Accountability via Delegation in Dynamic Elections

John Duggan∗ Jean Guillaume Forand†

May 28, 2019

Abstract

We study the possibility that elections achieve desirable policy outcomes by bringing the incentives of politicians in line with those of a representative voter, in the context of a general dynamic environment. As our normative benchmark, we take the solution of the dynamic programming problem facing the voter, as if she chose policy directly. We show that when politicians are highly office motivated, there exist equilibria in which all politician types implement an optimal policy rule of the voter. More generally, there exist equilibria in which the politician type corresponding to the representative voter acts as a “faithful delegate” by implementing an optimal rule when elected. We then show that when voters are patient or politicians are highly office motivated, the presence of such a faithful delegate allows the voter to achieve, asymptotically or exactly, optimal policies. Finally, we demonstrate the possibility of multiple equilibria with undesirable normative properties, and we provide relatively narrow conditions that preclude the possibility of such equilibria.

Keywords: Electoral Accountability; Dynamic Models; Median Voter

1 Introduction

The electoral process has the potential, by subjecting incumbents to periodic review by voters, to discipline office holders and bring policy choices in line with voters’ preferences. This is so even if politicians do not share these preferences, so long as the value of holding office provides a sufficient incentive for incumbents to put aside their own policy preferences and to compete with the option of a challenger. When elections take place in a dynamic environment, two distinctive challenges to the efficacy of the electoral mechanism present themselves, both stemming from the absence of intertemporal commitment. First, in any given election, candidates may find it difficult to make credible promises about their policy choices in future environments, so that even a candidate who would be willing to bind herself to popular policies in order to gain reelection has no way of doing so. Second, voters also have no way of committing to future reelection standards, so they cannot incentivize politicians by offering reelection in exchange for desirable policies. Because office holders’ expectations of future electoral prospects drive their current actions in office, and voters’
expectations of politicians’ future policy choices drive their current electoral decisions, political accountability is inherently vulnerable to this dual commitment problem.

In this paper, we consider a general dynamic model of elections, determine conditions under which policy choices are responsive to voter preferences, and, in case these conditions are violated, demonstrate the possibility of corresponding dynamic political failures. In each period of the model, a state is given and an incumbent office holder chooses policy from a feasible set; then a challenger is drawn and an election is held; and then a new state is realized, and so on. The state determines current policy preferences, and the state and policy choice determine the transition probability over next period’s state, as well as the distribution of the challenger’s type. Minimal structure on states and policies is assumed: states are discrete, policies lie in a compact metric space, and stage utilities and transition probabilities are assumed only to be continuous. We assume that information is symmetric between voters and the office holder, so that a politician’s type is observed by voters once the politician takes office—but the challenger’s type is not directly observed before the election. Thus, elections pit a known incumbent against a relatively unknown challenger. We study Markov electoral equilibria, which adapt the notion of stationary Markov perfect equilibrium to our political environment. Our results examine the possibility that equilibrium policies solve the dynamic programming problem of the representative voter in the hypothetical scenario that he chooses policy directly, the natural analogue of the classical median voter theorem from the static Downsian setting.

We establish existence of a Markov electoral equilibrium in which the politician type corresponding to the representative voter acts as a faithful delegate, in the sense that her policy choices solve the representative dynamic programming problem. We then show that the presence of a faithful delegate can allow the voter to achieve, asymptotically or exactly, optimal policies in equilibrium. This responsiveness can operate through two different channels. First, voter patience can overcome the commitment problems inherent in elections by allowing the voter to “wait it out,” rejecting incumbents until a faithful delegate takes office; using this logic, we show that the voter’s equilibrium payoffs at recurrent states converge to the optimum as the voter becomes patient. Second, exact optimality can be achieved for any given rate of time discounting, as long as (i) the transition probability on states is independent of policy, (ii) incumbents have available policy choices that deliver reelection, and (iii) politicians are highly office motivated. Here, the existence of a faithful delegate serves to discipline politicians with policy preferences at odds with the voter’s, with high office incentives reinforcing that effect. While the conclusion of the latter result is strong, so are its assumptions, and we demonstrate that political failures can occur in the absence of either (i) or (ii). For example, when the state transition depends on current policy choices, it is possible that even though one politician type does act as a faithful delegate, other types can manipulate the state, creating a political hold-up problem, and forcing the voter to reelect them after suboptimal policies.

Therefore, under broad conditions, there is a selection of equilibria in which the optimal payoff of the voter is approximated or achieved, and the accountability problem reduces to the possibility of additional equilibria that generate suboptimal policies. Given the results described above, the central difficulty is the possibility that the politician type corresponding
to the voter does not act as a faithful delegate. To address this, we show that if politicians are purely policy motivated, then the incentives of the politician type corresponding to the voter are always aligned with the voter’s, and the politician indeed chooses policies that solve the representative dynamic programming problem in every equilibrium. Thus, if politicians are policy motivated, then the voter’s payoff approximates the optimum for every selection of equilibria, as the voter becomes patient. This holds in the model with general policy space, states, and utilities; in particular, the positive result obtains even if we allow policy choices to affect the evolution of future states in an arbitrary way. In addition, to admit the possibility that politicians have a preference for holding office per se, we assume politicians have positive commitment power, in the sense that if an incumbent chooses a particular policy in a given state, then there is a positive probability that the state remains in place and the politician is bound to the same policy. We show that if the evolution of the state and challenger is exogenous, then, again, the politician type corresponding to the voter is a faithful delegate in every equilibrium. In light of the previous discussion, it follows that with positive commitment power, if (i) the transition on states and challenger types is independent of policy, (ii) incumbents can choose policies that deliver reelection, and (iii) politicians are highly office motivated, then the equilibrium multiplicity problem is precluded, and all politician types choose optimal policies for the voter.

The key to our positive results is that the politician type corresponding to the voter acts as a faithful delegate. Given this, to achieve asymptotic optimality of policies, it is enough if the voter is patient, permitting him to wait for a faithful delegate without relying on compliance of other politician types. To achieve exact optimality, it must be not only that the politician type corresponding to the voter is a faithful delegate, but that all other politician types solve the representative dynamic programming problem as well. This, in turn, relies on office motivation of politicians, so that reelection is sufficient to induce politicians to comply with the optimal policy choices of the voter—in this respect, high office incentives can facilitate responsiveness of politicians to voter preferences. As described above, the existence of a faithful delegate follows if politicians are policy motivated or if the evolution of the game is independent of current policy choices. However, in the absence of these assumptions, a wedge is introduced between the voter and the corresponding politician type, and this can create an incentive for the politician to manipulate the state, choosing current policies that are suboptimal for the voter to increase the chances of reelection in the future. Here, office incentives actually have a countervailing effect on voter welfare, and as a consequence, there may be equilibria in which no politician acts as a faithful delegate, so that the voter’s payoff does not achieve or approximate the optimum. This “curse of ambition,” along with the example of the political hold-up problem, illustrate the nature of possible political failures and the limitations of the electoral mechanism.

**Literature** A recurring theme of the dynamic political economy literature is that commitment problems are critical for understanding policy outcomes and evaluating electoral performance. The assumption that politicians can commit to policies in one-shot elections, standard since the work of Downs (1957), has often been contested, notably in the context

---

1This minimal form of ex post commitment on the part of politicians, which is consistent with a story of administrative or institutional costs of policy change, differs from the ex ante commitment in Downsian models and is in the spirit of the citizen-candidate approach.
of citizen-candidates models (Besley and Coate (1997) and Osborne and Slivinski (1996)).
Extending such commitment to sequences of policy choices is even more debatable (e.g.,
Alesina (1988)), and what is now a large literature studies the dynamic policy consequences
of office holders’ inability to make credible campaign proposals. For example, Acemoglu
et al. (2008) and Yared (2010) describe the distortions in tax policies that are necessary
to provide rent-seeking politicians with the incentives to limit their extractive activities
(see Acemoglu (2003) for an application to non-democratic states). Commitment failures
are accentuated in models with term limits, e.g., Banks and Sundaram (1998), Bernhardt
et al. (2004), and Besley and Case (1995), where subgame perfection directly implies that
politicians choose their ideal policies in the last term of office. Duggan (2017) shows that
this incentive leads to an upper bound on equilibrium payoffs of voters, irrespective of the
strength of office incentives, highlighting the role of the voters’ commitment problem.

Because voters cannot commit to reelect politicians after good policy choices, politicians
may anticipate government turnover and choose poor policies, reinforcing the voters’ incentive
to remove the politician. Persson and Svensson (1989) and Alesina and Tabellini (1990)
show that incumbents may distort public policies in order to “tie the hands” of potential
successors with different preferences. Besley and Coate (1998) find that politicians may fail
to implement Pareto-improving investments if they anticipate that future policy-makers will
not realize their returns. In a dynamic legislative bargaining setting, Battaglini and Coate
(2008) show that legislators’ uncertainty about being included in future governing coalitions
drives them to approve excessive pork barrel spending. In models of two-party competition,
Azzimonti (2011, 2015) shows that the prospects of government turnover can lead to
inefficiencies in either private or public capital accumulation, and Battaglini (2014) shows
that when voters’ preferences are time-varying, temporarily powerful districts can attract
inefficiently high levels of government spending. Bai and Lagunoff (2011) show that these
concerns are magnified when future office holders are determined by current policy choices,
so that they further distort policies in order to affect the identities of their successors.²

More broadly, our paper is related to the literature in economics and political science
that studies the possibility and limits of electoral accountability (for comprehensive surveys,
see Duggan and Martinelli (2017) and Ashworth (2012)). Existing work in this literature
often focuses on the effects of politicians’ private information, either in the form of adverse
selection or moral hazard, and imposes extensive structure on the electoral environment
to achieve tractability. In comparison, we assume symmetric information, but we consider
very general dynamic environments. The single-state version of our model is closely related
to the dynamic model of elections with adverse selection considered by Duggan (2000) and
Bernhardt et al. (2004).³ In that work, politicians are privately informed about their preferences, and there is no state variable; in equilibrium, after information is revealed by an
office holder’s initial policy choice (and assuming she is reelected), the politician’s policy

²Anticipated turnover in power has also been associated with political inefficiencies outside the realm
of electoral competition, e.g., with policy “gridlock” in legislatures (Bowen et al. (2014) and Dziuda and
Loeper (2016)) or with the creation of ineffective public administrations (Acemoglu et al. (2011)).
³See further applications include the analysis of competence (Meirowitz (2007)), parties (Bernhardt et al.
(2009)), valence (Bernhardt et al. (2011)), and taxation (Camara (2012)). Duggan (2014) provides a folk
theorem for the model when non-Markovian equilibria are permitted.
choice is expected to remain the same over time. Because of this, the equilibria of those adverse selection models are replicated in our model by specifying a single state and full commitment power on the part of the incumbent. Thus, the form of commitment we allow proxies for voter beliefs in the adverse selection framework. Fundamental differences between the models arise, however, when we move beyond the single-state model to allow multiple states and the more complex incentives they entail: whereas reputational issues blow up and render the adverse selection model intractable, our model remains viable.

Outline of the paper The remainder of the paper is organized as follows. Sections 2 and 3 set forth the model and describe the equilibrium concept, respectively. Section 4 introduces the analysis by considering the strong optimality criterion that equilibrium policy choices are symmetric and implement a single optimal policy rule. Section 5 contains our equilibrium existence result, and it provides conditions under which the presence of a faithful delegate allows the voter to approximate or attain his optimal payoff. Section 6 confronts the problem of multiple equilibria by providing conditions such that in all equilibria, some politician type acts as a faithful delegate, allowing results of the previous section to be applied; and in the absence of those conditions, we demonstrate the possibility of political failures. Section 7 concludes, and the Appendix contains proofs of results.

2 Dynamic Electoral Framework

Political environment A representative voter decides between an incumbent politician and a challenger in an infinite sequence of elections. The voter and an infinite pool of politicians are partitioned into a finite set $T$ of types, the voter’s type being $k$ and a politician’s typically denoted $t$. Politician types are initially private information and are independently (but not necessarily identically) distributed. Each period begins with a state $s$ and a politician who holds office, the state and the office holder’s type being publicly observed. The office holder chooses a policy $y$; a challenger whose type is private information is selected; an election is held; a new state is realized, and the winner’s type, along with the state, is observed; and the process repeats. We assume that states belong to an arbitrary countable set $S$; that policies lie in a compact metric space $Y$; and that in every state $s$, the set of feasible policies is a nonempty, closed (and therefore compact) subset $Y(s)$ of $Y$. The dependence of the feasible set on the state allows us to interpret $s$ as a state of the economy, which can affect the range of available policies.

In addition to choosing policy, the office holder chooses whether to run for reelection. Rather than model this decision using a separate variable, it is convenient to use $Y$ to represent choices of policy and the decision to run for reelection, and to use a copy of $Y$, denoted $Z$, to represent policy choices and the decision not to run.$^5$ We maintain the

$^4$The parallel between the frameworks extends to the model with a multidimensional policy space analyzed by Banks and Duggan (2008).

$^5$The option to forego reelection does not play a role in our results in this paper, but it is important for establishing the existence of equilibria, for which we apply a result from Duggan and Forand (2018). Indeed, even in our special case of that more general model, incumbents may strictly prefer to lose an election if future policy and voting outcomes are more favorable following a challenger’s victory. Clearly,
convention that $Y \cap Z = \emptyset$; we assume a mapping $\xi: Y \cup Z \rightarrow Z$ so that for all $y \in Y$, $\xi(y) = z$ is the element of $Z$ corresponding to $y$ and for all $z \in Z$, $\xi(z) = z$; we let $Z(s) = \xi(Y(s))$ be the feasible policy choices for an office holder who chooses not to seek reelection in state $s$. Let $X = Y \cup Z$ represent the space of simultaneous policy choices qua campaign decisions, and let $x \in X$ denote a generic choice of policy and campaign decision.

**Challengers** After the office holder chooses policy, a challenger is drawn at large from the pool of politicians who have never held office, so the challenger’s type is not observed by voters before the election. We maximize generality by allowing challenger selection to depend on the previous state and policy choice: let $q(t'|s, x)$ denote the probability that the challenger is type $t'$, given that the incumbent chooses policy $x$ in state $s$. We assume that the transition probability on challenger types, $q: T \times S \times X \rightarrow [0, 1]$, is continuous, and that it is independent of the incumbent’s campaign decision, i.e., $q(t'|s, y) = q(t'|s, \xi(y))$ for all $y \in Y$.

