \alpha t$ if $\sum_{s=t+1}^{t'} \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} > 0$. This contract is well-defined because $\overline{T}_t < \infty$. Notice that $U_{A,t}^{\alpha=0} \leq U_{A,t}^*$ and that $U_{A,t}^{\alpha=1} \geq U_{A,t}^*$. By continuity, there exists $\tilde{\alpha} \in [0,1]$ such that $U_{A,t}^{\tilde{\alpha}} = U_{A,t}^*$, as well as $U_{P,t}^{\tilde{\alpha}} = U_{P,t}^*$. Note that, by construction, because $\overline{T}_t \geq T_{t'}^{W_{t'}}$, $\kappa^{\tilde{\alpha}}$ is such that $T_t^{\tilde{\alpha}, W_t} \geq T_{t'}^{\tilde{\alpha}, W_{t'}}$ whenever $t' > t$ and $W_{t'} = W_t$. The proof that $\kappa^{\tilde{\alpha}}$ is individually rational for the agent is almost identical to that of the previous paragraph for the contract $\tilde{\kappa}$, and is omitted.

Step 5. We use this step to prove the one that follows. Fix an optimal contract κ^*, project history u^{t-1}, its superhistories u^t and $u^{t'}$, a superhistory $u^{t''}$ of u^t, and projects $\underline{w} \succ \overline{w}$. Suppose that (i) $u_{t'} = \overline{w}$, and (ii) $u_{t''} = \underline{w}$ and $\sum_{s=t+1}^{t''} \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} = 0$. We show that if $U_{A,t}^* > 0$ at u^t, then

$$\text{if } \kappa_{t'}^* < 1, \text{ then } \kappa_{t''}^* = 0.$$

To see this suppose, towards a contradiction, that $\kappa_{t'}^* < 1$ at $u^{t'}$ and that $\kappa_{t''}^* > 0$ at $u^{t''}$. Now consider an alternative contract $\tilde{\kappa}$, identical to κ^* except that (i) $\kappa_{t'}^* < \tilde{\kappa}_{t'} \leq 1$ at $u^{t'}$, (ii) $0 \leq \tilde{\kappa}_{t''} < \kappa_{t''}^*$ at $u^{t''}$, (iii) $\tilde{U}_{A,t} \geq 0$ and (iv)

$$\tilde{U}_{A,t-1} - U_{A,t-1}^* = \delta^{t'-(t-1)} \mathbb{P}_{t-1}(u^{t'})[\tilde{\kappa}_{t'} - \kappa_{t'}^*]\overline{w}_A - \delta^{t''-(t-1)} \mathbb{P}_{t-1}(u^{t''})[\kappa_{t''}^* - \tilde{\kappa}_{t''}]\underline{w}_A = 0. \tag{20}$$

Such a contract always exists, and furthermore $\tilde{\kappa}$ is individually rational for the agent. To see this, first note that, because $\tilde{U}_{A,t-1} = U_{A,t-1}^* \geq 0$, we have that $\tilde{\kappa}$ satisfies $(IR_{A,r})$ for all times $r \leq t-1$. Second, $\tilde{U}_{A,t} \geq 0$ so that $\tilde{\kappa}$ satisfies $(IR_{A,t})$. Third, $\tilde{\kappa}$ satisfies $(IR_{A,r})$ for all times $r > t$. This follows by an argument similar to the one in Step 1 and is omitted.

Finally, an argument as in Step 1 shows that (20) and the fact that $\underline{w} \succ \overline{w}$ imply that $\tilde{U}_{P,t-1} - U_{P,t-1}^* > 0$, yielding the desired contradiction.

Step 6. We show that, without loss of generality for optimal payoffs, we can restrict attention to contracts κ^* such that, given any time t, if either (i) $W_t \succ W_{t-1}$ or (ii) $W_t = W_{t-1}$ and $T_{t-1}^{W_{t-1}} > T_t^{W_t}$, then $U_{A,t}^* = 0$. To see part (i) of this claim, suppose, towards a contradiction, that there exist project history u^{t-1} and its superhistory u^t such that $W_t \succ W_{t-1}$ and $U_{A,t}^* > 0$. Because $W_t \succ W_{t-1}$, Step 2 implies that $u_t \in \mathcal{D}$, $\kappa_t^* > 0$ and

$$\mathbb{P}_t \left(\kappa_{t''}^* > 0, u_{t''} = W_t, \sum_{s=t+1}^{t''} \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} = 0 \right) > 0 \tag{21}$$

so that there exists superhistory $u^{t''}$ of u^t with $u_{t''} = W_t$, $\sum_{s=t+1}^{t''} \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} = 0$ and $\kappa_{t''}^* > 0$.

We now make two claims. First, we claim that project $w \in \mathcal{S}$ such that $W_t \succ w \succ W_{t-1}$ does not exist. Suppose it does, then, by Step 2,

$$\mathbb{P}_{t-1} \left(\kappa_{t'}^* > 0, u_{t'} = w, \sum_{s=t}^{t'} \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} = 0 \right) = 0$$

so that there exists a superhistory $u^{t'}$ of u^{t-1} with $u_{t'} = w$ and $\kappa_{t'}^* = 0$, which, by Step 5 and (21), is a contradiction because $W_t \succ w$. Second, we claim that

$$\mathbb{P}_{t-1} \left(\kappa_{t'}^* < 1, u_{t'} = W_{t-1}, \sum_{s=t}^{t'} \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} = 0 \right) = 0.$$

If not then there exists superhistory $u^{t'}$ of u^{t-1} with $u_{t'} = W_{t-1}$ and $\kappa_{t-1}^* < 1$, which, by Step 5 and (21), is a contradiction because $W_t \succ W_{t-1}$.

