BARGAINING POWER AND THE AGENDA

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Abstract. A two-issue, full information strategic bargaining model is considered in which the players differ in their bargaining power across issues. The effect of the order of issues within an agenda on the equilibrium outcome is analyzed for sequential implementation of issues for different bargaining cost specifications. *Journal of Economic Literature* Classification Number: C7

1. Introduction

From self-help books on car purchasing to fairly substantial works by lawyers, there exists quite a range of books by non-economists on how to bargain.[1] One key thread in all these works is a recognition that most interesting bargaining involves multiple distinct issues, and therefore the authors do not fail to provide advice on how to structure a multi-issue bargain in order to achieve the best outcome. These questions about the bargaining agenda receive different answers, sometimes even within the same book, however.

In the economics literature the question of the bargaining agenda has received surprisingly little attention until fairly recently. In the cooperative literature Kalai (1977), Herrero (1989) and Ponsati and Watson (1997) address the agenda. In the strategic bargaining literature Fershtman (1990) first contrasts the equilibrium outcomes under an agenda to those under a unified single issue bargain. Busch and Horstmann (1997b) further explores the causes of these differences. The second thread of the literature attempts to endogenize the agenda within a strategic setting.[2] All of these papers further our understanding of how and why the agenda may matter, and if these differences influence the equilibrium allocation if the agenda is determined strategically. Their focus, as that in the practitioner literature, is on how the agenda can create bargaining power — lead to a more successful outcome.

In this paper, a different question is addressed. Suppose that there are multiple issues to be settled and that the bargaining power a player enjoys on these issues, looked at in isolation, differs. Some issues lead to a large share of the surplus going to the player, others just a small share.[3] Are the insights of the previous literature, which considers issues on which the player has the same bargaining power, applicable in this setting? Is the agenda order which generates bargaining

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[1]For example Ramundo (1992), Lewis (1981), or Churchman (1995).

[2]Bac and Raff (1996), Busch and Horstmann (1994, 1997a, 1999b, 2000), Inderst (2000), Lang and Rosenthal (2001), Jun (1989), Flamini (2000), Weinberger (2000). Jun (1989) and Fershtman (2000) can also be viewed in this light.

[3]Muthoo (2000, p51) for example identifies bargaining power with the size of a player's equilibrium share.

power from symmetric issues also the one which preserves existing bargaining power best?

This investigation will also inform a debate within the practitioner literature which has received scant attention so far in the economic literature. An issue which appears of significant concern to practitioners is if hard or easy issues should be negotiated first. This raises, of course, the question of how one maps the notion of a "hard" issue into a game-theoretic model of bargaining, in particular since the practitioner literature does not define the term. Presumably the notion of a "hard" issue is linked to the equilibrium outcome of the model as much as to the actual issue at hand.

The only previous attempt to consider agenda order with hard and easy issues models hard issues as those for which there is the possibility of delay in agreement. Based on this definition, under which some issues are hard for both bargainers and others easy, Busch and Horstmann (2000) assume that there is incomplete information (about the size of the surplus) for one issue while the easy issue is taken to be a full information bargain. The story in their paper is based on a signalling explanation. If issues are implemented jointly only at the conclusion of all bargaining the question of hard versus easy is not relevant: The informed player's delay costs are unknown to the end if the hard or the easy issue is addressed first. Signalling occurs with the large enough issue in a manner similar to Busch and Horstmann (1999a) where the incomplete information is on the discount factor. However, if issues are implemented sequentially so that partial agreements are consumed immediately, easy issues will go first in a signalling equilibrium. This is due to the fact that the equilibrium allocation for the second issue is fixed if it is the easy issue. The easy issue is therefore used to send a signal and transform the subsequent hard issue in an easy issue (since a successful signal leads to a continuation path with full information.) In a sense, "bargaining momentum", a phrase from the practitioner literature, is created.

In contrast, this paper will consider full information environments only.[4] Also, under the definition that an issue is hard for a particular player if he will obtain a low share of the surplus from this issue, compared to the other issues available, an issue which is hard for one player is necessarily easy for his opponent. "Hard" is thus a wholly relative term, referring to a specific bargaining situation only.

The paper proceeds as follows. In section 2 the various approaches to bargaining power in the non-cooperative bargaining literature are reviewed. Common features of all models are collected in section 3. Section 4 will address the discounting formulation, section 5 a probability of breakdown formulation, while section 6 will summarize results for fixed bargaining costs. In section 7 we summarize the equilibria for endogenous agenda games, while section 8 concludes. All proofs and derivations are collected in an appendix.

2. Bargaining Power

In the strategic bargaining setting there are multiple ways in which the notion of bargaining power differences has been captured. The most extreme version is probably the fixed cost formulation of Rubinstein's (1982) model, where the player with the lower cost receives all the surplus when he makes an offer. If the high cost player makes the offer he may get at most the low cost player's cost.

[4]The paper most closely related to it is Flamini (2000).

In general, bargaining power is driven by the relationship between the costs the players face if they decide to reject an offer. In the commonly used discounting formulation this is captured by the (relative) size of players' discount factors. As is well known, if players discount the future at some rate r_i and the time between offers is Δ, then player i's discount factor is $\delta_i = exp(-r_i\Delta)$, and the equilibrium share of player 1, $(1-\delta_2)/(1-\delta_1\delta_2)$, converges to $r_2/(r_1+r_2)$ as $\Delta \to 0$. It follows that the more patient player 1 is compared to player 2, the higher is his equilibrium share and hence his bargaining power.