**State transitions** States are used to describe the political and/or economic environment in the current period. Given that an office holder chooses a policy $x$ in state $s$, a new state $s'$ is drawn with probability $p(s'|s, x)$: thus, states evolve according to a controlled Markov process. We assume that the transition probability $p: S \times S \times X \rightarrow [0, 1]$ is continuous and independent of the incumbent’s campaign decision, i.e., $p(s'|s, y) = p(s'|s, \xi(y))$ for all $y \in Y$.

**Commitment power** To understand the impact of the politicians’ commitment problem in sustaining suboptimal equilibria, we incorporate the possibility of a weak form of commitment that is consonant with the citizen-candidate approach. Specifically, if a type $t$ politician chooses a policy $x$ in a state $s$, and if she is subsequently reelected and the next state remains $s$ (i.e., $s' = s$), then with probability $\gamma_t \geq 0$, the politician is committed, or bound, to $x$; and with probability $1 - \gamma_t$, the politician is free to choose any feasible policy. In subsequent periods, the commitment process continues in this manner until either the state remains at $s$ and commitment is broken or the state transitions away from $s$ (i.e., $s' \neq s$). This form of commitment differs from the Downsian approach, in which both candidates commit to arbitrary platforms before an election; here, the incumbent who may be committed to a policy that has actually been implemented in a state after an election. Rather than relying on the credibility of campaign promises, our approach to commitment captures the need for an elected politician to put in place costly institutional and administrative structure (e.g., appointments of cabinet members, agency heads, or judgeships) to implement her intended policy program; alternatively, the “stickiness” of policy can reflect the cost of developing expertise that is specific to a state-policy pair, or it may reflect deals with unmodeled bureaucrats or political actors that are costly to break. Because the minimal form of commitment allowed can only apply when the state remains in place, we use $\gamma_t p(s'|s, x)$ to measure the commitment power of the type $t$ office holder from choosing $x$ in state $s$. Clearly, we capture the canonical citizen-candidate assumption of zero commitment.

---

6 Technically, $\xi$ restricted to $Y$ is an isometric embedding. It suffices to set $Z = Y \times \{1\}$ and to specify that $\xi(y) = (y, 1)$ for all $y \in Y$.

7 We give $S$ and $T$ the discrete topology, so our continuity assumption means that $q(t'|s, x)$ is continuous in $x$ for all $s \in S$ and all $t' \in T$. 

that eventuality can be ruled out if politicians are sufficiently office-motivated.
by setting $\gamma_t = 0$.

**Payoffs** The stage utility of a type $t$ citizen from policy $x$ in state $s$ is $u_t(s, x)$, while a politician who holds office receives an additional office benefit $b_t \geq 0$. We assume for simplicity that running for office is costless, i.e., for all $x \in Y$, $u_t(s, x) = u_t(s, \xi(x))$, and that $u_t: S \times X \to \mathbb{R}$ is bounded and continuous. Each type $t$ citizen discounts flows of payoffs by the factor $\delta_t \in [0, 1)$. Thus, given a sequence $(s_1, x_1, s_2, x_2, \ldots)$ of state-policy pairs, the discounted payoffs of a type $t$ citizen is

$$
\sum_{t=1}^{\infty} \delta_t^{t-1} [u_t(s_t, x_t) + I_t b_t],
$$

where $I_t$ is an indicator function taking value one if the citizen holds office in period $t$ and zero otherwise. We say the type $t$ politician is *policy motivated* if $\delta_t b_t = 0$, so that the politician does not value holding office in the future per se. We let $\underline{u}$ and $\bar{u}$ denote upper and lower bounds, respectively, on the payoff functions $u_t$, so that for all $s$, all $x$, and all $t$, we have $\underline{u} \leq u_t(s, x) \leq \bar{u}$. For convenience, we adopt the normalization that $\underline{u} = 0$ and $\bar{u} = 1$.

**Representative dynamic programming problem** Given our assumption of a representative voter, our normative benchmark in the analysis of accountability is the optimal value for the voter in the associated representative dynamic programming problem, in which the voter directly chooses any policy $x \in Y(s)$ in state $s$, receives utility $u_k(s, x)$, the next state $s'$ is realized from $p(\cdot|s, x)$, and so on. Under our maintained compactness and continuity conditions, this program has a unique value $V^*_k$, which solves the associated Bellman equation: for all $s$,

$$
V^*_k(s) = \max_{x \in Y(s)} u_k(s, x) + \delta_k \sum_{s'} p(s'|s, x) V^*_k(s').
$$

Let $\Phi^*_k(s)$ denote the set of optimal policies in state $s$, i.e.,

$$
\Phi^*_k(s) = \arg \max_{x \in Y(s)} u_k(s, x) + \delta_k \sum_{s'} p(s'|s, x) V^*_k(s').
$$

A *policy rule* is a mapping $\phi: S \to Y$ such that for all $s$, we have $\phi(s) \in Y(s)$, i.e., $\phi$ assigns a feasible policy to each state. By the optimality principle, a policy rule $\phi$ is optimal if and only if it selects from the correspondence of optimal policies, i.e., for all $s$, we have $\phi(s) \in \Phi^*_k(s)$.

### 3 Markov Electoral Equilibria

**Strategies** A stationary Markovian strategy for a type $t$ politician is a mapping $\pi_t: S \to \Delta(X)$, where $\Delta(X)$ is the set of Borel probability measures on $X$, and $\pi_t(\cdot|s)$ represents the mixture over policies used by the type $t$ politician when free in state $s$. We refer to $\pi_t$ as a *Markov policy strategy*, and $\pi = (\pi_t)_t$ denotes a profile of such strategies. A *voting strategy* is a Borel measurable mapping $\rho: S \times T \times X \to [0, 1]$, where $\rho(s, t, x)$ represents
the probability that a type \( t \) office holder is reelected following a free policy choice of \( x \) in state \( s \). The precise form of mixed voting we use is such that mixing occurs when the incumbent is free and chooses policy \( x \) in state \( s \); if the incumbent bound to \( x \) in state \( s \) (and thus reelected in the previous period after choosing \( x \) in state \( s \)), then the voter reelects the incumbent with probability one. This focus is not a constraint imposed on the voter; rather, by stationarity of the voter’s decision problem, it remains optimal to reelect the incumbent again when the politician is bound to a policy that was previously sufficient for reelection. We refer to \( \rho \) as a Markov voting strategy, and to \( \sigma = (\pi, \rho) \) as a Markov electoral strategy profile.

**Continuation values** Given a Markov electoral strategy profile \( \sigma \), we can define continuation values for a type \( t \) citizen. The discounted expected utility of the citizen from electing a type \( t' \) incumbent who chooses policy \( x \) in state \( s \) satisfies: for all \( x \in Y(s) \),

\[
V^I_t(s, t', x) = p(s|x, x) \left[ \gamma_{t'}(u_t(s, x) + \delta_{t'}V^I_t(s, t', x)) + (1 - \gamma_{t'})V^F_t(s, t') \right] + \sum_{s' \neq s} p(s'|s, x)V^F_t(s', t'),
\]

where \( V^F_t(s, t') \) is the expected discounted utility to the citizen from a type \( t' \) office holder who is free in state \( s \), calculated before a policy is chosen. In words, if the incumbent is reelected, then with probability \( p(s|x, x) \), the state remains \( x \), and in this case, with probability \( \gamma_{t'} \), the politician is bound to choose \( x \) again, and will be reelected; and with probability \( 1 - \gamma_{t'} \), the incumbent is free in \( s \). In all other states \( s' \), the incumbent is free. When an office holder chooses \( x \in Z(s) \) and thus not to stand for reelection, we have \( V^I_t(s, t', x) = V^C_t(s, x) \), where \( V^C_t(s, x) \) is the expected discounted utility of electing a challenger following the choice of \( x \) in state \( s \) and is defined by

\[
V^C_t(s, x) = \sum_{t'} q(t'|s, x) \sum_{s'} p(s'|s, x)V^F_t(s', t').
\]

That is, when a challenger is first elected, then the new office holder is free for every realization of next period’s state. Finally, \( V^F_t(s, t') \) is given by

\[
V^F_t(s, t') = \int_x \left[ u_t(s, x) + \delta_{t'}[\rho(s, t', x)V^I_t(s, t', x) + (1 - \rho(s, t', x))V^C_t(s, x)] \right] \pi_{t'}(dx|s).
\]

reflecting that the office holder chooses a policy \( x \) according to the policy strategy \( \pi_{t'}(\cdot|s) \), and is either reelected or replaced by a challenger.

In addition to payoffs from policies, a type \( t \) office holder evaluates future expected discounted office benefit from choosing policy \( x \) in state \( s \), conditional on being reelected, defined as follows: for all \( x \in Y(s) \),

\[
B_t(s, x) = p(s|x, x) \left[ \gamma_{t'}(b_t + \delta_{t'}B_t(s, x)) + (1 - \gamma_{t'})B^F_t(s) \right]
\]
where the expected discounted office benefit for a type $t$ office holder who is free in state $s$ is

$$B_t^F(s) = \int_{x'} [b_t + \delta_t \rho(s, t, x') B_t(s, x')] \pi_t(dx'|s),$$

reflecting the fact that the office holder receives $b_t$ in the current period and, conditional on choosing policy $x'$ and being reelected, receives $B_t(s, x')$ in the future. For all $x \in Z(s)$, set $B_t(s, x) = 0$.

**Reelection sets** Given a Markov electoral strategy profile $\sigma = (\pi, \rho)$ and policy choice $x$ in state $s$ by a type $t$ incumbent, the representative voter must consider the expected discounted utility of retaining the incumbent and must decide between her and a challenger. We therefore define for all states $s$ and all incumbent types $t$, the sets

$$P_k(s, t) = \{x \in Y(s) : V^I_k(s, t, x) > V^C_k(s, x)\}$$
$$R_k(s, t) = \{x \in Y(s) : V^I_k(s, t, x) \geq V^C_k(s, x)\}$$

of policies that yield the type $k$ voter an expected discounted utility strictly and weakly greater, respectively, than the expected discounted utility of a challenger. We refer to these as the *strict* and *weak reelection sets*, respectively. Note that choosing $x \in R_k(s, t)$ is necessary for reelection of a type $t$ incumbent in state $s$, and choosing $x \in P_k(s, t)$ is sufficient.

**Equilibrium concept** A Markov electoral strategy profile $\sigma$ is a *Markov electoral equilibrium* if policy strategies are optimal for all types of office holders and voting is consistent with incentives of the representative voter in all states; formally, we require that (i) for all $s$ and all $t$, $\pi_t(\cdot|s)$ puts probability one on solutions to

$$\max_{x \in X(s)} u_t(s, x) + b_t + \delta_t \rho(s, x, t)(V^I_t(s, t, x) + B_t(s, x)) + (1 - \rho(s, x, t))V^C_t(s, x),$$

and (ii) for all $s$, all $t$, and all $x$,

$$\rho(s, t, x) = \begin{cases} 1 & \text{if } x \in P_k(s, t) \\ 0 & \text{if } x \notin R_k(s, t), \end{cases}$$

where $\rho(s, t, x)$ is unrestricted if $x \in R_k(s, t) \setminus P_k(s, t)$. Intuitively, a type $t$ office holder maximizes current period utility plus future expected discounted payoff, which combines policy utility and office benefit (in case the politician is reelected) and the continuation value of a challenger (in case the politician loses). Duggan and Forand (2018) establish existence of Markov electoral equilibria in a more general framework that does not assume a representative voter and that allows general politician payoffs.

**Incumbency advantage** We sometimes focus on Markov electoral equilibria in which incumbents are always reelected. Formally, a strategy profile $\sigma$ exhibits *incumbency advantage* if for all $s$ and all $t$, we have $\int_x \rho(s, t, x) \pi_t(dx|s) = 1$. Such equilibria capture the
common feature of real-world elections that incumbents enjoy an electoral advantage over challengers, and they provide leverage on equilibria in which politicians make suboptimal policy choices. A much weaker condition is that incumbents have available at least one policy choice that ensures reelection with probability one: say $\sigma$ satisfies \textit{incumbency privilege} if for all $s$ and all $t$, there exists $x \in Y(s)$ such that $\rho(s, t, x) = 1$. Intuitively, incumbency privilege is the ability to choose reelection, and incumbency advantage is the ability and willingness to do so. Clearly, if $\delta_t b_t$ is high for each type, then all politician types are willing, and incumbency privilege implies incumbency advantage.

\textbf{Optimal retention} In the definition of Markov electoral equilibrium, the voter plays a passive role: he can check office holders retrospectively but takes future electoral decisions between incumbents and challengers as given. The following preliminary result, which we exploit later, shows that in equilibrium, it is as though the voter chooses an optimal reelection rule that, in addition to determining the winner of the current election, also dictates a probability of reelection for all future states, policies, and incumbent types. Given policy strategies for politicians, the corresponding \textit{representative optimal retention problem} is to choose such a rule to maximize the voter’s expected discounted payoff; let $V^I_k(s, t, x)$ denote the voter’s optimal value for this problem when facing a politician of type $t$ that has chosen policy $x$ in state $s$. In words, in any Markov electoral equilibrium, the voting strategy solves the voter’s corresponding optimal retention problem.

\textbf{Proposition 3.1.} Let $\sigma$ be a Markov electoral equilibrium. Then for all $s$, all $t$, and all $x \in Y(s)$,

$$V^I_k(s, t, x) = \rho(s, t, x)V^I_k(s, t, x) + (1 - \rho(s, t, x))V^C_k(s, x).$$

\section{Implementing Optimal Policy Rules}

In this section, we introduce the analysis by focusing on the strongest social optimality criterion, namely, that all politician types follow a given optimal policy rule for the voter. Specifically, we say that a Markov electoral equilibrium $\sigma = (\pi, \rho)$ \textit{implements} a policy rule $\phi: S \rightarrow Y$ if for all $s$ and all $t$, the strategy $\pi_t$ of type $t$ politicians places probability one on $\phi(s)$, i.e., $\pi_t(\{\phi(s)\}|s) = 1$, so that in every state $s$, every politician type $t$ chooses the designated policy $\phi(s)$. The notion of implementation used here implies that all politicians use symmetric, pure policy strategies, and clearly the existence of such equilibria will hold only under relatively stringent conditions, and if $\phi$ is optimal for the voter, then the welfare properties of such an equilibrium are strong. We show that existence holds under a joint restriction on politicians’ time discounting and office benefit: if office incentives are sufficiently high, then for every optimal policy rule, there is a Markov equilibrium that implements it, and furthermore, we can construct this equilibrium so that it exhibits incumbency advantage. The remaining sections of the paper are devoted to the analysis of equilibria in which politicians may use non-symmetric policy strategies.
We measure office incentives of the type \( t \) office holder in state \( s \) by
\[
\beta_t(s) = \frac{b_t + \delta_t - 1}{1 - \delta_t} + \min_{x \in Y(s)} p(s|x, x),
\]
and “sufficiently large” means that office incentives satisfy \( \delta_t \beta_t(s) \geq 1 \). Note that this inequality holds whenever \( \delta_t b_t \), the direct measure of politicians’ utility from holding office in the future, satisfies \( \delta_t b_t \geq 1 \). However, even when \( \delta_t b_t \) is low, if \( \frac{\delta_t b_t}{1 - \delta_t} > 1 \), then office incentives are large when the probability of staying in state \( s \) is close enough to one in every state (which, for example, holds trivially in the single-state model). To recap, \( \delta_t \beta_t(s) \geq 1 \) holds if office benefit is high, or if politicians are sufficiently patient, or if politicians are not too impatient while the inertia of the state is great enough.