Now consider the alternative contract $\kappa^{\alpha,\beta}$, identical to κ^* except that (i) $T_{t'}^{\alpha,\beta,W_{t'}} = \beta \geq t$ for all superhistories $u^{t'}$ of u^t with $W_{t'} = W_t$ and $\sum_{s=t+1}^{t'} \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} = 0$ and (ii) $W_{t''}^{\alpha,\beta} = W_t$ and $T_{t''}^{\alpha,\beta,W_t} = \alpha \geq t$ for all superhistories $u^{t''}$ of u^{t-1} with $\sum_{s=t}^{t''} \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} = 0$. We have that $U_{A,r}^{t,t} \geq U_{A,r}^*$ and $\lim_{\alpha,\beta\to\infty} U_{A,r}^{\alpha,\beta} \leq U_{A,r}^*$ for $r = t-1, t$. By continuity, there exist $\tilde{\beta} \leq \tilde{\alpha} < \infty$ such that $U_{A,t-1}^{\tilde{\alpha},\tilde{\beta}} = U_{A,t-1}^*$ and $U_{P,t-1}^{\tilde{\alpha},\tilde{\beta}} = U_{P,t-1}^*$, and either (a) $U_{A,t}^{\tilde{\alpha},\tilde{\beta}} = 0$ and $\tilde{\beta} \leq \tilde{\alpha}$ or (b) $U_{A,t}^{\tilde{\alpha},\tilde{\beta}} > 0$ and $\tilde{\beta} = \tilde{\alpha}$, which implies $W_{t-1}^{\tilde{\alpha},\tilde{\beta}} = W_t^{\tilde{\alpha},\tilde{\beta}}$ by the two claims above.

Contract $\kappa^{\tilde{\alpha},\tilde{\beta}}$ is clearly individually rational for the agent at all times $r \leq t$. The proof that contract $\kappa^{\tilde{\alpha},\tilde{\beta}}$ is individually rational for the agent at all times $r > t$ is similar to those of Steps 3 and 4, and is omitted. Finally, the proof of part (ii) of the claim is similar, and is omitted.

Step 7. The payoff-equivalent modifications operated on optimal contract κ^* described in Steps 3-6 were constructed history by history. Note that any contract can be identified with a point in $[0,1]^\infty$, a compact set in the product topology. Therefore, given an optimal contract $\kappa^{*,1}$, we can construct a sequence $\{\kappa^{*,n}\}_{n\geq 1}$ in $[0,1]^\infty$ such that (i) for each n, $\kappa^{*,n+1}$ is obtained from $\kappa^{*,n}$ by some operation from Steps 3-6 at some history and (ii) given any time t, there exists N such that, for all $n \geq N$, $\kappa_{t'}^{*,n} = \kappa_{t'}^{*,N}$ for all histories $u^{t'}$ with $t' \leq t$. This sequence must then have a subsequence converging to κ^*, some optimal contract satisfying all the properties of Steps 3-6.

Step 8. Given any time t and associated threshold project W_t as defined in Step 2, we have defined in Steps 3-7 a time threshold $T_t^{W_t}$ that respects the conditions of Proposition 1. Now given any $w \succ W_t$, define $T_t^w = \infty$, and given any $W_t \succ w$, define

$$T_t^w = \begin{cases} \min\{T_{t'}^{W_{t'}} : u^{t'} \text{ is a subhistory of } u^t \text{ and } w = W_{t'}\} & \text{if this is well-defined,} \\ \min\{t' : u^{t'} \text{ is a subhistory of } u^t \text{ and } W_{t'} \succ w\} & \text{otherwise.} \end{cases}$$

Note that because, by Step 2, W is non-decreasing (with respect to $\succ$), and because, by Steps 4 and 6, $T_t^{W_t}$ is non-increasing in t, our construction ensures that, for each $w \in \mathcal{S}$, the threshold T_t^w is non-increasing in t.

Step 9. Let $\underline{w} = \min_\succ \mathcal{S}$. We show that given an optimal contract κ^*, we have that $T_t^{\underline{w}} = \infty$ if $\sum_{s=1}^t \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} = 0$. To see this, suppose that there exists a history u^t with $\sum_{s=1}^t \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} = 0$ and, towards a contradiction, $T_t^{\underline{w}} < \infty$. Consider an alternative contract $\tilde{\kappa}$, identical to κ^* except that $\tilde{\kappa}_{t'} = 0$ at all histories $u^{t'}$ with $u_{t'} \in \mathcal{S}$ and $\sum_{s=1}^{t'} \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} = 0$. To see that $\tilde{\kappa}$ is individually rational for the agent, first note that if $\sum_{s=1}^r \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} > 0$ for some r, then $\tilde{U}_{A,r} = U_{A,r}^* \geq 0$. Second, if $\sum_{s=1}^r \kappa_s^* \mathbb{I}_{u_s \in \mathcal{D}} = 0$ for some r, then, because $\tilde{\kappa}_r u_{A,r} \geq 0$, it follows recursively by the previous point that we have

$$\begin{aligned}\tilde{U}_{A,r} &\geq \delta \mathbb{E}_r U^*_{A,r+1}\\ &\geq 0.\end{aligned}$$

Finally, we have that $\tilde{U}_{P,1} > U^*_{P,1}$, yielding the desired contradiction.