Another approach which generates bargaining power in a very direct way is to consider a game in which one player makes multiple offers in a row. Consider a game with identical time preferences in which player 1 makes k offers in a row and player 2 makes l offers i a row. Then the larger is k relative to l, the larger is player 1's share and hence his bargaining power. His equilibrium share is $(1-\delta^k)/(1-\delta^{k+l})$ which converges to $k/(k+l)$ as $\delta \longrightarrow 1$.[5]

An alternative interpretation of the bargaining friction in alternating offers bargaining is as a risk of breakdown. The continuation probabilities then take the role of the discount factors (Binmore *et al* (1986).) Inasmuch as players attach different subjective probabilities to breakdown, or to the extent that the actual probabilities depend on the identity of the player who makes or rejects an offer, the resulting equilibrium shares look just as if players had different discount factors. The more optimistic player, or the one for whom the game is more likely to continue after a rejection, obtains a larger share of the surplus.

Another way for a player to have bargaining power is to give him additional control over the surplus. For example, in a formulation in which "money burning" is allowed, so that one player may choose to destroy some fraction of the surplus, (in addition to common discounting) the solution features a compound term of the time discount and destruction rate which replaces the player's opponent's discount factor term in the standard solution.[6]

All these "micro"-models of bargaining power boil down to the fact that one player has a lower delay cost than another, and consequently obtains a larger share. The various solutions even "look" the same and can all be mapped back into the (most familiar) discounting formulation. Once multiple sequential issues are added into the model, however, these models may differ in how one would "naturally" extend them. For this reason we will investigate a discounting formulation, a model with risk of breakdown, as well as the fixed cost model in what follows. As the results show, the different bargaining cost specifications are not equivalent any more if multiple issues and agenda offers are introduced.

3. Model Description and Notation

Two players, 1 and 2, bargain over the division of the surplus of two distinct issues, X and Y. The amount of surplus from each issue is common knowledge, and is normalized to 1. Bargaining is via strictly alternating offers, with one offer per discrete time period $t = 1, 2, 3, \dots$. That means that one offer is made per time period, with player 1 offering in odd periods $\{1, 3, 5, \dots\}$ and player 2 offering in even periods $\{2, 4, 6, \dots\}$. This rule also applies to accepted partial offers. That

[5]The most extreme example of this model is the one-sided offer model, in which one player makes all the offers and gets all of the surplus.

[6]If money burning is a choice, it also leads to the existence of multiple equilibria, see Avery and Zemsky (1994) and Busch *et al* (1998).

is, if in some odd period player 1 makes an offer on only one issue to player 2 and player 2 accepts, then player 2 will make the next offer (on the remaining issue.)

Offers are specified in terms of the share of the surplus which is to be allocated to player 1. So an offer on X by either player is a $x \in [0,1]$, an offer on Y is a $y \in [0,1]$. Two games are contrasted, one in which offers must be made first on X, until agreement, then on Y, the other with the reversed order of issues. In both cases accepted offers are binding and remove the issue in question from "the table". The game ends as soon as an agreement on both goods exists.

An outcome in either game is specified in terms of an allocation $((x,t),(y,\tau))$. This notation signifies agreement at time t on x and agreement at time τ on y, where x,y are the shares of X,Y which player 1 receives. Player 2 therefore receives $1-x$ and $1-y$. By convention, $t(\tau)=\infty$ if no agreement is reached on $X(Y)$.

As is standard in bargaining, preferences are over allocations only, are risk neutral and separable across issues. We consider sequential implementation only. Under this rule exchange occurs as soon as agreement is reached on a particular good and regardless of whether agreement is ever reached on the other good.

We will maintain the following assumption on players' bargaining powers:

Assumption 1. *Player 1 has the relative bargaining advantage on X while player 2 has the relative bargaining advantage on Y, i.e. if the issues where bargained in isolation $x > y$ in both players' offers.*

The equilibrium concept used throughout is that of subgame perfect Nash equilibrium (SPE), and the term equilibrium henceforth refers to SPE.

4. Discounting

The most common interpretation of the bargaining friction in alternating offers bargaining is discounting. Under a discounting formulation the cost of bargaining derives from the delay in consumption implied by a rejection. Note that in the standard one issue setup $u(x,t) = \delta^t x$, so so that the marginal rate of substitution between future and current consumption is $1/\delta$. In the current case we have $U((x,t),(y,\tau)) = v(x,t)+w(y,\tau)$ due to the separability assumption. It then is not unreasonable to assume that $v(\cdot)$ and $w(\cdot)$ feature different inter temporal MRSs. Let $\delta_p^i \in (0,1)$, $i \in \{X,Y\}$, $p \in \{1,2\}$. Then under sequential implementation, an allocation $((x,t),(y,\tau))$ leads to utilities of

$$\begin{aligned} u_1((x,t),(y,\tau)) &\geq (\delta_1^X)^{t-1}x + (\delta_1^Y)^{\tau-1}y, \\ u_2((x,t),(y,\tau)) &\geq (\delta_2^X)^{t-1}(1-x) + (\delta_2^Y)^{\tau-1}(1-y). \end{aligned}$$

In order to simplify the presentation a symmetry assumption is imposed, so that

Assumption 2. $\delta_1^X = \delta_2^Y = \overline{\delta} \;>\; \underline{\delta} = \delta_2^X = \delta_1^Y.$

This assumption implies that players' comparative bargaining advantages align with their absolute bargaining advantages.[7]

The key fact which drives sequential implementation bargaining on multiple issues is the independence of the bargaining game on later issues from the bargained

[7] If in addition $2\overline{\delta} > 1 + \underline{\delta}\,\overline{\delta}$ then neither player can obtain more than 1/2 of his hard issue under any circumstance.

outcome of prior ones.[8] This is due to the fact that the already agreed upon issue has also been consumed and thus no costs arise from it from any potential hold-up. The second issue, whichever it may be, will therefore be allocated according to its Rubinstein (1982) outcome. The first issue bargain in turn is solved by backward induction.