Next, we demonstrate the positive implications of sufficiently large office incentives. In addition to implementing an optimal policy rule, we do so using equilibria that satisfy incumbency advantage, thereby maximizing the impact of office incentives. In the next section, Theorem 5.1 establishes that even if office incentives are zero, there exist equilibria in which the type \( k \) politician solves the representative dynamic programming problem; thus, office incentives are used here only to discipline the choices of other politician types.

**Proposition 4.1.** Assume that for all \( s \) and all \( t \), \( \delta_t \beta_t(s) \geq 1 \). If a policy rule \( \phi \) is optimal, then there is a Markov electoral equilibrium \( \sigma \) exhibiting incumbency advantage that implements \( \phi \).

The construction in the proof of Proposition 4.1 specifies that an incumbent is reelected in any state if and only if she follows the policy rule \( \phi \), and it hinges on the following logic: in any state \( s \), a type \( t \) office holder can guarantee reelection by following the rule \( \phi \), generating a stream of future office benefits with payoff \( \frac{\delta_t b_t}{1 - \delta_t} \); by deviating to \( x \neq \phi(s) \), the politician can achieve a policy gain in the current period, but at the cost of being replaced by a challenger, who also follows the rule. Because the distribution of the state can depend on \( x \), it is also possible that the deviation to \( x \) generates future policy gains—not by changing policy choices of the challenger (which are taken as given in equilibrium), but by increasing the likelihood of states that are advantageous to the current office holder. But if office incentives are sufficiently large, then no feasible increase in expected discounted policy gains compensates the incumbent for sacrificing reelection.

An implication of Proposition 4.1 is that when politicians are highly office motivated, the problem of electoral accountability reduces to the possibility of equilibrium multiplicity: the theorem leaves open the question of whether there are other equilibria in which politicians do not respond to electoral incentives by solving the problem of the representative voter. To address the possibility of such equilibria, we focus on equilibria satisfying incumbency privilege, and we assume a small amount of commitment power: we show that only optimal policy rules can be implemented by an equilibrium satisfying incumbency privilege.\(^8\) In the sequel, Corollaries 6.1 and 6.2 state corresponding results on the necessity of optimal policies, with the latter delivering exact optimality and assuming positive commitment

\(^8\)In fact, the result requires only that the type \( k \) politician has the ability to choose a policy that ensures reelection with probability one.
power. A difference between that corollary and the result here is that Corollary 6.2 uses high office incentives to discipline politician types \( t \neq k \) to follow the lead of the faithful delegate; in contrast, the next result implicitly avoids this by considering equilibria that implement a given rule, and thus it does not rely on strong office incentives.

**Proposition 4.2.** Assume that commitment power is positive, i.e., for all \( s \) and all \( x \in Y(s) \), i.e., we have \( \gamma_{kp}(s,x) > 0 \). For every policy rule \( \phi \), if there is a Markov electoral equilibrium \( \sigma \) satisfying incumbency privilege that implements \( \phi \), then \( \phi \) is optimal.

Put differently, in environments in which incumbents’ policy choices have some credible persistence, only optimal rules can be implemented by equilibria in which the representative voter displays a minimal deference towards type \( k \) incumbents. The idea is that if an equilibrium implements \( \phi \), and \( \phi \) is not optimal for the representative voter, then there is some state \( s \) such that \( \phi(s) \) is suboptimal given future policy choices, i.e., there is some \( x \in Y(s) \) such that

\[
 u_k(s,x) + \delta_k \sum_{s'} p(s'|s,x) V^\phi_k(s') > V^\phi_k(s).
\]

Then a type \( k \) incumbent can deviate by choosing \( x \) in state \( s \), and because the politician is committed to \( x \) if the state remains at \( s \), the representative voter then has a strict incentive to reelect the incumbent; thus, this deviation ensures reelection and clearly improves the politician’s policy payoff. However, this single-state deviation leaves open the possibility that future states in which the incumbent loses reelection become more likely, decreasing the politician’s expected office benefit and possibly offsetting policy gains from the deviation. To address this, we consider a deviation to a strategy, say \( \tilde{\pi}_k \), such that if the type \( k \) politician is free in any state \( s' \), then she chooses an alternative \( \tilde{x}_{s'} \) that secures reelection with probability one; such a policy exists by the assumption of incumbency privilege. We then argue that because each \( \tilde{x}_{s'} \) is acceptable to the voter, i.e., \( V^I_k(s', k, \tilde{x}_{s'}) \geq V^C_k(s', \tilde{x}_{s'}) \), this deviating strategy increases the type \( k \) politician’s expected discounted policy payoff in state \( s \), and because it also ensures reelection in every subsequent period, it is a profitable deviation—a contradiction. Thus, if \( \phi \) is implementable, then it is optimal.

## 5 Optimal Policies via Faithful Delegates

We have shown that when office incentives are high, optimal policy rules can be implemented by Markov electoral equilibria exhibiting incumbency advantage; and when commitment is positive, these are the only policy rules that can be implemented in this way. More generally, however, Markov electoral equilibria may specify that different politician types mix over different policy choices. In such cases, the implementation problem gives way to a non-trivial delegation problem for the representative voter, who must compare an incumbent to the meaningfully distinct option of a challenger. In this section, we first provide conditions under which the policy choices of the type \( k \) politician are optimal for the representative voter. We then show that this limited form of policy responsiveness can be leveraged by the representative voter to achieve, either asymptotically or exactly, equilibrium payoffs.
equal to his optimal value on a large set of states. Because incumbents’ policy choices must compete against those of likely challengers, and these challengers who may be type $k$, this type’s faithfulness can in principle spill over to other politicians.

We begin with an existence result for equilibria in which the type $k$ politician solves the dynamic programming problem of the representative voter and is reelected with probability one in every state. Existence of such equilibria follows from Proposition 4.1 when office incentives are large, for under that condition, there exist equilibria in which all politician types achieve the optimal value of the representative voter and all incumbents are reelected on the path of play. Of course, equilibria of this form do not exist generally, but we establish that there is always at least one Markov electoral equilibrium in which at least the type $k$ politician follows an optimal policy rule for the voter. Formally, given a Markov electoral equilibrium $\sigma$, we say the type $k$ politician is a faithful delegate if, in every state, the politician chooses an optimal policy for the voter: for all $s$, $V^F_k(s, k) = V^*_{k}(s)$.

**Theorem 5.1.** There is a Markov electoral equilibrium $\sigma$ in which the type $k$ politician is a faithful delegate and is reelected with probability one, i.e., for all $s$, $\int \rho(s, k, x) \pi_k(dx | s) = 1$.

The result is an application of Theorem 1 of Duggan and Forand (2018), which allows the state transition to depend on the incumbent’s type and the electoral outcome. To apply that result, we transform our model by specifying that if a type $k$ incumbent is removed from office, then the game moves to a bad state and remains at that state thereafter. Theorem 1 of Duggan and Forand (2018) then delivers a Markov electoral equilibrium $\sigma'$ of the transformed model in which type $k$ politicians are always reelected, regardless of policy choice. This removes the wedge between the incentives of the type $k$ politician and representative voter, and dynamic programming arguments can be used to deduce that the equilibrium strategy $\pi'_k$ of the type $k$ politician is optimal for the voter. We then map this equilibrium to a strategy profile $\sigma$ of the original model, maintaining policy choice strategies and modifying $\rho'$ so that for all $s$ and all $x$, the type $k$ politician is reelected with probability one if and only if $V^F_k(s, k, x) \geq V^C_k(s, x)$. This preserves equilibrium conditions of $\sigma'$, and we conclude that $\sigma$ is an equilibrium in which the type $k$ politician is a faithful delegate and is reelected with probability one. Clearly, Theorem 5.1 leaves open the possibility that there exist other equilibria such that the type $k$ politician fails to choose optimal policies for the representative voter; we address this issue in Section 6.

In our next result, we establish conditions under which patient voters approximately achieve their optimal payoff if the type $k$ politician is a faithful delegate, even if no other types choose desirable policies. Specifically, under the weak assumption that the probability of a type $k$ challenger has a positive lower bound, we establish that as the representative voter becomes patient, if the type $k$ politician is a faithful delegate in equilibrium, then the equilibrium payoff of the voter approaches the optimum value at every state satisfying a “strong recurrence” criterion; that is, at such states, equilibria are approximately optimal for the voter. The logic behind this asymptotic welfare result is that when the type $k$ politician is a faithful delegate, one possible voting strategy (which may not be optimal for the representative voter) is to simply retain any type $k$ office holder and reject all other types. Intuitively, as the representative voter becomes arbitrarily patient, the loss from this strategy becomes negligible. By Proposition 3.1, the equilibrium voting strategy solves the
representative voter’s optimal retention problem, and so it can do no worse than this simple rule; thus, the voter’s equilibrium payoffs must approach the optimal level.

Two assumptions are key for the success of the previous simple voting strategy at a given state. First, the state must recur with probability one under any profile of policy strategies by politicians. Second, the probability that a type $k$ politician is selected as challenger in that state must be uniformly bounded away from zero. To formalize the first assumption described above, we define a notion of ergodicity that is general but somewhat complex. A simpler formulation is possible if we assume that the set $S$ of states is finite and that the transition probability is positive, in the sense that for all $s$, all $x \in Y(s)$, and all $s'$, $p(s'|s,x) > 0$; then every state satisfies our ergodicity condition, and our policy responsiveness result applies to all states. More generally, for all $m$, let

$$\Psi^m(s) = \{(s_0, \ldots, s_m) \mid s_0 = s = s_j \text{ for some } j = 1, \ldots, m\}$$

be the set of paths $s = (s_0, \ldots, s_m)$ of states of length $m + 1$ such that $s = s_0$ recurs at least once. For all sequences $s = (s_0, \ldots, s_m)$ of states, let

$$\Xi^m(s) = \left\{ (x_0, \ldots, x_m) \mid x_0 \in Y(s_0) \text{ and } x_j \in Y(s_j) \text{ for all } j = 1, \ldots, m \right\}$$

be the set of feasible paths $x = (x_0, \ldots, x_m)$ of policies consistent with $s$. Then define

$$p^m(s) = \sum_{s \in \Psi^m(s)} \min_{x \in \Xi^m(s)} \left\{ \prod_{j=1}^m p(s_j|x_{j-1}, x_{j-1}) \mid x \in \Xi^m(s) \right\}$$

as the minimum probability that $s$ is realized within $m$ periods of $s$ being previously realized. Finally, we say $s$ is strongly recurrent if $\lim_{m \to \infty} p^m(s) = 1$.

Next, we establish that at every strongly recurrent state for which the probability of drawing a type $k$ challenger has a positive lower bound, the representative voter’s equilibrium payoff from an arbitrary politician type converges to the optimum as the type $k$ citizen becomes patient.

**Theorem 5.2.** Let $\delta_k = \delta \to 1$, and let $\{\sigma^\delta\}$ be corresponding Markov electoral equilibria such that the type $k$ politician is a faithful delegate. Then for all strongly recurrent states $s$ such that $\min_x q(k|s,x) > 0$ and for all $t$, we have

$$\lim_{\delta \to 1} \frac{V^F_k(s,t)}{V^\ast_k(s,k)} = 1,$$

where $V^F_k(s,t)$ denotes the expected discounted payoff to the representative voter from electing a free type $t$ politician in state $s$ given strategy profile $\sigma^\delta$, and $V^\ast_k(s,k)$ denotes the voter’s optimal value in state $s$ for the representative dynamic programming problem with discount factor $\delta$.

This asymptotic result formalizes, in an electoral accountability setting, the adage that “good things come to those who wait,” as the representative voter can take repeated draws
of challengers until a faithful delegate is realized. Less obvious is that this logic can be applied in a Markov electoral equilibrium, where the voter has no ability to commit in advance to a particular voting rule; to address this, we leverage Proposition 3.1.

Theorem 5.2 has further implications. First, since Theorem 5.1 implies that for each \( \delta \), there does in fact exist a Markov electoral equilibrium \( \sigma^\delta \) such that the type \( k \) politician is a faithful delegate, we conclude that as the type \( k \) citizens become patient, there is always a selection of equilibria that approach the optimal value of the voter at strongly recurrent states. This reasoning establishes a strong result on the possibility—though not the necessity—of policy responsiveness as the representative voter becomes patient: if we select the best equilibria for the representative voter, then policy responsiveness is achieved asymptotically.

Second, we have expressed Theorem 5.2 in terms of limits of ratios of values, but it has direct implications for optimality of policy choices themselves. Namely, we will show that for every strongly recurrent state \( s \), the equilibrium distribution of policy choices at that state converge to the optimal policies of the representative voter, as the voter becomes patient. To formulate this result precisely, take as given a Markov electoral equilibrium \( \sigma \), an initial state \( s \), and an initial politician type \( t \) who is free at \( s \). Equilibrium strategies induce a stochastic process over state-policy pairs \((s', x')\) in each period, i.e., a probability measure over infinite sequences \((s^m, x^m)_{m=1}^{\infty}\), where \( s^1 = s \). Let \( \mu_{s,t}^m \) denote the marginal distribution on state-policy pairs \((s^m, x^m)\) in period \( m \), where \( s^1 = s \). We aggregate these marginals across time by geometric discounting to define the probability measure

\[
\mu_{s,t}(p_r | s) = (1 - \delta_k) \sum_{m=1}^{\infty} \delta_k^{m-1} \mu_{s,t}^m,
\]

which depends on the initial state and politician type. Given a strongly recurrent state \( s \), our summary statistic for the equilibrium policies in state \( s \) is then the conditional \( \mu_{s,t}(\cdot | s) \) of the aggregate measure \( \mu_{s,t} \), which is well-defined since \( \mu_{s,t}(X \times \{s\}) > 0 \). Thus, for a Borel subset \( A \subseteq X \) of policies, \( \mu_{s,t}(A|s) \) measures the probability, given initial state \( s \) and politician type \( t \), that future policy choices, conditional on being in state \( s \), belong to \( A \). We use this measure to aggregate across periods in a way that reflects the time preferences of the representative voter.