Claims for the Optimal Demand for Favours

Step A. Fix optimal contract κ^*, project history u^t, its superhistory $u^{t'}$, and projects $\overline{v} \succ \underline{v}$. Suppose that (i) $u_t = \overline{v}$ and (ii) $u_{t'} = \underline{v}$. We show that

$$\text{if } \kappa^*_t < 1, \text{ then } \kappa^*_{t'} = 0.$$

To see this suppose, towards a contradiction, that $\kappa^*_t < 1$ at u^t and that $\kappa^*_{t'} > 0$ at $u^{t'}$. Now consider an alternative contract $\tilde{\kappa}$, identical to κ^* except that (i) $\kappa^*_t < \tilde{\kappa}_t \leq 1$ at u^t, (ii) $0 \leq \tilde{\kappa}_{t'} < \kappa^*_{t'}$ at $u^{t'}$, and (iii)

$$\tilde{U}_{A,t} - U^*_{A,t} = -\left[\tilde{\kappa}_t - \kappa^*_t\right]|\overline{v}_A| + \delta^{t'-t}\mathbb{P}_t(u^{t'})\left[\kappa^*_{t'} - \tilde{\kappa}_{t'}\right]|\underline{v}_A| = 0. \tag{22}$$

Note that contract $\tilde{\kappa}$ satisfies $(IR_{A,r})$ at all times $r \leq t$. To show that $\tilde{\kappa}$ satisfies $(IR_{A,r})$ at all times $r > t$, we proceed recursively. First note that we have that $\tilde{U}_{A,r} = U^*_{A,r} \geq 0$ whenever (i) u^r is not a superhistory of u^t or (ii) $r > t'$. Second, at history $u^{t'}$, we have that $\tilde{\kappa}_{t'} u_{A,t'} > \kappa^*_{t'} u_{A,t'}$, so that, by the previous point, $\tilde{U}_{A,r} > U^*_{A,r} \geq 0$. Third, given any superhistory u^r of u^t with $t < r < t'$, the previous points ensure that $\tilde{U}_{A,r} \geq 0$. Finally, an argument as in Step 1 shows that (22) and the fact that $\overline{v} \succ \underline{v}$ imply that $\tilde{U}_{P,t} - U^*_{P,t} > 0$, yielding the desired contradiction.

Step B. Step A does not restrict optimal contracts at history u^t and its superhistory $u^{t'}$ if $u_t = u_{t'} \in \mathcal{D}$. We now show that, without loss of generality for optimal payoffs, we can restrict attention to contracts with the property that, for such histories, if $\kappa^*_{t'} > 0$, then $\kappa^*_t = 1$. To see this, fix optimal contract κ^*, history u^t and project v, and suppose that $u_t = v$ and $\kappa^*_t < 1$. Now consider an alternative contract $\tilde{\kappa}$, identical to κ^* except that

$$\tilde{\kappa}_t = \begin{cases} \kappa^*_t + \mathbb{E}_t\left[\sum_{s \geq t+1} \delta^{s-t}\kappa^*_s \mathbb{I}_{u_s = v}\right] & \text{if } \kappa^*_t + \mathbb{E}_t\left[\sum_{s \geq t+1} \delta^{s-t}\kappa^*_s \mathbb{I}_{u_s = v}\right] \leq 1, \\ 1 & \text{otherwise,} \end{cases}$$

and that, for any superhistory $u^{t'}$ of u^t with $u_{t'} = v$,

$$\tilde{\kappa}_{t'} = \begin{cases} 0 & \text{if } \kappa_t^* + \mathbb{E}_t\left[\sum_{s\geq t+1} \delta^{s-t}\kappa_s^* \mathbb{I}_{u_s=v}\right] \leq 1, \\ \left[1 - \frac{1-\kappa_t^*}{\mathbb{E}_t\left[\sum_{s\geq t+1}\delta^{s-t}\kappa_s^*\mathbb{I}_{u_s=v}\right]}\right]\kappa_{t'}^* & \text{otherwise.} \end{cases}$$

Note that such a contract always exists, and that, by construction,

$$\begin{aligned} \tilde{U}_{A,t} - U_{A,t}^* &= |v_A|\left[-\left[\tilde{\kappa}_t - \kappa_t^*\right] + \mathbb{E}_t\left[\sum_{s\geq t+1}\delta^{s-t}\mathbb{I}_{u_s=v}\left[\kappa_s^* - \tilde{\kappa}_s\right]\right]\right] \\ &= 0 \\ &= \tilde{U}_{P,t} - U_{P,t}^*. \end{aligned}$$

Furthermore, for any superhistory $u^{t'}$ of u^t with $u_{t'} = v$, we have that either (i) $\tilde{\kappa}_t = 1$ and $\tilde{\kappa}_{t'} \geq 0$, or (ii) $\tilde{\kappa}_t < 1$ and $\tilde{\kappa}_{t'} = 0$. Note that contract $\tilde{\kappa}$ satisfies $(IR_{A,r})$ for all $r \leq t$. To see that $\tilde{\kappa}$ satisfies $(IR_{A,r})$ at all $r > t$, note that because $\tilde{\kappa}_r \leq \kappa_r^*$ for all superhistories u^r of u^t, it follows that $\tilde{\kappa}_r u_{A,r} \geq \kappa_r^* u_{A,r}$ for all $r > t$, and hence $\tilde{V}_{A,r} \geq V_{A,r}^* \geq 0$.