First, assume we are in the X then Y agenda. The unique equilibrium then has player 1 make an offer on X, of [9]

$$\min\begin{cases} 1, \\ \dfrac{(1-\underline{\delta})\left(1-\overline{\delta}^3-\underline{\delta}^2-\underline{\delta}^3+\overline{\delta}\left(1-\underline{\delta}+\underline{\delta}^2+\underline{\delta}^3\right)\right)}{\left(1-\underline{\delta}\,\overline{\delta}\right)^2}. \end{cases}$$

The min is caused by the fact that player 1, who is the stronger player on the first issue, X, may be constrained in his offer by the feasibility constraint of not being able to obtain more than all of that first issue.[10] In either case this offer is followed by an offer on Y by player 2 of $\underline{\delta}\,(1-\overline{\delta})/(1-\underline{\delta}\,\overline{\delta})$. Notice how the fact that player 1 is in a good bargaining position on issue X allows him to obtain a large allocation of issue X, larger, in fact than what his allocation would be without the second issue. Apparently player 1 experiences an increase in bargaining power relative to player 2 on the first issue when it is to be followed by a second issue.

If issue Y is to be bargained on first the equilibrium offer on Y is

$$\max\begin{cases} \dfrac{1-\overline{\delta}+\underline{\delta}-2\,\overline{\delta}\,\underline{\delta}+\overline{\delta}^2\,\underline{\delta}-\underline{\delta}^3+\overline{\delta}\,\underline{\delta}^3}{1-\underline{\delta}\,\overline{\delta}} \\ \dfrac{(1-\overline{\delta})\left(1-\overline{\delta}^2\left(1+\overline{\delta}\right)(1-\underline{\delta})+\underline{\delta}-\overline{\delta}\,\underline{\delta}-\underline{\delta}^3\right)}{\left(1-\underline{\delta}\,\overline{\delta}\right)^2}. \end{cases}$$

The first term arises if player 2 is constrained to offer player 1 a share of 0 on Y. In that case player 1 makes an offer that has player 2 just indifferent to rejecting and coming back with an offer of $y = 0$. The second term in the max is the standard (unique) interior solution to the two equations defining the equilibrium offers. The Y offer by player 1 is followed in either case by an offer on X by player 2 of $\overline{\delta}\,(1-\underline{\delta})/(1-\underline{\delta}\,\overline{\delta})$.

Notice that player 1 in this case experiences a deterioration of his bargaining power on Y, evidenced by the fact that his equilibrium share of Y is lower than if Y were bargained in isolation.[11]

It is now easy to verify that a player gets a higher payoff under the agenda in which the issue on which he is stronger is first. We summarize this in the following proposition.

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[8] While implementation is often not discussed in conjunction with the agenda in the practitioner literature, any references to concessions having a value since one can demand concessions in return in the future ("in the name of fairness") cannot have sequential implementation in mind. Subgame perfection imposes that "bygones are bygones".

[9] This and all other derivations are in the appendix.

[10] The condition on $(\underline{\delta}, \overline{\delta})$ for this to occur is provided in the appendix.

[11] Focusing on the second term in the maximum, it is less than the standard share if $1 - \overline{\delta}^2\left(1+\overline{\delta}\right)(1-\underline{\delta})+\underline{\delta}-\overline{\delta}\,\underline{\delta}-\underline{\delta}^3 < 1-\underline{\delta}\,\overline{\delta}$ or $-\overline{\delta}^2\left(1+\overline{\delta}\right)(1-\underline{\delta})+\underline{\delta}-\underline{\delta}^3 < 0$. This is satisfied as long as $\underline{\delta} < \overline{\delta}^2(1+\overline{\delta})$, which is also the condition for this second term to be applicable.

Proposition 1. *(Easy first under sequential implementation with discounting) A player gets his highest payoff in a fixed order agenda bargaining game with sequential implementation if the issue on which he has the highest bargaining power is first in the agenda.*

This proposition shows that a player will gain a bargaining power advantage if the agenda is structured such that his easy issue, that is the one on which he has relatively more bargaining power, is addressed first. Why is this the case?

4.1. **Exploring the Results.** What are the bargaining costs on the first issue in an agenda bargain? A rejection of a current offer will have three effects: First, it causes delay in the consumption of the first issue (the standard single issue effect.) Second, it will make the opponent the first mover on the second issue, so that the first mover advantage is foregone on the second issue. Third, it causes delay in the consumption of the second issue. The result must hinge on the latter two, since they are related to the second issue. Note that under a discounting formulation delay costs are proportional to the equilibrium share a player expects. Therefore the player with the higher bargaining power on the second issue will be more subject to the delay cost from the second issue. Consider the cost arising from issue Y for player 1 from rejecting 2's offer on X: instead of $(1-\overline{\delta})/(1-\underline{\delta}\,\overline{\delta})$ in the next period, he will receive $\underline{\delta}\,(1-\overline{\delta})/(1-\underline{\delta}\,\overline{\delta})$ two periods hence, for a utility loss of $\underline{\delta}(1-\underline{\delta}^2)(1-\overline{\delta})/(1-\underline{\delta}\,\overline{\delta})$. The corresponding utility loss for player 2 from rejecting 1's offer is $\overline{\delta}\ (1-\overline{\delta}^2)(1-\underline{\delta})/(1-\underline{\delta}\,\overline{\delta})$. Since $\underline{\delta}(1+\underline{\delta}) < \overline{\delta}(1+\overline{\delta})$ player 1 has the lower delay costs from issue Y in a rejection of an offer on X. This occurs even though he has the lower discount factor on issue Y and thus is more impatient to get to issue Y. This lower delay cost increases 1's bargaining power in the X bargain. Conversely, if Y is the first issue in the agenda player 1 will suffer a further decrease in his bargaining power on Y.