To analyze optimality of policy choices as the voter becomes patient, let \( \Phi^{*,\delta}(s) \) denote the representative voter’s optimal policies in state \( s \) given discount factor \( \delta \), i.e,

\[
\Phi^{*,\delta}(s) = \arg \max_{x \in Y(s)} u_k(s, x) + \delta \sum_{s'} p(s' | s, x) V^{*,\delta}_k(s'),
\]

where \( V^{*,\delta}_k(s, k) \) is the voter’s optimal payoff in state \( s \) in the representative dynamic programming problem with discount factor \( \delta \). The next corollary establishes that for every strongly recurrent state \( s \) and every politician type \( t \), the equilibrium probability distribution on policy choices at \( s \) allocates probability mass close to the optimal policies \( \Phi^{*,\delta}(s) \) as the voter becomes patient. More precisely, we show that for all \( \epsilon > 0 \), the equilibrium probability, conditional on \( s \), of policy choices in the open ball of radius \( \epsilon \) around the set \( \Phi^{*,\delta}(s) \) of optimal policy choices, denoted \( B_{\epsilon}(\Phi^{*,\delta}(s)) \), converges to one as \( \delta_k \to 1 \).
Corollary 5.1. Let $\delta_k = \delta \to 1$, and let $\{\sigma^\delta\}$ be corresponding Markov electoral equilibria such that the type $k$ politician is a faithful delegate. Then for all strongly recurrent states $s$ such that $\min_x q(k|s,x) > 0$, all $t$, and all $\epsilon > 0$, we have
\[
\lim_{\delta \to 1} \mu^\delta_{s,t}(B_\epsilon(\Phi^*,\delta(s))|s) = 1,
\]
where $\mu^\delta_{s,t}$ is the aggregate probability measure on state-policy pairs in (5) given $\delta$.

Theorem 5.2 and Corollary 5.1 are asymptotic in nature, showing that optimality is approximated when the representative voter is sufficiently patient. Next, we provide conditions under which, for arbitrary discount factors, competition with faithful delegates of type $k$ incentivizes all other politician types to choose policies that are exactly optimal. To this end, we focus on equilibria exhibiting incumbency advantage, and, importantly, we add the assumption that the state transition is independent of policy choices, which simplifies the representative dynamic programming problem considerably: a policy rule $\phi^*$ is optimal for the voter if and only if in each state $s$, the policy $\phi^*(s)$ maximizes the voter’s stage utility $u_k(s,x)$ over $x \in Y(s)$. Our conclusion holds on a class of states that includes the strongly recurrent ones but substantially more: it covers any state $s$ that is non-trivial, in the sense that there exist a state $\tilde{s}$ and policy $x$ such that $p(s|\tilde{s},x) > 0$. Under these assumptions, if the type $k$ politician is a faithful delegate, then every politician type chooses a utility-maximizing policy for the representative voter in every non-trivial state, providing a sharp result on responsiveness of politicians to electoral incentives that does not rely on voter patience.

Theorem 5.3. Assume that $p(\cdot|s,x)$ is policy independent and $\min_x q(k|s,x) > 0$ for all $s$. Let $\sigma$ be a Markov electoral equilibrium exhibiting incumbency advantage such that the type $k$ politician is a faithful delegate. Then $V_k^F(s,t) = V_k^*(s)$ for all non-trivial $s$ and all $t$.

The idea of the proof is that in an equilibrium exhibiting incumbency advantage, every politician type chooses a policy that secures reelection, so that for all $s$, all $t$, and all $x$ in the support of $\pi_t(\cdot|s)$, we have $V_k^I(s,t,x) \geq V_k^C(s,x)$. Interpreting, roughly, $V_k^C(s,x)$ as the “average” payoff from a new office holder, it follows that after equilibrium policy choices, every incumbent is either average or above average; but since no type chooses a below average policy, this in fact implies that all types are average. In particular, the voter’s payoff from reelecting a type $t$ incumbent in state $s$ is equal to the payoff from reelecting the type $k$ politician. This does not directly imply that the voter’s payoff from the type $t$ politician achieves the optimal value following state $s$; rather, it implies that the optimal value is achieved following state $s$. If $s$ is non-trivial, however, then we can choose a state $\tilde{s}$ such that $p(s|\tilde{s}) > 0$ and apply the same logic in state $\tilde{s}$, with the conclusion that the voter’s optimal value is indeed achieved in $s$ (as well as in all states $s'$ that state $\tilde{s}$ can transition to). But this intuition is still incomplete. It holds if the representative voter’s ranking of politician types following state $s$ is independent of policies: otherwise, it is possible for an incumbent

\footnote{A state $s$ that fails to satisfy this condition can occur only if it is the initial state of the game, after which the state transitions away from it with probability one and visits only non-trivial states thereafter; thus, the scope of our result is broad.}

16
of type $t$ to be better than the average challenger following her choice of $x$ in $s$, even if, ex ante, all politician types $t' \neq t$ would yield strictly higher payoffs to the voter starting from $s$. To ensure this, we assume the state transition is independent of policy choices.

Theorem 5.3, unlike Theorem 5.2, delivers optimal choices by all politician types, and thus elections must not only induce the type $k$ politician to act as a faithful delegate, but they must also discipline other politician types to follow her lead. This is implicitly achieved by considering equilibria exhibiting incumbency advantage. Next, we note that in an equilibrium in which all politicians are able to choose policies that ensure reelection, it is enough if we add the willingness to do so, in the form of high office incentives, to the conditions of Theorem 5.3.

**Corollary 5.2.** Assume that for all $s$, $p(\cdot | s, x)$ is policy independent and $\min_x q(k|s, x) > 0$, and that for all $t$, $\delta_t b_t \geq 1$. Let $\sigma$ be a Markov electoral equilibrium satisfying incumbency privilege such that the type $k$ politician is a faithful delegate. Then $V_k^F(s, t) = V_k^*(s)$ for all non-trivial $s$ and all $t$.

The assumption that state transitions are independent of policy and the restriction to equilibria exhibiting incumbency advantage in Theorem 5.3 are strong, but the result is tight: when either assumption is dropped, there are examples of equilibria in which the other is satisfied, yet some politician types do not achieve the voter’s optimal value in all non-trivial states. Our first example provides a model with policy-dependent state transitions that admits a suboptimal equilibrium exhibiting incumbency advantage: it illustrates a situation in which an incumbent’s policy choice can lead to a state at which the incumbent is suboptimal for the representative voter, but at which a challenger is expected to fare even more poorly. This confronts the voter with a hold-up problem, forcing the voter to reelect the incumbent; thus, even if the type $k$ politician is a faithful delegate, other politician types can exploit the opportunity to manipulate the state in order to secure reelection.

**Example 1 (Political hold-up problem).** Let the state space be $S = \{s_{-1}, s_1\}$ and the type space be $T = \{-1, k, 1\}$, and let $t$ range over $\{-1, 1\}$. We interpret $s_{-1}$ as a progressive state and $s_1$ as a conservative state: these could represent, for example, the organization of government activities along welfare state or laissez-faire principles, respectively. Correspondingly, politicians of type $t$ are partisans (either progressive or conservative), while citizens of type $k$ are moderates. Feasible policies are independent of states and allow for both partisan and moderate policies: $Y = \{\hat{x}_{-1}, \hat{x}_k, \hat{x}_1\}$. State transitions are such that implementing partisan policies generates matching partisan states, while implementing moderate policies leads to state persistence: $p(s_t | s_1, \hat{x}_t) = p(s_{-1} | s_1, \hat{x}_{-1}) = p(s_1 | s_1, \hat{x}_1) = 1$. Challenger selection probabilities are independent of states and policies: a challenger is type $k$ with probability $0 < q < 1$ and is type $t$ with probability $\frac{1}{2} (1 - q)$. Voter $k$ prefers moderate policies in all states, but otherwise prefers partisan policies that match the state: $u_k(s_1, \hat{x}_k) > u_k(s_t, \hat{x}_t) > u_k(s_1, \hat{x}_{-1})$. In words, the voter has no inherent preference for a welfare state or a laissez-faire government, as both can be administered in moderate fashion. However, transitions from one form of government to the other require costly investments. The policy utilities of partisan types are independent of states: $u_t(\hat{x}_t) > u_t(\hat{x}_k) > u(\hat{x}_{-1})$. 

17
Furthermore, moderate policies are optimal for the voter in both states:

$$V^*_k(s_t) = \frac{u_k(s_t, \hat{x}_k)}{1 - \delta_k}.\]

We assume that $q$ and $\delta_k$ jointly satisfy

$$q \leq \frac{(1 - \delta_k)[u_k(s_t, \hat{x}_t) - u_k(s_t, \hat{x}_{-t})]}{2u_k(s_t, \hat{x}_k) - (1 + \delta_k)u_k(s_t, \hat{x}_t) + (1 - \delta_k)u_k(s_t, \hat{x}_{-t})} \in (0, 1),$$

so that the representative voter’s incentives to dismiss incumbents in the hope of drawing a challenger of type $k$ are not too strong.

We claim that there exists a Markov electoral equilibrium in which all politicians choose their stage-ideal policy (and hence the type $k$ politician is a faithful delegate) and are always reelected. Note that in this equilibrium,

$$V^F_k(s_t, t) = \frac{u_k(s_t, \hat{x}_t)}{1 - \delta_k} < V^*_k(s_t),$$

$$V^F_k(s_t, -t) = u_k(s_t, \hat{x}_{-t}) + \delta_k\frac{u_k(s_{-t}, \hat{x}_{-t})}{1 - \delta_k} < V^*_k(s_t),$$

so that in all states, the voter strictly prefers moderate politicians but reelects all partisan politicians. The reason for this is that by choosing progressive policies in any state, a progressive politician makes opting for the challenger too risky for the voter: the costs of progressive policies, and the corresponding transition to the progressive state, are sunk, and the probability that the challenger is conservative, and therefore unsuited to governing in the new state, is high. For the same reason, moreover, the voter would reelect a conservative politician if she were to engineer a transition to the conservative state by implementing conservative policies in the progressive state.

Our second example provides a model with policy-independent state transitions that admits a suboptimal equilibrium that does not satisfy incumbency advantage. Incumbency advantage, intuitively, implies that in equilibrium, all politician types are both willing and able to compete for power against the challenger pool; otherwise, if some politician types cannot retain office following any policy, then she has no incentive to choose the representative voter’s ideal policies.

**Example 2 (Uncompetitive politicians).** Our purpose can be met in a simple model of rent-seeking with a single state (i.e., $S = \{s\}$). Suppose that citizens are either type $k$ or are the “bad” type $t$, and that office-holding politicians can either work or shirk: $Y = \{\overline{x}, \overline{z}\}$, where $\overline{x}$ represents working and $\overline{z}$ represents shirking. Challenger selection probabilities are independent of policies: a challenger is type $k$ with probability $0 < q < 1$. Politicians of type $k$ fully commit to policies, but bad politicians have limited commitment power: $\gamma_k = 1$ and $0 < \gamma_t < q$. Citizens of type $k$ prefer working to shirking and bad-type citizens have the opposite preferences: $u_k(\overline{x}) > u_k(\overline{z})$ and $u_t(\overline{x}) < u_t(\overline{z})$. Finally, note that

$$V^*_k(s) = \frac{u_k(\overline{x})}{1 - \delta_k}.\]
We claim that there is a Markov electoral equilibrium that does not exhibit incumbency advantage in which the type $k$ is a faithful delegate (choosing to work in both states) but bad politicians shirk. In this equilibrium, type $k$ incumbents are reelected if they work, while a challenger is elected if the type $k$ politician shirks or if the incumbent is bad. Note that in this equilibrium, the voter’s expected discounted utility from a bad politician fails to achieve his optimal value:

$$V^F_k(s, t) < u_k(x) + \frac{\delta u_k(x)}{1 - \delta} < V^a_k(s).$$

This equilibrium does not exhibit incumbency advantage, because the voter strictly prefers to elect the challenger rather than reelect a bad incumbent following any policy choice. Bad incumbents cannot compete against challengers even by working, because $\gamma_k < q$ implies that a bad politician is less likely to continue working if reelected than the challenger is likely to be of type $k$. This is true even if $\delta b_k$ is large, so that bad politicians would in principle be willing to work in exchange for reelection.

6 The Possibility of Delegation Failure

Our results in the preceding section establish the existence of equilibria in which elections attain (or approximate) the voter’s optimum from the representative dynamic programming problem. Thus, as was the case in Section 4 for the implementation of optimal policy rules, the problem of accountability reduces to the possibility of multiple equilibria. By Theorems 5.2 and 5.3, the attainment of the voter’s optimal payoff, in an asymptotic or exact sense, hinges on the existence of a faithful delegate. In this section, we investigate conditions under which type $k$ politicians are guaranteed to be faithful delegates across a wide class of equilibria, fulfilling the key requirement of Theorems 5.2 and 5.3, and ensuring the effectiveness of elections.

We begin with an example that offers an important cautionary note: if policies influence the evolution of the state, and if office incentives are high, then there can exist Markov electoral equilibria in which the policy choices of the type $k$ politician are suboptimal, due to equilibrium incentives to manipulate future states at the cost of undesirable policies in the current period. Thus, although office incentives can have the positive effect of disciplining politicians to follow the lead of a faithful delegate, as in Proposition 4.1 and Corollary 5.2, they can have the countervailing, negative effect of introducing a wedge between the representative voter and the type $k$ politician. The example is simple and only has three states, where we interpret one state as perpetual war and the other two as peaceful states on the brink of war. In the latter states, the representative voter prefers peace to be maintained, but the type $k$ politician moves to war in order to secure reelection; remarkably, these reelection incentives drive a wedge between the preferences of the representative voter and the type $k$ politician, who both agree that peace is the preferred state. Underlying this is the assumption that office incentives are large—it is sufficient that $b_k$ is large or $\delta_k$ is close to one—along with the voter’s expectation that if the type $k$ politician is reelected in a peaceful state, then rather than serve as a faithful delegate, she will move to war in
the following period. Thus, the political ambitions of the type $k$ politician can lead to the pathological choice of suboptimal policies in certain states.