Step C. The procedure from Step B, which modifies contract κ^* at a single history in a payoff-invariant way, can be extended simultaneously to all histories as in Step 7.

Step D. Given an optimal contract κ^* along with any $v \in \mathcal{D}$ and any history u^t, define

$$\overline{t}^v = \sup\{t' : u^{t'} \text{ is a subhistory or superhistory of } u^t,\ v_{t'} = v \text{ and } \kappa_{t'}^* > 0\}$$

if this is well-defined and $\overline{t}^v = \infty$ otherwise, as well as

$$T_t^v = \begin{cases} \overline{t}^v + \kappa_{\overline{t}^v}^* & \text{if } \overline{t}^v < \infty, \\ \infty & \text{otherwise.} \end{cases}$$

By construction, the resulting time thresholds $\{T_t^v\}_{v\in\mathcal{D}}$ are non-increasing. Furthermore, by the results of Steps A-C, it follows that, for all t,

$$\kappa_t^* = \begin{cases} 1 & \text{if } T_t^{v_t} \geq t+1, \\ T_t^{v_t} - t & \text{if } t < T_t^{v_t} < t+1, \\ 0 & \text{if } T_t^{v_t} \leq t. \end{cases}$$

Step E. Fix optimal contract κ^* and project history u^t such that $v_t \succ \overline{W}_{t-1}$. We show that $\kappa_t^* > 0$. Suppose, towards a contradiction, that $\kappa_t^* = 0$. Then by Part 1 of Proposition 1 (which we have established), at superhistory u^{t+1} of u^t with $u_{t+1} = \overline{W}_{t-1}$, $T_{t+1}^{\overline{W}_{t-1}} = T_{t-1}^{\overline{W}_{t-1}} > t+1$ and hence $\kappa^*_{t+1} < 1$. Now consider an alternative contract $\tilde{\kappa}$, identical to κ^* except that (i) $\tilde{\kappa}_t > 0$, (ii) $\kappa^*_{t+1} < \tilde{\kappa}_{t+1} \leq 1$ and (iii) $\tilde{U}_{A,t} = U^*_{A,t}$. Such a contract always exists. To see that $\tilde{\kappa}$ is individually rational for the agent, first note that it is individually rational for the agent at all histories except at u^{t+1}. Second, for u^{t+1}, because $\tilde{\kappa}_{t+1} u_{A,t+1} \geq 0$, it follows recursively by the previous point that we have

$$\begin{aligned}\tilde{U}_{A,t+1} &\geq \delta \mathbb{E}_{t+1} U^*_{A,t+2} \\ &\geq 0.\end{aligned}$$

By (iii), we have that

$$\begin{aligned}\tilde{U}_{A,t} - U^*_{A,t} &= -\tilde{\kappa}_t |v_{A,t}| + \delta \overline{W}_{A,t-1} \left[\tilde{\kappa}_{t+1} - \kappa^*_{t+1}\right] \mathbb{P}_t \left(u_{t+1} = \overline{W}_{t-1}\right) \\ &= 0.\end{aligned} \tag{23}$$

But then, it follows that

$$\begin{aligned}\tilde{U}_{P,t} - U^*_{P,t} &= \tilde{\kappa}_t v_{P,t} - \delta \left|\overline{W}_{P,t-1}\right| \left[\tilde{\kappa}_{t+1} - \kappa^*_{t+1}\right] \mathbb{P}_t \left(u_{t+1} = \overline{W}_{t-1}\right) \\ &= \tilde{\kappa}_t |v_{A,t}| \left[\frac{v_{P,t}}{|v_{A,t}|} - \frac{\left|\overline{W}_{P,t-1}\right|}{\overline{W}_{A,t-1}}\right] \\ &> 0,\end{aligned}$$

where the second equality follows from substituting (23) and the inequality follow because $v_t \succ \overline{W}_{t-1}$, contradicting the optimality of κ^*.

Step F. Fix optimal contract κ^* and project history u^t such that $\underline{W}_{t-1} \succ v_t$. We show that $\kappa^*_t = 0$. Suppose, towards a contradiction, that $\kappa^*_t > 0$. Then by Part 2 of Proposition 1 (which we have established), at superhistory u^{t+1} of u^t with $u_{t+1} = \underline{W}_{t-1}$, $T_{t+1}^{\underline{W}_{t-1}} \leq T_{t-1}^{\underline{W}_{t-1}} < t+2$ and hence $\kappa^*_{t+1} > 0$. Now consider an alternative contract $\tilde{\kappa}$, identical to κ^* except that (i) $0 \leq \tilde{\kappa}_t < \kappa^*_t$, (ii) $0 \leq \tilde{\kappa}_{t+1} < \kappa^*_{t+1}$ and (iii) $\tilde{U}_{A,t} = U^*_{A,t}$. Such a contract always exists and that $\tilde{\kappa}$ is individually rational for the agent follows by an argument as in Step E. By (iii), we have that

$$\begin{aligned}\tilde{U}_{A,t} - U^*_{A,t} &= \left[\kappa^*_t - \tilde{\kappa}_t\right] |v_{A,t}| - \delta \underline{W}_{A,t-1} \left[\kappa^*_{t+1} - \tilde{\kappa}_{t+1}\right] \mathbb{P}_t \left(u_{t+1} = \underline{W}_{t-1}\right) \\ &= 0.\end{aligned} \tag{24}$$