Both, the first mover advantage and the proportional delay costs are important for this result. To see this, let us shut down each in turn.

Consider first the effect of the assumption that delay costs on the second issue accrue while the players bargain on the first. What if the delay costs only apply while players bargain on an issue? The appropriate definition for players' utilities for an allocation $((x,t),(y,\tau))$, $\tau > t$ in the X then Y agenda then becomes

$$\begin{aligned} u_1((x,t),(y,\tau)) &= (\delta_1^X)^{t-1}x + (\delta_1^Y)^{\tau-1-t}y, \\ u_2((x,t),(y,\tau)) &= (\delta_2^X)^{t-1}(1-x) + (\delta_2^Y)^{\tau-1-t}(1-y), \end{aligned}$$

for example. In that case, repeating the computation on delay costs attributable to the second issue leads to identical relative costs between players. Yet it is straight forward to verify that an X then Y agenda will lead to an allocation $((x,1),(y,2))$ with

$$x = \frac{1-\underline{\delta}}{1-\underline{\delta}\,\overline{\delta}} + \frac{(1-\underline{\delta})^2(1-\overline{\delta})}{(1-\underline{\delta}\,\overline{\delta})^2}, \quad y = \frac{\underline{\delta}(1-\overline{\delta})}{1-\underline{\delta}\,\overline{\delta}},$$

focusing on an interior solution. Following an Y then X agenda in contrast will lead to an allocation $((x,2),(y,1))$ with

$$x = \frac{\overline{\delta}(1-\underline{\delta})}{1-\underline{\delta}\,\overline{\delta}}, \quad y = \frac{1-\overline{\delta}}{1-\underline{\delta}\,\overline{\delta}} + \frac{(1-\underline{\delta})(1-\overline{\delta})^2}{(1-\underline{\delta}\,\overline{\delta})^2},$$

again focusing on the interior solution. From this it follows that the preferred agenda order for player 1 continues to be X then Y. This demonstrates that it is not only the cost of delaying the second issue which causes this preference ordering.

On the other hand, suppose that we alter the game so that the player who makes the initial offer on the second issue is chosen randomly, with equal probabilities for each player (otherwise all offers remain strictly alternating.) This is the formulation in Fershtman (1992), for example, and there clearly is no first mover advantage on the second issue. In that case, from the perspective of the initial bargain, player 1 expects a share of $(1+\underline{\delta})(1-\overline{\delta})/(2-2\underline{\delta}\,\overline{\delta})$ on issue Y. The utility loss to player 1 due to Y from rejecting an offer on X then is $\underline{\delta}(1-\underline{\delta})(1-\overline{\delta})(1+\underline{\delta})/(2-2\underline{\delta}\,\overline{\delta})$. The corresponding cost to player 2 is $\overline{\delta}(1-\overline{\delta})(1-\underline{\delta})(1+\overline{\delta})/(2-2\underline{\delta}\,\overline{\delta})$. Clearly player 1 has the lower cost again. Focusing again on the interior solutions it is easy to verify that the utility difference for player 1 between an X first and a Y first agenda is

$$\frac{(1-\underline{\delta})(1-\overline{\delta})\left(\overline{\delta}^3+\overline{\delta}^2(2-\underline{\delta})+\overline{\delta}(2+\underline{\delta}^2)-\underline{\delta}(2+2\underline{\delta}+\underline{\delta}^2)\right)}{2(1-\underline{\delta}\,\overline{\delta})^2}$$

which is positive.[12] It follows that it is not the first mover advantage by itself which causes the easy first agenda to be preferred.

It is interesting to note that another slight modification of the game yields a complete decoupling of issues. In particular, if we specify that the same player who makes the accepted first offer also makes the initial offer on the second issue. Now bargaining is not strictly alternating any more and the first mover advantage works against a player making an offer, since only if his current offer is accepted will the player have the first move on the second issue. Under this assumption the second issue actually does not contribute any costs at all to rejections on the first issue. This is due to the fact that for a single issue a player, by definition, is just indifferent between having his opponent make him the equilibrium offer or delaying by one period to make his own equilibrium offer. When considering the rejection decision on the first issue, the player now has the choice to accept, in which case his opponent will make him an offer on the second issue one period hence, or reject, in which case he himself will make the offer on the second issue, but two periods hence. The payoff from the second issue is thus identical in both scenarios. The equilibrium offers on the first issue are consequently the same as if the issue were bargained on in complete isolation.

Consequently, player 1 will make both offers in equilibrium, and they are $(1-\underline{\delta})/(1-\underline{\delta}\,\overline{\delta})$ on X and $(1-\overline{\delta})/(1-\underline{\delta}\,\overline{\delta})$ on Y independent of agenda order. Note also that this does not mean that the allocation will be identical to a joint bargain. While in both procedures the same player makes the accepted offer on both issues, the joint offer bargaining game yields an equilibrium allocation of

$$(x,y)=\left(1,\frac{1-\overline{\delta}}{1+\underline{\delta}}\right).$$

Here we refer to the game in which offers are tuples (x,y) and need to be accepted or rejected in their entirety. Note that player 1 obtains all of X but a very small share of Y.

[12] Expanding and rearranging the third term of the numerator yields $(\overline{\delta}^3-\underline{\delta}^3)+2(\overline{\delta}^2-\underline{\delta}^2)+(2-\overline{\delta}\,\underline{\delta})(\overline{\delta}-\underline{\delta})$ which determines the sign of the expression and is clearly positive.