**Example 3 (Curse of ambition).** Assume the state space is $S = \{s_w, s_1, s_{-1}\}$, where $s_w$ is a state of perpetual war, and $s_j$ is a state on the brink of war, where $j$ ranges over $\{-1, 1\}$. There are two citizen types, $T = \{k, \ell\}$, and the challenger is equally likely to be of either type. The set of feasible policies in state $s_j$ is $Y(s_j) = \{x_p, x_w\}$, where $x_p$ is interpreted as maintaining the peace, and $x_w$ is moving to war. In state $s_w$, there is a single feasible policy, $\overline{x}$, so that policy plays no role. The state transition is such that perpetual war $s_w$ is an absorbing state, and at the brink of war, in state $s_j$, the state transition depends on policy: the choice of $x_p$ moves the state to $s_{-j}$ with probability one, so that peace is maintained; and the choice of $x_w$ leads to $s_w$ with probability one. Formally, $p(s_w|s_w, \overline{x}) = p(s_{-j}|s_j, x_p) = p(s_w|s_j, x_w) = 1$. The payoffs to the type $k$ citizen are such that peace is preferred to war: $u_k(s_j, x_p) = u_k(s_j, x_w) > u_k(s_w, \overline{x})$, so that the optimal policy rule for the representative voter is to choose $x_p$ in both states $s_1$ and $s_{-1}$, thereby maintaining peace. For our purposes, we can assume that the type $\ell$ politicians have the same preferences; the precise form of $u_\ell$ is immaterial, as long as the type $\ell$ politician prefers peace to war. Assume, however, that the type $k$ politician has office incentives that are strong enough to offset the cost of war relative to peace:

$$
\delta_k b_k > (2 - \delta_k) \left( u_k(s_j, x_p) - (1 - \delta_k) u(s_j, x_w) - \delta_k u_k(s_w, \overline{x}) \right) > 0. \tag{6}
$$

We claim that there is a Markov electoral equilibrium such that the type $k$ politician always chooses war, while the type $\ell$ politician chooses peace, i.e., $\pi_k(x_w|s_j) = \pi_\ell(x_p|s_j) = 1$; and the type $k$ politician is never reelected at the brink of war but is reelected in the war state, while the type $\ell$ politician is always reelected, i.e., $\rho(s_j, k, x) = 0$ and $\rho(s_j, \ell, x) = \rho(s_w, k, x) = \rho(s_w, \ell, \overline{x}) = 1$. To see that these strategies form an equilibrium, the key observation is that the type $k$ politician’s choice in state $s_j$ is optimal: choosing $x_w$ leads to war and a discounted stream of future office benefits, while choosing $x_p$ maintains peace for one period, after which the politician is replaced, losing future office benefits.

Writing the politician’s payoff from $x_w$ in $s_j$ as

$$
W = u_k(s_j, x_w) + b_k + \frac{\delta_k}{1 - \delta_k} \left( u_k(s_w, \overline{x}) + b_k \right),
$$

the payoff from $x_p$ in $s_j$ is then

$$
P = u_k(s_j, x_p) + b_k + \delta_k \left( \frac{u_k(s_j, x_p)}{2(1 - \delta_k)} + \frac{W}{2} \right).
$$

Optimality of $x_w$ requires that $W \geq P$, which is implied by (6). Clearly, the type $\ell$ politician’s choice is optimal, since they choose their ideal action in state $s_j$ and are reelected. Moreover, it is optimal for the representative voter to replace a type $k$ politician in state $s_j$, even if the politician chooses $x_p$, and it is optimal to reelect the type $k$ politician in the war state. Indeed, in state $s_j$, if the type $k$ politician chooses $x_p$ and is reelected, then in the next period, the state transitions to $s_{-j}$, and the politician chooses $x_w$, moving the state to perpetual war; and if the politician is removed, then with probability one half, the challenger
is type $\ell$, who maintains peace and is reelected thereafter. As well, in state $s_w$, the voter is indifferent between reelecting a type $k$ incumbent and a randomly drawn challenger.

Although Example 3 is simple, the equilibrium we select is robust: it obtains regardless of commitment power, it exhibits incumbency advantage (notably, this is the case even for the unfaithful representative politician), and equilibrium conditions hold strictly, so the strategies specified survive as best responses if the model is perturbed in any number of ways. The example removes commitment power by assuming that if peace is maintained in one peaceful state, then the state transitions to the other peaceful state with probability one; but we can assume that at the brink of war, the state remains in place with positive probability, and that politicians have small, positive commitment power. Finally, to compare the example to the optimality result of Proposition 4.2, note that the equilibrium constructed does not implement a policy rule, because the type $k$ politician chooses $x_w$ at the brink of war in state $s_j$, whereas the type $\ell$ politician chooses the peaceful policy $x_p$, demonstrating that that result hinges on the restriction to symmetric policy strategies.

In Example 3, the type $k$ politician fails to act as a faithful delegate, but the type $\ell$ politician always chooses peace, and thus chooses the optimal policy for the voter. It is a simple matter to adjust the example, by introducing a state that is antecedent to the brink of war, in which neither politician type acts as a faithful delegate. Specifically, we add a peaceful state, $s_p$, that transitions to the brink of war, $s_1$, with probability one, after which play follows Example 3. Because the peaceful state is transient, each politician type simply chooses their ideal policy, so that the type $k$ politician chooses the policy desired by the voter, while the type $\ell$ politician does not. Interestingly, because the type $k$ politician will incite war in state $s_1$, the voter will never reelect a type $k$ incumbent in state $s_p$, and thus the example illustrates the possibility that the type $k$ politician’s future office incentives undermine her present electoral fortunes in the peaceful state—so that the politician is in fact cursed by her own ambition.

**Example 4 (Curse of ambition, continued).** We augment Example 3 by adding a peaceful state, $s_p$, which we can think of as a prelude to the brink of war. Assume that there are two feasible policies, $\hat{x}_k$ and $\hat{x}_\ell$, in state $s_p$, and that the state automatically moves from $s_p$ to $s_1$, i.e., $p(s_1|s_p, \pi) = 1$. Specify utilities as in Example 3, with the addition that the ideal policy of the type $k$ politician in $s_p$ is $\hat{x}_k$, and the ideal policy of the type $\ell$ politician in $s_p$ is $\hat{x}_\ell$, i.e., $u_k(s_p, \hat{x}_k) > u_k(s_p, \hat{x}_\ell)$ and $u_\ell(s_p, \hat{x}_k) < u_\ell(s_p, \hat{x}_\ell)$. We define equilibrium strategies as before, but with the additional specifications that the incumbent chooses their ideal policy in state $s_p$, and that the representative voter replaces the type $k$ politician and retains the type $\ell$ citizen in state $s_p$. Optimality of this choice is straightforward: if a type $k$ politician is reelected in state $s_p$, then once at the brink of war, the politician will move to the war state $s_w$; but if the politician is replaced by a challenger, there is a positive probability that the replacement will be type $\ell$, in which case peace is maintained thereafter.

We end with two results providing conditions under which the distortions highlighted in Examples 3 and 4 are precluded, so that in equilibrium, the type $k$ politician chooses policies that are optimal from the point of view of the representative voter. We first show that if type $k$ politicians are policy motivated, then the incentives of the politician and the
voter are aligned, and the politician acts as a faithful delegate.

**Theorem 6.1.** Assume $\delta_kb_k = 0$. Let $\sigma$ be a Markov electoral equilibrium. Then the type $k$ politician is a faithful delegate.

The proof relies on dynamic programming arguments. If $\delta_kb_k > 0$, then there is a wedge between the preferences of the representative voter and the type $k$ politician: even though they have the same policy preferences, the politician is willing to sacrifice some policy utility for some increased chance of reelection. When either $\delta_k = 0$ or $b_k = 0$, this wedge disappears, and we show that the conditions for Markov electoral equilibrium imply that the strategies $(\pi_k, \rho)$ of the type $k$ politician and representative voter solve the best response problem for a unitary, type $k$ agent. This implies that the policy strategy $\pi_k$ must be optimal for the representative voter, and in particular, the type $k$ politician is a faithful delegate.

Theorem 6.1 holds independently of the power of commitment, but it assumes away office incentives. Next, we establish that when commitment power is positive, and states and challengers evolve independently of policy choices, the type $k$ politician is again a faithful delegate—now, regardless of office incentives. By assuming policy independence of the state and challenger transition probabilities, we remove the possibility of state manipulation, and the assumption of positive commitment power again aligns the preferences of the type $k$ politician and voter. Together, these results imply that the political inefficiency illustrated in Example 3 can only arise when the type $k$ politician has both positive office incentives and the ability to influence the evolution of the state through policy choices; thus, those features of the examples are essential.

**Theorem 6.2.** Assume that for all $s$, $p(\cdot|s, x)$ and $q(\cdot|s, x)$ are policy independent, and that commitment power is positive, i.e., for all $s$, we have $\gamma_k p(s|s) > 0$. Let $\sigma$ be a Markov electoral equilibrium. Then the type $k$ politician is a faithful delegate.

Theorems 5.1 and 5.2 establish the existence of electoral equilibria that are asymptotically optimal for the representative voter. To rule out the possibility of asymptotically suboptimal equilibria, it suffices to provide conditions under which the type $k$ politician is a faithful delegate in every equilibrium, which is the topic of Theorem 6.1. We present the joint implications of these observations next.

**Corollary 6.1.** Assume that the type $k$ politician is policy motivated, i.e., $b_k = 0$. Let $\delta_k = \delta \to 1$, and let $\{\sigma^s\}$ be corresponding Markov electoral equilibria. Then for all strongly recurrent states $s$ such that $\min_x q(k|s, x) > 0$ and for all $t$, equation (4) holds.

In relation to the literature on electoral modelling, Corollary 6.1 provides a partial analogue to the median convergence result of policy-motivated candidates in the Downsian model (cf. Calvert (1985)), where the unique Nash equilibrium is that both candidates locate at the median voter’s ideal point. An obvious difference between results is that Corollary 6.1 is framed in a dynamic model in which convergence is asymptotic, whereas the median voter result is static and exact. In addition, the median voter result requires commitment, whereas ours does not. Finally, the median convergence result in the Downsian model is robust to mixed motivations, i.e., $b_k > 0$, whereas Example 3 demonstrates that in a dynamic,
citizen-candidate environment, office incentives can lead to political inefficiency. Our result is also analogous to Theorem 8 of Banks and Duggan (2008) for the single-state model, which allows $b_k = 0$ and shows that as citizens become patient, the set of policy choices leading to reelection collapses to the median voter’s ideal point, and thus long run policies eventually approach the median.\footnote{Related results are obtained by Forand (2014), in a model with commitment, and by Van Weelden (2013), in a single-state model without commitment but with complete information about politicians’ types.} Our result extends this finding to the general model, with arbitrary states, policies, and preferences, with or without commitment power.

From Theorem 5.3, we know when transitions are policy independent, the presence of faithful delegates disciplines all non-representative politicians in all Markov electoral equilibria exhibiting incumbency advantage. Combining Corollary 5.2 and Theorem 6.2, we have a last corollary excluding suboptimal equilibria in dynamic elections: assuming policy-independent transitions and strong office incentives, if the type $k$ politician has positive commitment power, then in every Markov electoral equilibrium exhibiting incumbency privilege, every politician type chooses an optimal policy for the representative voter in every state.

**Corollary 6.2.** Assume that for all $s$, $p(\cdot|s,x)$ and $q(\cdot|s,x)$ are policy independent and $\min_k q(k|s,x) > 0$, that for all $t$, $\delta_t b_t \geq 1$, and that commitment power is positive, i.e., for all $s$, we have $\gamma_k p(s|s) > 0$. Let $\sigma$ be a Markov electoral equilibrium exhibiting incumbency privilege. Then $V_k^F(s,t) = V_k^s(s)$ for all non-trivial $s$ and all $t$.

Corollary 6.2 provides a partial analogue to the classical Downsian median voter theorem with office-motivated candidates, in which the unique Nash equilibrium dictates that the candidates locate at the median voter’s ideal point. Clearly, our result is derived in a dynamic model, and we need only a positive amount of ex post commitment, whereas the Downsian model is static and assumes full, ex ante commitment. In a dynamic model with a single state, a closer analogue of the latter results is Theorem 7 of Banks and Duggan (2008), which shows that when office incentives are large, the unique equilibrium is such that every politician type chooses the ideal point of the median voter. In the model of that paper, voter beliefs proxy for commitment power, and when office incentives are large all equilibria exhibit incumbency advantage; thus Corollary 6.2 extends their theorem to the general model with arbitrary states, policies, and preferences.

## 7 Conclusion

The results of this paper inform us of the possibilities for—and limits of—electoral accountability in delivering responsive policy choices by politicians in a general dynamic electoral framework. In sum, our results support the view that elections can be an effective mechanism for holding politicians accountable and delivering responsive policy choices. When office incentives are strong, there always exist equilibria that implement optimal policy rules in a strong sense, and commitment power sharpens this result: when commitment power is positive, there are no suboptimal policy rules that can be implemented by an equilibrium in which politicians have the option of reelection. More generally, there always exist equilibria in which the type $k$ politician acts as a faithful delegate. The payoffs of the voter
are asymptotically optimal along any sequence of such equilibria as patience increases; and for an arbitrary rate of time discounting, the voter’s payoff attains the optimum in any such equilibrium, if the state and challenger transitions are policy independent, politicians have the option of re-election, and politicians are highly office motivated. These positive results must be qualified, however. In general, there is the possibility of additional, suboptimal equilibria. The possibility of such equilibria is obviated when politicians are purely office motivated, in which case patience leads to optimality in the limit, regardless of the selection of equilibria. In addition, we show that suboptimal equilibria can be excluded when the future environment is independent of current policy, and politicians have positive commitment power. In the absence of this structure, examples illustrate possible political failures—due to the political hold-up problem or to the curse of ambition—and thereby the limitations of the electoral mechanism.