But then, it follows that

$$\begin{aligned}
\tilde{U}_{P,t} - U^*_{P,t} &= -\left[\kappa^*_t - \tilde{\kappa}_t\right] v_{P,t} + \delta \left|\underline{W}_{P,t-1}\right| \left[\kappa^*_{t+1} - \tilde{\kappa}_{t+1}\right] \mathbb{P}_t \left(u_{t+1} = \underline{W}_{t-1}\right) \\
&= \left[\kappa^*_t - \tilde{\kappa}_t\right] |v_{A,t}| \left[\frac{\left|\underline{W}_{P,t-1}\right|}{\underline{W}_{A,t-1}} - \frac{v_{P,t}}{|v_{A,t}|}\right] \\
&> 0,
\end{aligned}$$

where the second equality follows from substituting (24) and the inequality follow because $\underline{W}_{t-1} \succ v_t$, contradicting the optimality of κ^*. $\square$

Proof of Proposition 3. We proceed in a number of steps.
Step 1. Fix project $v' \in \mathcal{D}$ and suppose that $u_1 = v'$. We define the reduced problem

$$\max_{\kappa \in \mathcal{K}} U_{P,1} \quad \text{subject to} \quad U_{A,1} \geq 0. \tag{25}$$

First, note that a standard argument establishes that $U^*_{A,1} = 0$ at any solution to (25). Second, note that problem (25) only requires individual rationality at $t = 1$. Therefore, if the solution to (25) also satisfies $(IR_{A,t})$ at all times $t > 1$, then it must be part of an optimal contract conditional on $u_1 = v'$.
Step 2. We show that there exists a solution κ^* to (25) of the following threshold type: for each $v \in D$, there exists $T^v \geq 0$ such that, given any history u^t with $u_t = v$,

$$\kappa^*_t = \begin{cases} 1 & \text{if } T^v \geq t+1, \\ T^v - t & \text{if } t < T^v < t+1, \\ 0 & \text{if } T^v \leq t, \end{cases}$$

and for each $w \in S$, there exists $T^w \geq 0$ such that, given any history u^t with $u_t = w$,

$$\kappa^*_t = \begin{cases} 1 & \text{if } T^w \leq t, \\ t+1-T^w & \text{if } t < T^w < t+1, \\ 0 & \text{if } T^w \geq t+1. \end{cases}$$

The critical difference with the corresponding expressions with a general process u from Proposition 1 is that the time thresholds $(\{T^v\}_{v\in\mathcal{D}}, \{T^w\}_{w\in\mathcal{S}})$ are fixed and independent of histories. The proof of this claim follows from arguments closely mirroring those of Steps 3 and C of Proposition 1, and is omitted. In fact, these arguments are simplified in this case because the only individual rationality constraint for the agent in problem (25) is for the initial history.

Finally, we normalise these time thresholds so that (i) $T^v = 0$ if and only if $\kappa_t^* = 0$ for all $t \geq 1$ with $u_t = v$, that is, if and only if the principal never demands favour v under κ^*, and that (ii) $T^w = 0$ if and only if $\kappa_t^* = 1$ for all $t \geq 1$ with $u_t = w$, that is, if the principal always supplies favour w under κ^*.

Step 3. We show that there exists a solution to (25) with the following properties: there exist $v^* \in \mathcal{D}$ and $w^* \in \mathcal{S}$ such that

1. $T^v = 0$ if $v^* \succ v$ and $T^v = \infty$ if $v \succ v^*$.
2. $T^w = 0$ if $w^* \succ w$ and $T^w = \infty$ if $w \succ w^*$.
3. Given any $v \in \mathcal{D}$, if $T^v < \infty$, then $T^w = 0$ for all $v \succ w$. Also, if $T^v > 0$ then $T^w = \infty$ for all $w \succ v$.

The threshold projects are defined as

$$v^* = \begin{cases} \min_{\succ}\{v \in \mathcal{D} : T^v > 0\} & \text{if well-defined,} \\ \max_{\succ} \mathcal{D} & \text{if } T^v = 0 \text{ for all } v \in \mathcal{D}, \end{cases}$$

and

$$w^* = \begin{cases} \max_{\succ}\{w \in \mathcal{S} : T^w < \infty\} & \text{if well-defined,} \\ \min_{\succ} \mathcal{S} & \text{if } T^w = \infty \text{ for all } w \in \mathcal{S}. \end{cases}$$

In words, v^* is the worst project among those which the principal ever demands as a favour, and w^* is worst project among those the principal ever supplies as a favour (if applicable). Note that Item 3 implies that if $T^{v^*} < \infty$, then $T^{w^*} = 0$, and that if $T^{w^*} > 0$, then $T^{v^*} = \infty$.

The proof of this claim follows from arguments closely mirroring those of Steps 1 and A of Proposition 1, and is omitted. Again, these arguments are simplified in this case because the only individual rationality constraint for the agent in problem (25) is for the initial history.