Finally, notice that as the bargaining friction vanishes, modelled as the time between offers approaching zero, the equilibrium utility of player 1 for both the $X-Y$ and $Y-X$ agendas converges to[13]

$$\frac{\overline{r}}{\underline{r}+\overline{r}}+\frac{\underline{r}}{\overline{r}+\underline{r}}=1.$$

It is therefore identical, in the limit, to the joint bargain utility. We can conclude that for a bargaining power specification which allows different issues to be discounted at different rates, the agenda is a phenomenon which is important only if frictions exist. In a frictionless environment the agenda has no influence on the allocation.

5. Continuation Probabilities

The other common interpretation of the bargaining friction is as a probability of breakdown. How does this specification extend to a multiple issue setting? The first question is if breakdown applies to only one or to all issues. The most natural extension here is probably that breakdown affects all issues, so that breakdown on an early issue in an agenda affects all subsequent issues. We also need to specify if there is a probability of breakdown "between" issues. If breakdown is a bargaining friction related to bargaining disagreement, the it seems reasonable to assume that after an acceptance of an offer the next issue bargaining round is reached with certainty.[14] Differential bargaining power now means that each player has a (subjective) probability of breakdown that depends upon which issue is on the table at any given moment.[15] Denote the continuation probabilities by $\rho_1^X, \rho_1^Y, \rho_2^X, \rho_2^Y$ and assume that $\rho_1^X > \rho_2^X$ while $\rho_1^Y < \rho_2^Y$ so that Assumption 1 is satisfied. For simplicity we again impose symmetry, and let

Assumption 3. $\rho_1^X = \rho_2^Y = \overline{\rho} \; > \; \underline{\rho} = \rho_2^X = \rho_1^Y$.

Suppose an X then Y agenda order has to be followed. Should agreement on X be reached, the bargain on issue Y is standard and results in an agreement on

$$\overline{y}=\frac{1-\overline{\rho}}{1-\overline{\rho}\underline{\rho}} \quad \text{or} \quad \underline{y}=\underline{\rho}\frac{1-\overline{\rho}}{1-\overline{\rho}\underline{\rho}}$$

if player 1, respective 2, makes the initial offer. In the bargain over issue X the equilibrium offer of player 1 is

$$\overline{x}=\min\left\{\begin{array}{l}1,\\ \dfrac{2-(3-\overline{\rho})\underline{\rho}}{1-\overline{\rho}\underline{\rho}}\end{array}\right..$$

If a Y then X agenda order is imposed instead, the equilibrium offers are

$$\overline{y}'=\max\left\{\begin{array}{l}2(1-\overline{\rho}),\\ \dfrac{2-(3-\underline{\rho})\overline{\rho}}{1-\overline{\rho}\underline{\rho}},\end{array}\right. \qquad \text{and} \qquad \underline{x}'=\frac{\overline{\rho}(1-\underline{\rho})}{1-\overline{\rho}\underline{\rho}}.$$

[13] Here we let $\underline{\delta}=exp(-\overline{r}\Delta)$ and $\overline{\delta}=exp(-\underline{r}\Delta)$ and let $\Delta \longrightarrow 0$.

[14] Flamini (2000) considers between issue bargaining breakdown.

[15] Note that, since only the breakdown risk after a rejection is important, and only to the rejecting player, subjective probabilities and actual probabilities which differ lead to the same conclusions.

Comparing the payoff to player 1 from each of these agenda orders we find that player 1 prefers the X then Y agenda. We summarize this in the following proposition.

Proposition 2. *(Easy first under risk of breakdown)*
A player gets his highest payoff in a fixed order agenda bargaining game with issue dependent subjective breakdown risks if the issue on which he has the highest bargaining power is first in the agenda.

In contrast to the discounting case, this ranking persists in the limit as the bargaining friction vanishes. Letting λ be the instantaneous continuation probability, player 1's limiting payoffs in an $X-Y$ and $Y-X$ agenda are $\overline{\lambda}/(\underline{\lambda}+\overline{\lambda})$ and $\underline{\lambda}/(\underline{\lambda}+\overline{\lambda})$ respectively, with $\overline{\lambda} > \underline{\lambda}$. Here player 1 is inherently strong when bargaining on X since he believes that the game will most likely continue. Player 2, in contrast, attaches a lower probability to this event. Since the fixed future payoff to be achieved from the second issue is subject to these continuation probabilities, it is as if the players bargained over an issue X which is larger. First mover considerations are irrelevant to this analysis. Indeed, if we consider a game where an accepted offer by a player leads to that player making the first offer on the next issue, the equilibrium utilities for player 1 are identical to those above.

Finally note that this model is identical to a discounting model in which players discount future payoffs at a rate which depends on the issue currently under consideration. With time preference derived from a utility function this is a problematic assumption, while in this setting it arises "naturally".

6. Fixed bargaining costs

The alternative formulation in Rubinstein's (1982) alternating offers bargaining game assumes fixed delay costs. In practice this means that bargaining costs are independent of the allocation. The usual way to think about these fixed costs is as offer costs: a player incurs this cost if he rejects and has to prepare a counter offer. In this interpretation agreed upon issues do not incur further costs. Neither are there any costs related to future issues incurred in prior bargains.

Here we consider only one particular extension of the fixed cost model to multiple issues. We assume that preparing a counter offer is the only thing that costs money, and that players have different such costs for each issue, which differ across players.

Suppose, then, that the utility to the players from an agreement on (x, y) is

$$U_1(x, y) = x + y \quad \text{and} \quad U_2(x, y) = (1 - x) + (1 - y).$$

Bargaining costs are incorporated as a fixed cost of making a counter offer. Let these costs be denoted as c_i^j, $i \in 1, 2$, $j \in X, Y$, so that c_1^X, for example, is player 1's cost of making a counter offer on issue X. Assumption 1 now translates into

Assumption 4. *Player 1 has the relative bargaining advantage on X while player 2 has the relative bargaining advantage on Y:* $c_1^X < c_2^X$, $c_2^Y < c_1^Y$.