A Proofs of Results

Proof of Proposition 3.1. We first describe the representative optimal retention problem in more detail. Given any state \( s \), incumbent type \( t \), and policy choice \( x \), the voter chooses between the incumbent or a challenger, and in either case, a new state \( s' \) is realized from \( p( \cdot | s, x) \). If the incumbent is chosen, then a policy is realized from \( \pi_t( \cdot | s') \), and the voter receives utility \( u_k(s', x') \); and if the challenger is chosen, then the politician’s type \( t' \) is realized from \( q( \cdot | s, x) \), and a policy \( x' \) is realized from \( \pi_{t'}(\cdot | s') \), giving the voter a utility of \( u_k(s', x') \). The voter again decides to retain the current incumbent or replace the politician with another challenger, and so on. For all \( s, t \) and \( x \in Y(s) \), the Bellman equation for this dynamic programming problem is

\[
\nabla_k^I(s, t, x) = \max \left\{ p(s|x) \left[ \gamma_t(u_k(s, x) + \delta_k \nabla_k^I(s, t, x)) + (1 - \gamma_t)^k \nabla_k^F(s, t) \right], \nabla_k^C(s, x) \right\},
\]

where \( \nabla_k^C \) and \( \nabla_k^F \) are defined as in (2) and (3), with \( \nabla_k^I \) substituted for \( V_k^I \).

Returning to the proof of Proposition 3.1, for each \( s, t \), and \( x \in Y(s) \), define

\[
\tilde{V}_k^I(s, t, x) = p(s, t, x) V_k^I(s, t, x) + (1 - p(s, t, x)) V_k^C(s, x).
\]

It suffices to show that \( \tilde{V}_k^I \) solves the following functional equation:

\[
\tilde{V}_k^I(s, t, x) = \max \left\{ p(s|x) \left[ \gamma_t(u_k(s, x) + \delta_k \tilde{V}_k^I(s, t, x)) + (1 - \gamma_t)^k \tilde{V}_k^F(s, t) \right], \tilde{V}_k^C(s, x) \right\},
\]

where \( \tilde{V}_k^C \) and \( \tilde{V}_k^F \) are defined as in (2) and (3), with \( \tilde{V}_k^I \) substituted for \( V_k^I \). We refer to the modified equations as (2’) and (3’).

First, we claim that \( \tilde{V}_k^C(s, x) = V_k^C(s, x) \) and \( \tilde{V}_k^F(s, t) = V_k^F(s, t) \) for all \( s, t \) and \( x \). To see this, note that \( p(s, t, x) \tilde{V}_k^I(s, t, x) = p(s, t, x) V_k^I(s, t, x) \). This follows because
Therefore, after substitution we have

\[ \hat{V}_k^I(s, t, x) \neq V_k^I(s, t, x) \] if and only if \( V_k^I(s, t, x) < V_k^C(s, t) \), in which case \( \rho(s, t, x) = 0 \). Therefore, equations (2') and (3') are in fact identical to (2) and (3), yielding the claim.

Second, to show that \( \hat{V}_k^I \) solves the desired functional equation, consider any \( s, t, x \in Y(s) \). Suppose that \( V_k^I(s, t, x) \geq V_k^C(s, x) \), so that \( \hat{V}_k^I(s, t, x) = V_k^I(s, t, x) \). Using our claim, we have

\[
V_k^I(s, t, x) = p(s|x) \left[ \gamma_t u_k(s, x) + \delta_k \hat{V}_k^I(s, t, x) + (1 - \gamma_t) \hat{V}_k^F(s, t) \right] + \sum_{s' \neq s} p(s'|s) \hat{V}_k^F(s', t),
\]

and since \( V_k^I(s, t, x) = \max\{V_k^I(s, t, x), V_k^C(s, x)\} \), we are done. Now suppose that \( V_k^I(s, t, x) < V_k^C(s, x) \), so that \( \rho(s, t, x) \neq 0 \) and, using our claim, \( \hat{V}_k^I(s, t, x) = V_k^C(s, x) = \hat{V}_k^C(s, x) \). Suppose in order to deduce a contradiction that

\[
\hat{V}_k^I(s, t, x) < \max \left\{ p(s|x) \left[ \gamma_t u_k(s, x) + \delta_k \hat{V}_k^I(s, t, x) + (1 - \gamma_t) \hat{V}_k^F(s, t) \right] + \sum_{s' \neq s} p(s'|s) \hat{V}_k^F(s', t), \hat{V}_k^C(s, x) \right\}.
\]

Therefore, after substitution we have

\[
V_k^C(s, x) < p(s|x) \left[ \gamma_t u_k(s, x) + \delta_k V_k^C(s, x) + (1 - \gamma_t) V_k^F(s, t) \right] + \sum_{s' \neq s} p(s'|s) V_k^F(s', t),
\]

or equivalently,

\[
V_k^C(s, x) < \frac{p_t(s|x)[\gamma_t u_k(s, x) + (1 - \gamma_t) V_k^F(s, t)] + \sum_{s' \neq s} p(s'|s) V_k^F(s', t)}{1 - \gamma_t \delta_k p_t(s|x)} = V_k^I(s, t, x),
\]
a contradiction.

\[ \square \]

**Proof of Proposition 4.1.** Let \( \phi \) be an optimal policy rule. Define policy strategies such that for all \( s \) and all \( t \), \( \pi_t(\{\phi(s)\}|s) = 1 \), i.e., each type of politician chooses \( \phi(s) \) in each state \( s \), and define the voting strategy such that for all \( s, t, x \in Y(s) \), we have \( \rho(s, t, x) = 1 \) if \( x = \phi(s) \), and \( \rho(s, t, x) = 0 \) otherwise. Obviously, \( \sigma = (\pi, \rho) \) is a Markov strategy profile that exhibits incumbency advantage. To show that \( \rho \) satisfies the conditions for Markov electoral equilibrium, first note that \( V_k^F(s, t) = V_k^*(s) \) for all types \( t \). Furthermore, for every policy \( x \in Y(s) \), we have

\[
V_k^I(s, t, x) - V_k^C(s, x) = V_k^I(s, t, x) - \sum_{s'} p(s'|s) V_k^*(s')
\]

\[
= \gamma_t p(s|x) \left[ u_k(s, x) + \delta_k V_k^I(s, t, x) - V_k^*(s) \right] \leq 0,
\]

25
with equality if $x$ is optimal for the voter. Thus, after the choice of $\phi(s)$ in state $s$, the voter is indifferent between reelecting an incumbent following the or replacing her with a challenger; and for all other policy choices $x \neq \phi(s)$, the voter weakly prefers the challenger. We conclude that $\rho$ satisfies the optimality condition required by Markov electoral equilibrium.

To verify that policy strategies $\pi_t$ are optimal for all politicians in all states, let $V_t^\phi(s)$ denote the expected policy utility to the type $t$ office holder from following the rule $\phi$ in state $s$ and thereafter:

$$ V_t^\phi(s) = u_t(s, \phi(s)) + \delta_t [p(s|s, \phi(s))V_t^\phi(s) + \sum_{s' \neq s} p(s'|s, \phi(s))V_t^\phi(s')]. $$

Then the total expected payoff from following the rule $\phi$ in state $s$, and holding office in perpetuity, is $V_t^\phi(s) + \frac{b_t}{1-\delta_t}$.

The expected payoff from deviating to $x \neq \phi(s)$ in state $s$, and being replaced by a challenger, is no more than

$$ \bar{u} + \delta_t [p(s|s, \phi(s))V_t^\phi(s) + (1 - p(s|s, \phi(s)))\bar{u}], $$

where we use the fact that the challenger continues to implement the rule $\phi$. Then it is optimal for the office holder to follow $\phi$ if

$$ V_t^\phi(s) + \frac{\delta_t b_t}{1-\delta_t} \geq \bar{u} + \delta_t [p(s|s, \phi(s))V_t^\phi(s) + (1 - p(s|s, \phi(s)))\bar{u}], $$

or equivalently, if

$$ (1 - \delta_t p(s|s, \phi(s)))V_t^\phi(s) + \frac{\delta_t b_t}{1-\delta_t} \geq \bar{u} + \delta_t (1 - p(s|s, \phi(s)))\bar{u}. $$

Using $u = 0$ and $\bar{u} = 1$, a sufficient condition for this is

$$ \frac{\delta_t b_t}{1-\delta_t} \geq 1 + \delta_t \left(1 - \min_{x \in Y(s)} p(s|s, x)\right), $$

which is equivalent to $\delta_t \beta_t(s) \geq 1$.

Proof of Proposition 4.2. Let $\sigma = (\pi, \rho)$ be a Markov electoral equilibrium exhibiting incumbency privilege that implements a policy rule $\phi$, and suppose toward a contradiction that $\phi$ is not optimal for the voter. Then there is some state $s$ such that $\phi(s)$ is suboptimal given future policy choices, i.e., there exist $x \in Y(s)$ and $\epsilon > 0$ such that

$$ u_k(s, x) + \delta_k \sum_{s'} p(s'|s, x)V_k^\phi(s') > V_k^\phi(s) + \epsilon. \quad (7) $$

Consider any history of the game such that the state in the current period is $s$. The type $k$ politician’s equilibrium payoff in this state is no greater than

$$ V_k^\phi(s) + \frac{b_k}{1-\delta_k}. $$
Let $\tilde{\pi}_k^m$ be a non-stationary deviation for the type $k$ politician defined as follows. First, we say a state transition is *liberating* if after the transition, the politician is free; that is, the transition from $s'$ to $s''$ is liberating if either $s' = s''$ and the politician is uncommitted to her policy at $s'$ (which occurs with probability $1 - \gamma_k$) or $s' \neq s''$. Now, we specify $\tilde{\pi}_k^m$ as follows: (1) In the current period, the politician chooses $x$. (2) After each of the next $m$ liberating transitions, if the state transitions to $s' = s$, then $\tilde{\pi}_k^m(s'|s)$ places probability one on $\tilde{x}_s = x$; and if the state transitions to $s' \neq s$, then, using the assumption of positive commitment power, let $\tilde{\pi}_k^m(s'|s')$ place probability one on a policy $\tilde{x}_{s'}$ that secures reelection, i.e., $\rho(s', k, \tilde{x}_{s'}) = 1$. (3) After $m + 1$ liberating transitions, the politician chooses policy according to the original policy strategy, i.e., in state $s'$, she chooses $\phi(s')$. Thus, $\tilde{\pi}_k^m$ modifies $\pi_k$ for at least $m + 1$ periods, after which the politician returns to $\pi_k$.

We claim that after any deviation to policy $x$ in state $s$, the type $k$ politician is reelected with probability one. Note that

$$V_k^I(s, k, x) - V_k^C(s, k) = \gamma_k p(s|s, x)[(u_k(s, x) + \delta_k \sum_{s'} p(s'|s, x)V_k^\phi(s')) - V_k^\phi(s)] > 0,$$

where the strict inequality uses (7) and the assumption of positive commitment power, $\gamma_k p(s|s, x) > 0$. Thus, $\rho(s, k, x) = 1$, as claimed. By construction, the policy choices $\tilde{x}_{s'}$ after the next $m$ state transitions also secure reelection for the politician.

Given any state $s'$, let $\tilde{V}_k^{m,m}(s')$ denote the expected discounted policy payoff (not including office benefits) of the type $k$ politician calculated after the choice of $\tilde{x}_{s'}$ in state $s'$, assuming that a total of $n$ liberating transitions have occurred prior to that choice, and that the politician uses $\tilde{\pi}_k^m$ thereafter. When $n = m$, the state has transitioned $m$ times, and so following the next liberating transition, the politician implements $\phi$. Thus, for all $s'$, we have

$$\tilde{V}_k^{m,m}(s') = p(s'|s', \tilde{x}_{s'})[\gamma_k (u_k(s', \tilde{x}_{s'}) + \delta_k \tilde{V}_k^{m,m}(s')) + (1 - \gamma_k)V_k^\phi(s')] + \sum_{s'' \neq s'} p(s''|s', \tilde{x}_{s'})V_k^\phi(s'').$$

Note that for the equilibrium $\sigma$, the expected discounted utility from a free, type $k$ politician in any state $s'$ is $V_k^F(s', k) = V_k^\phi(s')$, and thus $\tilde{V}_k^{m,m}$ solves the functional equation (1) with $t = t' = k$ and $x = \tilde{x}_{s'}$, which implies that $\tilde{V}_k^{m,m}(s') = V_k^I(s', k, \tilde{x}_{s'})$. Because $\tilde{x}_{s'}$ belongs to the reelection set, we have

$$0 \leq V_k^I(s', k, \tilde{x}_{s'}) - V_k^C(s', \tilde{x}_{s'}) = \gamma_k p(s'|s', \tilde{x}_{s'})[u_k(s', \tilde{x}_{s'}) + \delta_k \tilde{V}_k^{m,m}(s') - V_k^\phi(s')],$$

and by the assumption of positive commitment power, this implies that the type $k$ politician’s expected discounted policy payoff from following $\tilde{\pi}_k^m$ in state $s'$ weakly exceeds the policy payoff from following $\pi_k$: for all $s'$,

$$u_k(s', \tilde{x}_{s'}) + \delta_k \tilde{V}_k^{m,m}(s') \geq V_k^\phi(s').$$

Now, for an induction proof, assume that for some $n \in \{1, 2, \ldots, m\}$, we have: for all $s'$,

$$u_k(s', \tilde{x}_{s'}) + \delta_k \tilde{V}_k^{m,m}(s') \geq V_k^\phi(s'). \tag{8}$$
Note that for all \( s' \), we have
\[
\tilde{V}_{k}^{m,n-1}(s') = p(s'| s', \tilde{x}_{s'})[\gamma_k(u_k(s', \tilde{x}_{s'}) + \delta_k \tilde{V}_{k}^{m,n-1}(s')) + (1 - \gamma_k)\tilde{V}_{k}^{m,n}(s')] \\
+ \sum_{s'' \neq s'} p(s''| s', \tilde{x}_{s'})\tilde{V}_{k}^{m,n}(s'') \\
\geq p(s'| s', \tilde{x}_{s'})[\gamma_k(u_k(s', \tilde{x}_{s'}) + \delta_k \tilde{V}_{k}^{m,n-1}(s')) + (1 - \gamma_k)V_{k}^{\phi}(s')] \\
+ \sum_{s'' \neq s'} p(s''| s', \tilde{x}_{s'})V_{k}^{\phi}(s''),
\]
where the inequality follows from the induction hypothesis. This implies (by manipulating terms so that \( \tilde{V}_{k}^{m,n-1}(s') \) appears only on the left-hand side of the inequality) that \( \tilde{V}_{k}^{m,n-1}(s') \geq V_{k}^{I}(s', k, \tilde{x}_{s'}) \). Again, because \( \tilde{x}_{s'} \) belongs to the reelection set, we have
\[
0 \leq V_{k}^{I}(s', k, \tilde{x}_{s'}) - V_{k}^{C}(s', \tilde{x}_{s'}) \leq \gamma_k p(s'| s', \tilde{x}_{s'})[u_k(s', \tilde{x}_{s'}) + \delta_k \tilde{V}_{k}^{m,n-1}(s') - V_{k}^{\phi}(s')],
\]
and by the assumption of positive commitment power, this implies that (8) holds for \( n - 1 \), and by induction, it holds for \( n = 0 \).