Step 4. Given $\overline{v}, \underline{v} \in \mathcal{D}$, consider the associated solutions $\overline{\kappa}^*$ and $\underline{\kappa}^*$ to the problem (25) with $u_1 = \overline{v}$ and $u_1 = \underline{v}$, respectively. We show that if either

$$(i)\ \overline{v}^* \succ \underline{v}^* \text{ or } (ii)\ \overline{v}^* = \underline{v}^* \text{ and } \overline{T}^{\overline{v}^*} < \underline{T}^{\underline{v}^*},$$

then either

$$(i)\ \overline{w}^* \succ \underline{w}^* \text{ or } (ii)\ \overline{w}^* = \underline{w}^* \text{ and } \overline{T}^{\overline{w}^*} \leq \underline{T}^{\underline{w}^*}.$$

To see this, suppose that either (i) $\overline{v}^* \succ \underline{v}^*$ or (ii) $\overline{v}^* = \underline{v}^*$ and $\overline{T}^{\overline{v}^*} < \underline{T}^{\underline{v}^*}$. Note that, by Item 1 in Step 3, we have in both cases (i) and (ii) that $\overline{T}^v \leq \underline{T}^v$ for all $v \in \mathcal{D}$, with at least one inequality strict, so that, in words, the contract $\overline{\kappa}^*$ is strictly more generous in terms of what it demands from the agent than $\underline{\kappa}^*$. Now suppose, towards a contradiction, that either (i) $\underline{w}^* \succ \overline{w}^*$ or (ii) $\underline{w}^* = \overline{w}^*$ and $\underline{T}^{\underline{w}^*} < \overline{T}^{\overline{w}^*}$. Note that, by Item 2 in Step 3, we have that $\overline{T}^w \geq \underline{T}^w$ for all $w \in \mathcal{S}$, with at least one inequality strict, so that, in words, the contract $\underline{\kappa}^*$ is strictly more generous in terms of what it supplies to the agent than $\overline{\kappa}^*$. First, let $\tilde{v}$ be such that $\overline{T}^{\tilde{v}} < \underline{T}^{\tilde{v}}$, which by assumption must exist. Then, we have $\overline{T}^{\tilde{v}} < \infty$ and $\underline{T}^{\tilde{v}} > 0$. Second, fix $\tilde{w}$ such that $\overline{T}^{\tilde{w}} > \underline{T}^{\tilde{w}}$, which by (our contradiction) assumption must exist. Then, we have $\overline{T}^{\tilde{w}} > 0$ and $\underline{T}^{\tilde{w}} < \infty$. Third, $\overline{T}^{\tilde{v}} < \infty$ and $\overline{T}^{\tilde{w}} > 0$, by Item 3 of Step 3, implies $\tilde{w} \succ \tilde{v}$. Fourth, $\underline{T}^{\tilde{v}} > 0$ and $\underline{T}^{\tilde{w}} < \infty$, by Item 3 of Step 3, implies $\tilde{v} \succ \tilde{w}$, yielding the desired contradiction.

Step 5. The previous point allows us to rank the solutions to (25) for various $v' \in \mathcal{D}$ for which $u_1 = v'$ in terms of how generous they are to the agent. Specifically, fix $\overline{v}, \underline{v} \in \mathcal{D}$ and consider the associated solutions $\overline{\kappa}^*$ and $\underline{\kappa}^*$ to the problem (25) with $u_1 = \overline{v}$ and $u_1 = \underline{v}$, respectively. If either (i) $\overline{v}^* \succ \underline{v}^*$, or (ii) $\overline{v}^* = \underline{v}^*$ and $\overline{T}^{\overline{v}^*} < \underline{T}^{\underline{v}^*}$, or (iii) $\overline{v}^* = \underline{v}^*$, $\overline{T}^{\overline{v}^*} = \underline{T}^{\underline{v}^*}$ and $\overline{w}^* \succ \underline{w}^*$, or (iv) $\overline{v}^* = \underline{v}^*$, $\overline{T}^{\overline{v}^*} = \underline{T}^{\underline{v}^*}$, $\overline{w}^* = \underline{w}^*$ and $\overline{T}^{\overline{w}^*} < \underline{T}^{\underline{w}^*}$, then we say that the contract $\overline{\kappa}^*$ is *more generous to the agent* than contract $\underline{\kappa}^*$. In words, Step 4 says that when these conditions are met, then $\overline{\kappa}^*$ demands less of every project $v \in \mathcal{D}$, and supplies more of every project $w \in \mathcal{S}$, than $\underline{\kappa}^*$. Fix any project u such that $u_1 = u$. and let $\overline{U}_{i,1}$ denote the payoff to i from contract $\overline{\kappa}^*$, and $\underline{U}_{i,1}$ denote the payoff to i from contract $\underline{\kappa}^*$. It follows that if $\overline{\kappa}^*$ is more generous to the agent than $\underline{\kappa}^*$, then we have that $\overline{U}_{A,1} \geq \underline{U}_{A,1}$. An implication is that contract $\overline{\kappa}^*$ must still satisfy $(IR_{A,1})$ if $u_1 = \underline{v}$, but that contract $\underline{\kappa}^*$ does not satisfy $(IR_{A,1})$ if $u_1 = \overline{v}$.

Step 6. We remove from the set $\mathcal{D}$ any project v for which the solution to problem (25) with $u_1 = v$ is the no-production contract following all histories u^t with $u_t \in \mathcal{D} \cup \mathcal{S}$. For all histories in which one of these project is available, we set the production probabilities to 0 in the optimal contract we are constructing. Note that by the construction of problem (25), no individually rational contract delivers a higher payoff to the principal following any such history.