This definition captures the notion of bargaining power in that the equilibrium shares on issue X in isolation, for example, would be $(1, 0)$ in player 1's offer and $(1 - c_1^X, c_1^X)$ in player 2's offer.

Now consider the agenda in which players first bargain on issue X then on issue Y. Once issue X is allocated the only cost of bargaining on issue Y is given by c_1^Y and c_2^Y, so we immediately know that issue Y is allocated as $(c_2^Y, 1 - c_2^Y)$ if player

1 makes the offer and as $(0,1)$ if player 2 makes the offer. Solving backwards in the usual fashion it is easy to see that issue X will be allocated as $(1,0)$ if player 1 makes the first offer but as $(1-c_1^X-c_2^Y, c_1^X+c_2^Y)$ if player 2 makes the offer. The additional term c_2^Y in this expression stems from the additional cost to player 1 of foregoing the first mover advantage on issue Y, should he choose to make a counter offer.

Conversely, if issue Y comes first in the agenda and issue X second, we obtain an initial allocation of issue Y in 1's offer of $(c_1^X+c_2^Y, 1-c_1^X-c_2^Y)$, and in 2's offer of $(0,1)$. Again the additional cost (c_1^X) which player 1 manages to extract from player 2 is player 2's cost of loosing the first mover advantage on the following issue, X, when he chooses to make a counter offer.

The equilibrium utility allocations in the 2 agendas are therefore

$$\begin{aligned} X \text{ then } Y \text{ agenda} &: \quad U_1 = 1,\, U_2 = 1. \\ Y \text{ then } X \text{ agenda} &: \quad U_1 = 1 + c_2^Y,\, U_2 = (1 - c_2^Y). \end{aligned}$$

In the first case each player simply receives the full value of his easy issue, since he is the one making the initial offer on that issue. In the second case player 1 makes the initial offer on his hard issue, which allows him to extract player 2's direct counter offer costs, c_2^Y as well as indirect costs c_1^X. Player 1 does loose the opportunity to move first on his easy issue, but note how the associated utility loss (c_1^X) is precisely equal to player 2's indirect cost, so on net player 1 looses nothing from this.

It follows that it is more beneficial for player 1 to follow a hard then easy agenda. Where we to start the game with player 2 the same obtains, that is, player 2 would prefer an X then Y agenda if he is the one making the first offer. This result, then, is in stark contrast to the previous cost specifications.

This difference cannot be attributed to the fact that in this fixed cost specification no costs are incurred on either prior or future issues. Instead, the result is driven solely by the loss of the first mover advantage in strictly alternating offers bargaining. To see this consider the allocations under this bargaining cost specification if a player who has his first offer accepted gets to make the offer on the next issue: In the X then Y agenda $((x,t),(y,\tau)) = ((1,1),(c_2^Y,2))$ while in the Y then X agenda $((x,t),(y,\tau)) = ((1,2),(c_2^Y-c_1^X,1))$, provided $c_2^Y - c_1^X > 0$. Otherwise player 2 will be strong enough to reject 1's initial offer and move to the next period where he will offer on both issues. As is apparent from the allocations, player 1 now prefers the X then Y agenda. This confirms that the first mover advantage outweighs the bargaining costs in the strictly alternating offers setting. This is, of course, driven by the fact that a weak player gets no payoff unless he makes the first offer.

7. Endogenizing the agenda

Most of the recent literature in agenda bargaining is concerned with the question if an agenda can occur as an equilibrium phenomenon in games where the players may use partial offers, but don't have to.[16] In general the answer is negative: Quite robustly the equilibrium will feature joint offers on all outstanding issues. The same is the case here. In a recent paper In and Serrano (2000) have also introduced a new game in which the players have to make partial offers, but the order of issues in this

[16] See footnote 2 for a list of papers.

agenda is open, so that the question arises if a player can, in equilibrium, obtain his preferred agenda order. In their model, based on a probability of breakdown bargaining friction and player utilities in which players attach different importance to issues, they obtain a multiplicity of equilibria for this formulation. Without their differential marginal rates of transformation across issues we do not have this feature here. Instead there is a unique equilibrium in which each player makes offers according to his preferred agenda order.

7.1. **Discounting.** Consider the discounting formulation first. If players are restricted to make offers on only one issue but the initial issue they offer on is not specified, there is a unique equilibrium in which player 1 will choose to make offers on X first while player 2 chooses to make offers on Y first. This is driven by the fact that each player gets the best possible continuation payoff, subject to providing the opponent with an acceptable level of utility, from delaying the issue he is weaker on and offering on his easy issue first. The resulting equilibrium allocation is $((\hat{x}, 1), (\hat{y}, 2))$ with

$$\hat{x} = \frac{1 + \overline{\delta} - 2\,\underline{\delta}\,\overline{\delta}^2 - \underline{\delta}^3(1 - \overline{\delta}))}{(1 + \overline{\delta})(1 - \underline{\delta}\,\overline{\delta})}, \quad \hat{y} = \frac{\underline{\delta}\,(1 - \overline{\delta})}{1 - \underline{\delta}\,\overline{\delta}}.$$

Note that player 1 looses utility compared to the case of a fixed agenda, as evidenced by the smaller X share he obtains in equilibrium. This is due to the fact that in his counter offer player 2 can switch to an agenda order which is better for him, so that player 1 has to offer a higher utility to player 2 in order to get his initial offer accepted. The following proposition summarizes this result.

Proposition 3. *If implementation is sequential and the players have to follow an agenda, but are free to choose the order of issues, the first mover determines the equilibrium agenda and will choose to make an offer on his easy issue first. The equilibrium strategies of the other player also have him make offers on his easy issue first.*

Finally, we can consider a game in which the agenda is fully endogenous. In this game the kind of offer a player makes is only constrained by what issues are still outstanding. In the absence of any previous agreement an offer can therefore be on either one of the issues or on both issues together. However, in the equilibrium no single issue offers are made, i.e., no agenda is used.