A similar argument shows that the type \( k \) politician’s expected discounted policy payoff, calculated the period after the initial deviation to \( x \) in state \( s \), weakly exceeds the payoff from implementing \( \phi \). The former is
\[
\tilde{V}_{k}^{m,0}(s) = p(s| s, x)[\gamma_k(u_k(x, s) + \delta_k \tilde{V}_{k}^{m,0}(s)) + (1 - \gamma_k)(u_k(s, x_{s}) + \delta_k \tilde{V}_{k}^{m,1}(s))] \\
+ \sum_{s' \neq s} p(s'| s, x)(u_k(s', x_{s'}) + \delta_k \tilde{V}_{k}^{m,1}(s')) \\
\geq p(s| s, x)[\gamma_k V_{k}^{\phi}(s) + (1 - \gamma_k)V_{k}^{\phi}(s)] + \sum_{s' \neq s} p(s'| s, x)V_{k}^{\phi}(s'),
\]
where the inequality uses the above induction argument, and this implies
\[
\tilde{V}_{k}^{m,0}(s) \geq \sum_{s'} p(s'| s, x)V_{k}^{\phi}(s'). \tag{9}
\]
Combining (7) and (9), we conclude that the type \( k \) politician’s expected discounted policy payoff from the deviation \( \tilde{\pi}_{k} \), calculated at the initial deviation to \( x \) in state \( s \), satisfies
\[
\tilde{V}_{k}^{m,0}(s) \geq u_k(s, x) + \delta_k \tilde{V}_{k}^{m,0}(s) \geq u_k(s, x) + \delta_k \sum_{s'} p(s'| s, x)V_{k}^{\phi}(s') \\
\geq V_{k}^{\phi}(s) + \epsilon
\]
for all \( m \).

Finally, the type \( k \) politician’s payoff from the deviation, accounting for at least \( m + 1 \) periods of office benefit, is no less than
\[
u_k(s, x) + \delta_k \tilde{V}_{k}^{m,0}(s) + \frac{(1 - \delta^{m+1})b_k}{1 - \delta},
\]
28
and the politician’s payoff from following $\pi_k$ is no greater than $V^\phi_k(s) + \frac{b_k}{1 - \delta_k}$. Since
\[
\liminf_{m \to \infty} \left[ u_k(s, x) + \delta_k \bar V^m_k(s) + \frac{(1 - \delta^{m+1})b_k}{1 - \delta} \right] > V^\phi_k(s) + \frac{b_k}{1 - \delta_k},
\]
it follows that for high enough $m$, the deviation to $\tilde \pi_k$ is profitable, a contradiction. We conclude that if $\phi$ is implemented by a Markov electoral equilibrium, then the policy rule is optimal. 

**Proof of Theorem 5.1.** As explained in the text, we imbed our model in the more general framework of Duggan and Forand (2018), modifying our model by adding a bad absorbing state $s_b \notin S$, and then applying Theorem 1 of that paper. The augmented set of states is $\tilde S = S \cup \{s_b\}$. We then specify the state transition such that for all $s, s' \in S$, all $t \in T$, all $x \in Y(s)$, and all $e \in \{0, 1\}$,
\[
\tilde \pi_t(s'|s, x, e) = \begin{cases} 
1 & \text{if } s' = s_b, t = k, \text{ and } e = 0, \\
& \text{or if } s = s_b, \\
p(s'|s, x) & \text{else.}
\end{cases}
\]
That is, the state transition is otherwise the same as in our model, but if a type $k$ politician is removed from office, then the state transitions to the absorbing bad state. We define stage utility functions as in our model, but as assign a bad payoff to the voter in the bad state:
\[
\tilde u_k(s, x) = \begin{cases} 
-2 & \text{if } s = s_b, \\
u_k(s, x) & \text{else,}
\end{cases}
\]
where we recall that stage utility is bounded between $\underline u = 0$ and $\bar u = 1$. With this specification, Theorem 1 of Duggan and Forand (2018) yields a Markov electoral equilibrium $\tilde \sigma = (\tilde \pi, \tilde \rho)$, and we map this into our model by defining $\pi_t$ as the restriction of $\tilde \pi_t$ to $S$, and we define $\rho$ so that for all $s \in S$ and all $x \in Y(s)$, if $t \neq k$, then $\rho(s, t, x) = \tilde \rho(s, t, x)$; and $\rho(s, k, x) = 1$ if $V^\rho_k(s, k, x) \geq \bar V^\rho_k(s, x)$, and $\rho(s, k, x) = 0$ otherwise.

In the equilibrium $\tilde \sigma$ of the augmented model, the type $k$ politician’s expected discounted office benefit at every state $s \neq s_b$ for every policy choice $x \in Y(s)$ is $\frac{b_k}{1 - \delta_k}$; in particular, it is constant with respect to the policy choice. Thus, the type $k$ politician is essentially policy motivated, i.e., $b_k = 0$. Using the result of Theorem 6.1, this implies that the continuation value of a free type $k$ politician achieves the optimal value of the voter, and the reelection set of the type $k$ politician is nonempty: for all $s \neq s_b$, we have $V^\pi_k(s, k) = V^\ast_k(s)$ and $\bar R_k(s, k) \neq \emptyset$. Note that in $\tilde \sigma$, from any state $s \in S$, the bad state $s_b$ is reached with probability zero, i.e., for all $x \in Y(s)$, we have $\rho(s, k, x) = 1$. Indeed, consider any policy choice $x \in Y(s)$ by a type $k$ incumbent at state $s \neq s_b$. The voter’s discounted payoff from reelecting the incumbent is at least equal to $-2\delta_k/(1 - \delta_k)$, and this is strictly greater than the payoff of electing a challenger is $-2/(1 - \delta_k)$.

Next, note that for all $x \in \text{supp}(\pi_k(\cdot|s))$, we have $V^\pi_k(s, k, x) = \sum s' p(s'|s, x)V^\ast_k(s')$, and thus in the strategy profile $\sigma = (\pi, \rho)$ of the original model, the type $k$ politician is always reelected with probability one. This implies that continuation values from the augmented
model are preserved in the original model: for all $s \neq s_b$, all $t$ and $t'$, and all $x$, we have $V^f_t(s, t', x) = \tilde{V}^f_t(s, t', x)$ and $V^C_t(s, x) = \tilde{V}^C_t(s, x)$. In turn, this implies that $\sigma$ preserves equilibrium conditions, and so is a Markov electoral equilibrium of our model. Clearly, we have $R_k(s, k) \neq \emptyset$ and $V^F_k(s, k) = V^*_k(s)$ for all $s$, and thus the type $k$ politician is a faithful delegate, as required. □

Proof of Theorem 5.2. Fix $\delta$, and let $\sigma^\delta = (\pi^\delta, \rho^\delta)$ be a Markov electoral equilibrium given $\delta$. Since the type $k$ politician is a faithful delegate, we have $V^F_k(s, k) = V^*_k(s, k)$ for all $s$. Let $\tilde{\rho}$ denote the voting strategy defined as follows: for all $s$ and $x$, $\tilde{\rho}(s, t, x) = 1$ if $t = k$ and $\tilde{\rho}(s, t, x) = 0$ otherwise. Letting $\tilde{V}_k^F(s, t)$ denote the expected discounted payoff to the type $k$ voter from electing a free type $t$ politician in state $s$ given strategy profile $\tilde{\sigma}^\delta = (\pi^\delta, \tilde{\rho})$, we again have $\tilde{V}_k^F(s, k) = V^*_k(s, k)$. The profile $\tilde{\sigma}^\delta$ may not itself be an equilibrium, but because the equilibrium voting strategy solves the representative voter type’s optimal retention problem, it must be the case that for all $s$ and all $t$, we have $V^F_k(s, t) \geq \tilde{V}_k^F(s, t)$. Now, consider any strongly recurrent state $s$ such that $\alpha = \min_x q_k(s, x) > 0$. Let $p^m_n(s)$ denote the probability that $s$ is realized at least $n$ times in $m$ periods, conditional on beginning at $s$. For all $n$, since $s$ is strongly recurrent, we have $\lim_{m \to \infty} p^m_n(s) = 1$. Thus, regardless of policy choices, the probability that a type $k$ politician is drawn within $m$ periods is bounded below by $p^m_n(s)(1 - (1 - \alpha)^n)$ for all $n$. Given the equilibrium $\sigma^\delta$, the representative voter’s expected discounted utility from electing a free type $t$ politician satisfies

$$V^F_k(s, t) \geq \tilde{V}_k^F(s, t) \geq p^m_n(s)(1 - (1 - \alpha)^n)\delta^m V^*_k(s, k)$$

for all $m$ and all $n$, where we use the normalization $u_k \geq 0$. This implies

$$\frac{V^F_k(s, t)}{V^*_k(s, k)} \geq p^m_n(s)(1 - (1 - \alpha)^n)\delta^m$$

for all $m$. Given $\epsilon > 0$, we can choose $n(\epsilon)$ sufficiently high that $1 - (1 - \alpha)^n(\epsilon) > 1 - \epsilon$, and then we can choose $m(\epsilon)$ sufficiently high that $p^m_n(s)(1 - (1 - \alpha)^n(\epsilon)) > 1 - \epsilon$. Then, taking limits, we have

$$\sup_{m, n} p^m_n(s)(1 - (1 - \alpha)^n)\delta^m \geq (1 - \epsilon)\delta^{m(\epsilon)} \to 1 - \epsilon$$

as $\delta \to 1$. Since $\epsilon$ was arbitrary, the desired inequality follows. □

Proof of Corollary 5.1. Note that the normalized values $(1 - \delta)V^*(s')$ belong to the compact interval $[0, 1]$, and thus we can without loss of generality consider a subsequence $\nu^\delta = ((1 - \delta)V^*(s'))_{s' \in S}$ with pointwise limit $\nu$. Let $s$ be strongly recurrent and such that $\min_x q_k(s, x) > 0$, and consider any type $t$. Because payoffs are additively separable across time, we can write the representative voter’s (normalized) expected discounted payoff from $(s, t)$, given discount factor $\delta$, as

$$(1 - \delta)V^F_k(s, t) = (1 - \delta) \sum_{m=1}^{\infty} \delta^{m-1} \int u_k(s', x') p^m_{s, t}(d(s', x'))$$

30
Because $\mu_{s,t}^\delta$ belongs to the set $\Delta(Y)$, which is compact with the weak* topology, we can go to a further subsequence such that $\mu_{s,t}^\delta$ converges weak* to a limit $\mu$. Using Theorem 5.2, we have

$$v_s = \lim_{\delta \to 1} (1 - \delta)V_k^s,\delta(s) = \lim_{\delta \to 1} (1 - \delta)V_k^F,\delta(s, t) = \int u_k(s', x') d(s', x'). \quad (10)$$

Note that since $s$ is strongly recurrent, the marginal probability of $\mu_{s,t}^\delta$ and $\mu$ on $s$ is positive, and thus, because $S$ is discrete, the conditional measures $\mu_{s,t}^\delta(\cdot|s)$ converge weak* to $\mu(\cdot|s)$.

Suppose toward a contradiction that there exists $\epsilon > 0$ such that the limit does not hold, so that

$$\lim \inf_{\delta \to 1} \mu_{s,t}^\delta(B_{\epsilon}(\Phi(s)|\pi))|s < 1.$$

Since $Y(s)\setminus B_{\epsilon}(\Phi(s)|\pi)$ is compact, weak* convergence implies that

$$\mu(Y(s)\setminus B_{\epsilon}(\Phi(s)|\pi))|s > 0,$$

and thus there exists $\eta > 0$ and a Borel measurable subset $Y' \subseteq Y(s)\setminus B_{\epsilon}(\Phi(s)|\pi)$ such that $\mu(Y'|s) > 0$ and for all $x \in Y'$, we have

$$\sum_{s'} p(s'|s, x)\pi_{s'} + \eta \leq \max_{x \in Y(s)} \sum_{s'} p(s'|s, x')\pi_{s'}.$$

Since $s$ has positive marginal probability under $\mu$, this implies

$$\int u_k(s', x') d(s', x') < \max_{x \in Y(s)} \sum_{s'} p(s'|s, x')\pi_{s'}.$$ \quad (11)

Define the mapping $U_s: Y \times [0, 1] \times [0, 1]^S \to \mathbb{R}$ by

$$U_s(x|\delta, v) = (1 - \delta)u_k(s, x) + \delta \sum_{s'} p(s'|s, x)v_{s'},$$

and note that it is jointly continuous in $(x, \delta, v)$. By definition of optimal value, we have

$$v_{s}^\delta = \max_{x \in Y(s)} U(x|\delta, v^\delta).$$

By the theorem of the maximum, this maximized value is continuous, and taking $\delta \to 1$, we have

$$\int u_k(s', x') d(s', x') = \overline{v}_s = \max_{x \in Y(s)} U(x|1, \pi) = \max_{x \in Y(s)} \sum_{s'} p(s'|s, x)\pi_{s'}.$$

where the first equality follows from (10). This contradicts (11), however, and we conclude that $\mu_{s,t}^\delta(B_{\epsilon}(\Phi(s)|\pi))|s \to 1.$
Proof of Theorem 5.3. Since the equilibrium satisfies incumbency advantage, it must be that for all \( s \), all \( t \), and all \( x \in \text{supp}(\pi_t(\cdot|s)) \), we have \( \pi_t(R_k(s,t)|s) = 1 \) and \( \rho(s,t,x) = 1 \). Since the type \( k \) politician is a faithful delegate, it follows that for all \( s \) and all \( x \in \text{supp}(\pi_k(\cdot|s)) \), \( x \) achieves the optimal value for the representative voter. Since \( \rho(s,k,x) = 1 \), it follows that \( u_k(s,x) + \delta_k V_k^I(s,k,x) = V_k^I(s) \). Hence, for all \( x \in \text{supp}(\pi_k(\cdot|s)) \),

\[
V_k^I(s,k,x) - \sum_{s'} p(s'|s)V_k^F(s',k) = \gamma_k p(s|s) \left[ u_k(s,x) + \delta_k V_k^I(s,k,x) - V_k^F(s,k) \right] = \gamma_k p(s|s) [ V_k^*(s) - V_k^F(s,k) ] = 0. \tag{12}
\]