Step 7. Let $v^1 \in \mathcal{D}$ be the project for which the solution κ^{1*} to problem (25) with $u_1 = v^1$ is the most generous among all solutions to (25) with $u_1 = v'$ for some $v' \in \mathcal{D}$. By Step 6, we can assume that κ^{1*} is such that the associated time thresholds have $T^v > 0$ for some $v \in \mathcal{D}$ (and correspondingly $T^w < \infty$ for some $w \in \mathcal{W}$).

First, we show that we cannot have $v^* \succ v^1$. In words, it must be that, conditional on $u_1 = v^1$, the principal demands a favour with positive probability at $t = 1$ under κ^{1*}. To see this suppose, towards a contradiction, that $v^* \succ v^1$. Fix an initial history with $u_1 = v^1$, and define an alternative contract $\tilde{\kappa}$ such that (i) $\tilde{\kappa}_1 = 0$ and $\tilde{\kappa}_t = 0$ following any history u^t such that $u_{t'} \in \mathcal{S}$ for any subhistory $u^{t'}$ of u^t with $t' \geq 2$, and (ii) $\tilde{\kappa}$ implements κ^{1*} starting from t following all other histories. By the fact that u is a Markov process and by Step 5, we know that $\tilde{U}_{A,t} \geq 0$ following all histories listed in point (ii). In turn, because $\tilde{\kappa}_t u_{A,t} = 0$ for all histories listed in point (i), it follows that $\tilde{U}_{A,t} \geq 0$ for these histories. Therefore, $\tilde{U}_{A,1} \geq 0$. Because there exists some $w \in \mathcal{S}$ such that $T^w < \infty$, we have that $\tilde{U}_{1,P} > U^{1*}_{1,P}$, which contradicts the optimality of κ^{1*} in (25).

Second, we show that following any history u^t with $t \geq 2$, contract κ^{1*} satisfies $(IR_{A,t})$. To see this, let $U^{1*,t}_{A,t}$ be the payoff to the agent if κ^{1*} was implemented starting from t, and note that

$$\begin{aligned} U^{1*}_{A,t} &\geq U^{1*,t}_{A,t} \\ &\geq 0, \end{aligned}$$

where the first inequality follows by the fact that u is a Markov process and because contract κ^{1*} becomes more generous between times 1 and t, and the second inequality follows, again, by the fact that u is a Markov process and by Step 5.

Finally, note that no individually rational contract delivers a higher payoff to the principal than κ^{1*} at any history u^t with $u_t = v^1$, which follows by the construction of problem (25).

Step 8. Define the set of projects $V^1 = \{v^1\}$ with associated set of contracts $K^1 = \{\kappa^{1*}\}$. Now, inductively, fix a set of projects $V^{n-1} = \{v^1, \ldots, v^{n-1}\}$ and associated set of contract $K^{n-1} = \{\kappa^{1*}, \ldots, \kappa^{n-1*}\}$. Assume that (i) each κ^{i*} is individually rational following all histories, and that (ii) no individually rational contract delivers a higher payoff to the principal than κ^{i*} following any history u^t with $u_t = v^i$. Further assume that (iii) the time thresholds associated to the contracts in K^{n-1} are such that $T^{u,1} \leq T^{u,2} \leq \ldots \leq T^{u,n-1}$ for all $u \in \mathcal{D} \cup \mathcal{S}$. Fix any project $v' \in \mathcal{D} \setminus V^{n-1}$ and suppose that $u_1 = v'$. We define the reduced problem

$$\begin{aligned} \max_{\kappa \in \mathcal{K}} U_{P,1} \quad \text{subject to} \quad & U_{A,1} \geq 0, \\ & U_{A,t} \geq 0 \text{ at each } t > 1 \text{ at which } u_t \in V^{n-1}, \\ & \kappa = \kappa^{i*} \text{ at each } t > 1 \text{ with } u_t = v^i \in V^{n-1} \text{ and } U_{A,t} = 0. \end{aligned} \tag{26}$$

This problem corresponds closely to the problem (25): the goal is to find an optimal contract while ignoring all individual rationality constraints other than (i) the constraint at time $t = 1$ but also (ii) all constraints associated with the first arrival of an opportunity to demand favour $v^i \in V^{n-1}$. In the latter case, the problem (26) prescribes that the contract κ^{i*} be adopted at this history if the agent-feasibility constraint binds.

Step 9. As in Step 2 for problem (25), we show that there exists a solution to problem (26) with contract κ^* of the following threshold type: for each $v \in \mathcal{D} \setminus V^{n-1}$, there exists $T^v \geq 0$ such that, given any history u^t with (i) $u_t = v$ and (ii) $U_{A,t'} > 0$ for all $1 \leq t' \leq t$ for which $u_{t'} \in V^{n-1}$,

$$\kappa_t^* = \begin{cases} 1 & \text{if } T^v \geq t+1, \\ T^v - t & \text{if } t < T^v < t+1, \\ 0 & \text{if } T^v \leq t, \end{cases}$$

and for each $w \in \mathcal{S}$, there exists $T^w \geq 0$ such that, given any history u^t with (i) $u_t = w$ and (ii) $U_{A,t'} > 0$ for all $1 \leq t' \leq t$ for which $u_{t'} \in V^{n-1}$,

$$\kappa_t^* = \begin{cases} 1 & \text{if } T^w \leq t, \\ t+1-T^w & \text{if } t < T^w < t+1, \\ 0 & \text{if } T^w \geq t+1. \end{cases}$$

In words, the thresholds above are valid until the contract transitions to κ^{i*} for some $i = 1, \ldots, n-1$. As for Step 2, the proof of this claim follows from arguments closely mirroring those of Steps 3 and C of Proposition 1, and is omitted. Also as in Step 2, we normalise these time thresholds so that (i) $T^v = 0$ if and only if $\kappa_t^* = 0$ for all $t \geq 1$ with $u_t = v$, and that (ii) $T^w = 0$ if and only if $\kappa_t = 1$ for all $t \geq 1$ with $u_t = w$.