Proposition 4. *In the open agenda bargaining game with sequential implementation no agenda is used. The unique equilibrium has immediate agreement on a joint* (x, y) *offer by player 1 of*

$$\left(1, \frac{1 - \overline{\delta}}{1 + \underline{\delta}}\right).$$

Note that player 1 obtains all of X but a very small share of Y. Notice also that this is the equilibrium of a game in which players are restricted to make only joint offers. Indeed, in the limit the allocation approaches $(1, 0)$, so that each player receives all his utility from the issue on which he has the bargaining power advantage. Note that this limiting allocation coincides with that for the agenda games, derived previously.

7.2. **Probability of breakdown.** In order to extend the analysis of breakdown probabilities to models with endogenous agenda orders we first must take a stand on which continuation probability a player will assess, given the offer on the table and the offer on which he wishes to make an offer. In particular, suppose that player 2 has received an offer on X. If he rejects this offer and counters with an offer on Y, versus an offer on X again, what is the probability he assigns on being able to make such a counter offer? In what follows we will adopt the stance that the offer currently on the table determines the continuation probabilities. That is, if an offer on X is on the table ρ_i^X, $i = 1, 2$ are the continuation probabilities irrespective of the offer which is then made. The point of this is that if player 1 makes an offer on issue X, on which he is strong, then player 2 is weak, irrespective of which counter offer he will make.

Under this assumption the appendix shows that both players will make their initial offer on the issue on which they are strong, that is, their easy issue, in the game in which they have to make offers on only one issue. The equilibrium agenda therefore is determined by the first mover and is that player's preferred agenda order.

Proposition 5. *If players have to follow an agenda, but are free to choose the order of issues, the first mover determines the equilibrium agenda and will choose to make an offer on his easy issue first. The equilibrium strategies of the other player also have him make offers on his easy issue first.*

In the open agenda players are allowed to make offers on both issues simultaneously. In keeping with the above assumption that the issue on the table determines the continuation probability each player assesses, and further supposing that if any one issue leads to breakdown the whole bargain is considered over, it seems reasonable to assume that for joint offers the maximal breakdown probability is the relevant one. Since there is no probability of breakdown between an accepted issue and the next offer on the remaining one a joint bargain does not have any efficiency advantage. Consequently a player may obtain the same level of utility either by making an offer on only his easy issue, or on both issues. In both cases the opponent will assess the low continuation probability in case he rejects the offer. Only the equilibrium utilities are unique then, not the strategies.

Proposition 6. *In an open agenda bargaining game the first mover determines the equilibrium agenda. He is indifferent between a joint offer and an agenda in which his easy issue is dealt with first.*

8. Discussion and concluding remarks

If players differ in their bargaining power across issues the order of issues in an agenda has a critical effect on the bargaining power of a player in the agenda bargain. Not surprisingly, this effect depends on the precise formulation of the game and may explain why there is so much conflicting advice in the practitioner literature.

In the standard strictly alternating offers strategic bargaining game with discounting the discount factor of one player relative to that of his opponent determines the bargaining power. This formulation is quite general (see Fishburn and Rubinstein (1982)). One key effect of this formulation is that bargaining costs accrue on issues which have already been agreed upon but not yet implemented on the one hand, but also on future issues which have not even been discussed yet on the other. If issues are implemented sequentially, that is, partial agreements are implemented as soon as they are reached, the first of these effects is shut down. The bargain on the remaining issues can therefore not be influenced by the agreements on prior issues, or put differently, bargaining power can not flow up the agenda. Can it "flow down"? Our analysis has shown that it does so only for strictly positive bargaining frictions. In the limit the agenda order does not affect the allocation, so that it is only the first mover advantage which drives the result that an easy first agenda is most advantageous for a player.

If the bargaining friction is modelled as a probability of breakdown, the result is unchanged: An easy first agenda is most advantageous for a player. Now each player applies his probability for the first issue to the second as well, so that the bargaining power of a player on the first issue applies also to the expected payoff from the second issue. Essentially the players bargain over a larger issue (namely the first plus the utility offset from the second.) It now is beneficial to have the bargaining advantage on the "larger" issue, which is always the first. Note that the key difference to the discounting formulation is the treatment of the second issue payoff: in the discounting formulation this was discounted at the second issue discount factor. In the probability of breakdown formulation the second issue expected payoff is "discounted" by the continuation probability appropriate for the first issue bargain. Hence the difference.

In the most extreme case of bargaining advantage, the fixed cost model, the results are reversed. This is basically driven by the fact that a player, if he is not the first mover on his hard issue, obtains no payoff from it at all. The key consideration then becomes the first mover advantage.

In any case, the results indicate that players may wish to restrict bargaining to follow a particular agenda which is advantageous to them. How then is the bargaining agenda fixed? One approach is to have a pre-bargaining bargaining phase such as in Busch and Horstmann (1999b). Suppose that this were the case in our setup, with players bargaining on which fixed agenda order to use. If players have access to some randomization device and make offers in terms of probabilities of implementing each fixed agenda order, we would observe both agendas in equilibrium. If only pure offers are feasible the pre-bargain will have either agenda as an equilibrium outcome and will also allow for delay.[17] Thus the presence of an observed agenda can be explained as the outcome of a pre-bargaining phase in which players agree to follow an agenda. If randomization is not possible, a reasonable assumption in international bargaining, for example, then we might expect the agenda setting bargaining round to be more drawn out and controversial than the actual bargaining on the issues.