Now, given any state \( s \), let politician type \( \tilde{t} \) and policy \( \tilde{x} \in \text{supp}(\pi_{\tilde{t}}(\cdot|s)) \) satisfy

\[
V_k^I(s,\tilde{t},\tilde{x}) = \min \left\{ V_k^I(s,t,x) : (t,x) : t \in T, x \in \text{supp}(\pi_t(\cdot|s)) \right\},
\]

and suppose toward a contradiction that

\[
V_k^I(s,\tilde{t},\tilde{x}) < \int V_k^I(s,k,x')\pi_k(dx'|s). \tag{13}
\]

Then, using the fact that \( \rho(s,t',x) = 1 \) for all \( t' \) and all \( x \in \text{supp}(\pi_{t'}(\cdot|s)) \), we have

\[
V_k^C(s,\tilde{x}) = \sum_{t'} q(t'|s,\tilde{x}) \left[ p(s|s)(\gamma_{t'} V_k^F(s,t') + (1 - \gamma_{t'}) V_k^F(s,t') + \sum_{s' \neq s} p(s'|s)V_k^F(s',t') \right] \]

\[
= \sum_{t'} q(t'|s,\tilde{x}) \left[ p(s|s) \left( \gamma_{t'} \int [ u_k(s,t',x) + \delta_k V_k^I(s,t',x) ] \pi_{t'}(dx|s) \right) + (1 - \gamma_{t'}) V_k^F(s,t') \right] + \sum_{s' \neq s} p(s'|s)V_k^F(s',t') \]

\[
= \sum_{t'} q(t'|s,\tilde{x}) \int \left[ p(s|s) \left[ \gamma_{t'} (u_k(s,x) + \delta_k V_k^I(s,t',x)) + (1 - \gamma_{t'}) V_k^F(s,t') \right] + \sum_{s' \neq s} p(s'|s)V_k^F(s',t') \right] \pi_{t'}(dx|s) \]

\[
> V_k^I(s,\tilde{t},\tilde{x}),
\]

contradicting \( \rho(s,\tilde{t},\tilde{x}) = 1 \), with the inequality following from (13) and \( q(k|s,\tilde{x}) > 0 \). Therefore, it must be that for all states \( s \), types \( t \), and policies \( x \in \text{supp}(\pi_t(\cdot|s)) \), we have

\[
V_k^I(s,t,x) = \int V_k^I(s,k,x')\pi_k(dx'|s). \tag{14}
\]
Finally, we claim that $V^F_k(s, t) = V^*_k(s)$ for all non-trivial $s$ and all $t$. Indeed, note that $V^F_k(s, t) \leq V^*_k(s) = V^*_k(s, k)$. Suppose toward a contradiction that $V^F_k(s, t) < V^*_k(s, k)$, and, since $s$ is non-trivial, fix $\bar{s}$ such that $p(s|\bar{s}) > 0$. Then, for all $x \in \text{supp}(\pi_k(\cdot|s))$,

\[
V^I_k(\bar{s}, k, x) = \sum_{s'} p(s'|\bar{s}) V^F_k(s', k) > \sum_{s'} p(s'|\bar{s}) V^F_k(s', t) \\
= p(\bar{s}|\bar{s}) \left[ \gamma_t \int \left[ u_k(\bar{s}, x') + \delta_k V^I_k(\bar{s}, t, x') \right] \pi_t(dx'|\bar{s}) + (1 - \gamma_t) V^F_k(\bar{s}, t) \right] \\
+ \sum_{s' \neq \bar{s}} p(s'|\bar{s}) V^F_k(s', t) \\
= \int \left[ p(\bar{s}|\bar{s}) \left[ \gamma_t (u_k(s, x') + \delta_k V^I_k(s, t, x') + (1 - \gamma_t) V^F_k(s, t) \right] \\
+ \sum_{s' \neq \bar{s}} p(s'|\bar{s}) V^F_k(s', t) \right] \pi_t(dx'|\bar{s}) \\
= \int V^I_k(\bar{s}, t, x') \pi_t(dx'|\bar{s}),
\]

where the first equality follows from (12) and the inequality by supposition. This contradicts (14) and completes the proof of the claim. \hfill \Box

**Proof of Theorems 6.1 and 6.2.** Fix a Markov electoral equilibrium $\sigma = (\pi, \rho)$. A first remark is that if $V^F_k(s, k) = V^*_k(s)$ holds for all states $s$, then we must also have $R_k(s, k) \neq \emptyset$ for all states. To see this, suppose toward a contradiction that $R_k(s, k) = \emptyset$ for some state $s$. Note that for all $x \in \text{supp}(\pi_k(\cdot|s))$, the definition of Markov electoral equilibrium implies that

\[
x \in \text{argmax}_{x \in Y(s)} u_k(s, x) + \delta_k \sum_{s'} p(s'|s, x) V^*_k(s').
\]

Thus, we have

\[
V^I_k(s, k, x) = \frac{\gamma_k p(s|s, x) u_k(s, x) + (1 - \gamma_k)p(s|s, x) V^F_k(s, k) + \sum_{s' \neq s} p(s'|s, x) V^F_k(s', k)}{1 - \gamma_k \delta_k p(s|s, x)} \\
= \frac{\gamma_k p(s|s, x) [u_k(s, x) + \delta_k \sum_{s'} p(s'|s, x) V^F_k(s', k)]}{1 - \gamma_k \delta_k p(s|s, x)} \\
+ \frac{(1 - \gamma_k \delta_k p(s|s, x)) \sum_{s'} p(s'|s, x) V^F_k(s', k) - \gamma_k p(s|s, x) V^F(s, k)}{1 - \gamma_k \delta_k p(s|s, x)} \\
= \frac{\gamma_k p(s|s, x) [V^*_k(s) - V^F_k(s, k)]}{1 - \gamma_k \delta_k p(s|s, x)} + \sum_{s'} p(s'|s, x) V^*_k(s') \\
= \sum_{s'} p(s'|s, x) V^*_k(s') \\
\geq V^C_k(s, x),
\]

33
where the inequality follows since $V^F_k(s, t) \leq V^*_k(s)$ for all politician types $t$, as desired.

To prove Theorem 6.1, first assume that $\delta_k = 0$. Note that an arbitrary policy $x \in Y(s)$ maximizes

$$u_t(s, x') + b_t + \delta_t \left[ \rho(s, x', t)(V^I_t(s, t, x') + B_t(s, x')) + (1 - \rho(s, x', t))V^C_t(s, x') \right],$$

over $Y(s)$ if and only if

$$x \in \text{argmax}_{x' \in Y(s)} u_k(s, x'),$$

which holds if and only if

$$x \in \text{argmax}_{x' \in Y(s)} u_k(s, x') + \delta_k \sum_{s'} p(s'|s, x') V^*_k(s'),$$

so that $V^F_k(s, k) = V^*_k(s)$ for all $s$, as required.

Second, assume that $b_k = 0$. Viewing the type $k$ politicians and representative voter as a unitary agent, consider the best response problem of this agent given the equilibrium profile $\sigma = (\pi, \rho)$, formulated as a dynamic programming problem, as follows. The set of states of the problem is $s \times t \times X \times \{0, 1\}$, where state $(s, t, x, b)$ is interpreted so that the office holder is type $t$, if $b = 1$, then she is bound to $x$, and otherwise the politician is free. In state $(s, k, \pi, b)$, if $b = 0$, then the agent chooses any policy, say $x$; and if $b = 1$, then the agent must choose $x = \pi$. The agent receives utility $u_k(s, x)$.

The agent then chooses to remain in office or to leave office, possibly to return again or to be replaced by a different politician type. In the first case, a new $s'$ is drawn according to the distribution $p(\cdot|s, x)$, and if $s' \neq s$, then the new state is $(s', k, x, 0)$; if $s' = s$, then with probability $\gamma_k$, the new state is $(s, k, x, 1)$; and with probability $1 - \gamma_k$, the new state is $(s, k, x, 0)$. In the second case, where the agent leaves office, for each $t$, the probability of state $(s', t, x, 0)$ is $p(s'|s, x)q(t|s, x)$. Thus, with probability $q(k|s, x)$, the agent remains in office, free to choose any policy in state $(s', k, x, 0)$; and with complementary probability, some type $t \neq k$ politician takes office in state $(s', t, x, 0)$. Then a new policy $x'$ is drawn from the distribution $\pi_t(\cdot|s')$, and the $k$ agent receives utility $u_k(s', x')$.

The agent then decides to keep the type $t$ incumbent or remove the politician. In the first subcase, a new $s''$ is drawn from $p(\cdot|s', x')$, and if $s'' \neq s'$, then the new state is $(s'', t, x', 1)$; if $s'' = s'$, then with probability $\gamma_t$, the new state is $(s', t, x', 1)$, and the agent receives utility $u_k(s', x')$; and with probability $1 - \gamma_t$, the new state is $(s', t, x', 0)$. In the second subcase, for each $t'$, the probability of state $(s'', t', x', 0)$ is $p(s''|s', x')q(t'|s', x')$, and the process repeats.

We can define continuation values for this problem in the usual way, and in particular we write $\hat{V}^I_k(s, t, x, b)$ for the maximized utility of the $k$ agent in state $(s, t, x, b)$; we write $\hat{V}^F_k(s, t)$ for the expected utility of the agent when integrating over states $(s, t, x, 0)$ using $\pi_t(\cdot|s)$; and we write $\hat{V}^C_k(s, x)$ for the expected utility when integrating over states $(s', t, x, 0)$ using $p(\cdot|s, x)q(\cdot|s, x)$. For each $b \in \{0, 1\}$, these continuation values satisfy the functional equation

$$\hat{V}^I_k(s, t, x, b) = \max \left\{ p(s|s, x) \left[ \gamma_t(u_k(s, x) + \delta_k \hat{V}^I_k(s, t, x, b)) + (1 - \gamma_t)\hat{V}^F_k(s, t) \right] + \sum_{s' \neq s} p(s'|s, x)\hat{V}^F_k(s', t) \right\},$$

$$\hat{V}^C_k(s, x) = \max \left\{ p(s|s, x) \left[ \gamma_t(u_k(s, x) + \delta_k \hat{V}^C_k(s, t)) + (1 - \gamma_t)\hat{V}^F_k(s, t) \right] + \sum_{s' \neq s} p(s'|s, x)\hat{V}^F_k(s', t) \right\}.$$
and in particular, $\hat{V}_k(s, t, x, b)$ is independent of $b$, and these values coincide with the values in the voter’s optimal retention problem. By Proposition 3.1, these are in fact the equilibrium values, $V^*_{k}(s, t, x)$, $V^F_{k}(s, t)$, and $V^C_{k}(s, x)$.

The agent’s maximized utility, calculated at state $(s, k, x, 0)$, satisfies
\[
\hat{V}_k(s, k, x, 0) = \max_{x' \in X(s)} u_k(s, x') + \delta_k \max\{V^I(s, t, x'), \hat{V}^C_{k}(s, x')\},
\]
or, using the preceding observation and the equilibrium condition on $\rho$,
\[
\hat{V}_k(s, k, x, 0) = \max_{x' \in X(s)} u_k(s, x') + \delta_k \left[\rho(s, x', k)V^I(s, t, x') + (1 - \rho(s, x', k))V^C_{k}(s, x')\right].
\]
Therefore, by definition of Markov electoral equilibrium, $\pi_k(\cdot|s)$ places probability one on solutions to the $k$ agent’s optimal policies in each state $(s, k, x, 0)$. We conclude that $(\pi_k, \rho)$ is indeed optimal for the $k$ agent; in particular, we have $\hat{V}^F_{k}(s, k) = V^*_{k}(s)$ for all $s$, as required.

To prove Theorem 6.2, assume that $p(\cdot|s, x)$ and $q(\cdot|s, x)$ are independent of $x$ and $s$, and that $\gamma_k p(s|s) > 0$ for all $s$. Then we have
\[
x' \in \arg\max_{x \in Y(s)} u_k(s, x) + \delta_k \sum_{s'} p(s'|s)V^*_{k}(s'),
\]
if and only if
\[
x' \in \arg\max_{x' \in Y(s)} u_k(s, x'),
\]
We will show that $\pi_k(\cdot|s)$ places probability one on the solutions to the above problem in every state. For each state $s$, there are two cases. First, assume that $R(s, k) = \emptyset$, so that $\rho(s, k, x) = 0$ for all $x$. Hence, we have
\[
x' \in \arg\max_{x \in Y(s)} u_k(s, x) + \delta_k V^C_{k}(s', x),
\]
if and only if
\[
x' \in \arg\max_{x \in Y(s)} u_k(s, x),
\]
where we use the fact that $V^C_{k}(s', x)$ is independent of $x$ when $q(t'|s, x)$ is independent of $x$. Second, assume there exists $x \in R(s, k)$, and suppose toward a contradiction that $x \notin \arg\max_{x' \in Y(s)} u_k(s, x')$. In that case, for $\bar{x} \in \arg\max_{x' \in Y(s)} u_k(s, x')$, using $\gamma_k p(s|s) > 0$, we have
\[
V^I_{k}(s, k, \bar{x}) = \frac{p(s|s)[\gamma_k u_k(s, \bar{x}) + (1 - \gamma_k)\hat{V}^F_{k}(s, k)] + \delta_k \sum_{s' \neq s} p(s'|s)\hat{V}^F_{k}(s', k)}{1 - \gamma_k \delta_k p(s|s)} > V^I_{k}(s, k, x) \geq V^C_{k}(s, k),
\]
and the equilibrium condition on the voting strategy $\rho$ implies $\rho(s, k, \bar{x}) = 1$. Furthermore, optimality of $\pi_k(\cdot|s)$ implies that $\pi(\arg\max_{x \in Y(s)} u_k(s, x)|s) = 1$. We conclude that $\pi_k(\cdot|s)$ places probability one on maximizers of stage utility in every state, and thus $V^F_{k}(s, k) = V^*_{k}(s)$, for all $s$, as required. \qed
References


Camara, O., 2012. Economic policies of heterogeneous politicians.