Step 10. Given simple adaptations of the arguments in Steps 3-5 for problem (25), it can be shown that solutions to (26) for projects $v \in \mathcal{D} \setminus V^{n-1}$ can be ranked according to how generous they are to the agent. Furthermore, each of these solutions is not the no-production contract, because each solution in K^{n-1} is not the no-production contract (by Step 6).

Step 11. Let $v^n \in \mathcal{D} \setminus V^{n-1}$ be the project for which the solution κ^{n*} to problem (26) with $u_1 = v^n$ is the most generous among all solutions to (26) with $u_1 = v'$ for some $v' \in \mathcal{D} \setminus V^{n-1}$ that also have $T^{v'} > 0$. First, if no such project exists, then for all projects $v \in \mathcal{D} \setminus V^{n-1}$ and all histories u^t such that $u_{t'} \notin V^{n-1}$ for all $t' \leq t$, we set the production probabilities to 0 in the optimal contract we are constructing. Furthermore, by construction of problem (26), no individually rational contract delivers a higher payoff to the principal following any such history. Second, if instead project v^n is well-defined, then simple adaptations of the arguments in Step 7 for problem (25) ensure that contract κ^{n*} is such that (i) it satisfies $(IR_{A,t})$ following any history u^t and (ii) no individually rational contract delivers a higher payoff to the principal than κ^{n*} at any history u^t with $u_t = v^n$.

Step 12. The previous step concludes the inductive construction of the optimal contract. Note that for each contract in the collection K^n, the associated thresholds $(\{T^v\}_{v\in\mathcal{D}}, \{T^w\}_{w\in\mathcal{S}})$ define the collection of thresholds $\left(\{\tau^{v'v}\}_{v\in\mathcal{D}}, \{\tau^{v'w}\}_{w\in\mathcal{S}}\right)_{v'\in K^n}$ from the statement of the proposition. It only remains to be shown that these thresholds are ordered by their generosity to the agent: suppose that $\underline{v} = v^n$ and that $\overline{v} = v^{n-1}$, then it follows that $\overline{T}^u \leq \underline{T}^u$ for all $u \in \mathcal{D}\cup\mathcal{S}$. To see this, first note that, if we let $\{\underline{T}^{u,n-1}\}_{u\in\mathcal{D}\cup\mathcal{S}}$ be the thresholds associated to the solution to the version of problem (26) at stage $n-1$ (given sets V^{n-2} and K^{n-2}) with $u_1 = \underline{v}$, we have that $\overline{T}^u \leq \underline{T}^{u,n-1}$ for all $u \in \mathcal{D}\cup\mathcal{S}$. Second, our claim is established after we show that $\underline{T}^{u,n-1} \leq \underline{T}^u$ for all $u \in \mathcal{D}\cup\mathcal{S}$. To see this, note that given $u_1 = \underline{v}$, problem (26) differs from the version of this problem at stage $n-1$ only through the additional constraint that $\kappa = \overline{\kappa}$ following all histories with $u_t = \overline{v}$ and $U_{A,t} = 0$. Therefore, if we let $\underline{U}_{A,1}^{z,n-1}$ denote the agent's payoff in (26) at stage $z = n-1, n$ with $u_1 = \underline{v}$ to thresholds $\{\underline{T}^{u,n-1}\}_{u\in\mathcal{D}\cup\mathcal{S}}$, it follows that $\underline{U}_{A,1}^{n,n-1} \geq \underline{U}_{A,1}^{n-1,n-1} = 0$. Given this, it must be that thresholds $\{\underline{T}^u\}_{u\in\mathcal{D}\cup\mathcal{S}}$ are less generous than thresholds $\{\underline{T}^{u,n-1}\}_{u\in\mathcal{D}\cup\mathcal{S}}$, that is, that $\underline{T}^{u,n-1} \leq \underline{T}^u$ for all $u \in \mathcal{D}\cup\mathcal{S}$. $\square$

Proof of Corollary 3. Suppose that the project process u is *iid*, fix some history u^t along with a contract κ, consider the agent's payoff

$$U_{A,t} = \kappa_t u_{A,t} + \delta \mathbb{E}_t U_{A,t+1},$$

and note that $\mathbb{E}_t U_{A,t+1}$ is independent of u_t. It follows that the solution to problem (25) with $u_1 = \overline{v}$ is more generous to the agent than the solutions to (25) with $u_1 = \underline{v}$ if and only if $|\overline{v}_A| > |\underline{v}_A|$. The same property holds for solutions to problem (26), which together generate the ranking of projects $v \in \mathcal{D}$ in terms of their generosity to the agent. $\square$

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