Another approach is to let the first offer determine the agenda, that is, if the first mover makes an offer on issue X only all subsequent offers have to be on issue X until agreement is reached on X.[18] This arrangement of course presents a great advantage to the first mover, so that a serious model of how the first mover is

[17]This is the standard result on bargaining with fixed alternatives/increments, where it may not be possible for either player to make an offer that has the opponent just indifferent between accepting and rejecting.

[18]It sometimes is claimed that collective bargaining proceeds in this fashion, in the sense that issues not part of a first offer may not be brought in at a later point.

chosen will then be required.[19] In our model the agenda would then be that most advantageous to player 1.

A final approach is to endogenize the agenda within the main bargain by allowing players a choice on which issue to offer. Doing so, we find that, if no order of issues is fixed but offers are only possible on one issue at a time, each player will choose as his initial issue the one that is most advantageous to him in the fixed agenda. Finally, if there is no agenda exogenously imposed at all and players can make offers on any (sub)set of issues, including all issues at once, the result first suggested in Busch and Horstmann (1994) applies: No issue-by-issue agendas arise in equilibrium. As Ponsati and Watson (1997) show, this can be expected to be a very robust result.

The results thus confirm the importance of the agenda for the bargained outcome, and thus justify the resources expended on agreeing on, and fixing, the agenda to be followed in multi-issue bargaining settings. They also demonstrate that models with differential bargaining power, perhaps not that interesting in single issue models, are much more important in multi-issue models, especially in models with restricted agenda forms.

9. Appendix

Discounting Formulation:

Sequential Implementation, Fixed Order: As usual, we solve by backward induction. The second issue is bargained on when the first has been consumed. Let $\hat{x}$ denote the agreement on X, achieved at time t_x, and let $\overline{y}$ and $\underline{y}$ denote the offers on Y by 1 and 2, respectively. The two equations defining the usual Rubinstein bargaining solution for the second issue thus have an offset. In particular, for each $\tau > t_x$ we require:

$$u_2((\hat{x}, t_x), (\overline{y}, \tau)) \geq u_2((\hat{x}, t_x), (\underline{y}, \tau + 1)) \quad \longrightarrow \quad (1 - \overline{y}) \geq \overline{\delta}(1 - \underline{y}) \tag{1}$$

$$u_1((\hat{x}, t_x), (\underline{y}, \tau)) \geq u_1((\hat{x}, t_x), (\overline{y}, \tau + 1)) \quad \longrightarrow \quad \underline{y} \geq \underline{\delta}\, \overline{y}. \tag{2}$$

Supposing both equations hold as an equality we get the standard

$$(\overline{y}, \underline{y}) = \left(\frac{1 - \overline{\delta}}{1 - \underline{\delta}\, \overline{\delta}}, \frac{\underline{\delta}\, (1 - \overline{\delta})}{1 - \underline{\delta}\, \overline{\delta}} \right).$$

In the bargain on the first issue we now have the following set of inequalities (where stationarity of preferences is exploited):

$$u_2((\overline{x}, t), (\underline{y}, t + 1)) \geq u_2((\underline{x}, t + 1), (\overline{y}, t + 2)) \tag{3}$$

$$u_1((\underline{x}, t), (\overline{y}, t + 1)) \geq u_1((\overline{x}, t + 1), (\underline{y}, t + 2)) \tag{4}$$

Supposing that both hold as an equality we get a solution of

$$\overline{x} = \frac{(1 - \underline{\delta}) \left(1 - \overline{\delta}^3 - \underline{\delta}^2 - \underline{\delta}^3 + \overline{\delta}\, \left(1 - \underline{\delta} + \underline{\delta}^2 + \underline{\delta}^3\right)\right)}{\left(1 - \underline{\delta}\, \overline{\delta}\right)^2}, \tag{5}$$

$$\underline{x} = \frac{\overline{\delta} - \overline{\delta}^4\, (1 - \underline{\delta}) - \overline{\delta}^2\, (1 - \underline{\delta})^2 - \overline{\delta}\, \underline{\delta}^3 - \underline{\delta}\, \left(1 - \underline{\delta}^2\right)}{\left(1 - \underline{\delta}\, \overline{\delta}\right)^2}. \tag{6}$$

[19] This may be one of the important benefits of winning an election: in a parliamentary system such as Canada's, for example, the government has the right to set the agenda, i.e., choose in which order bills are introduced and voted on.

However $\overline{x}$ in (5) could become larger than 1, which is not feasible. Indeed, this happens if

$$\overline{\delta} \geq -\frac{1-\underline{\delta}+\underline{\delta}^2-\sqrt{1+2\,\underline{\delta}+3\,\underline{\delta}^2-6\,\underline{\delta}^3-3\,\underline{\delta}^4+4\,\underline{\delta}^5}}{2\,(1-\underline{\delta})}$$

In that case the best that player 1 can do is to ask for all of issue X. He can not keep player 2 indifferent to rejecting, and must give 2 too much utility, i.e., (3) remains a strict inequality. Player 2 on the other hand computes his counter offer from (4) with an equality. This yields the equilibrium offers of

$$(\overline{x}',\underline{x}') = \left(1, \left(\frac{\overline{\delta}-\underline{\delta}+\underline{\delta}\,\overline{\delta}-\overline{\delta}^2\,\underline{\delta}+\underline{\delta}^3-\overline{\delta}\,\underline{\delta}^3}{1-\underline{\delta}\,\overline{\delta}}\right)\right). \tag{7}$$

In the text we use player 1's offers only, and the constraint is captured by the min{} operator.

If issue Y is dealt with first we follow a similar series of steps. The unconstrained Y offer by player 1 then is

$$\overline{y} = \frac{\left(1-\overline{\delta}\right)\left(1-\overline{\delta}^2\,(1-\underline{\delta})-\overline{\delta}^3\,(1-\underline{\delta})+\underline{\delta}-\overline{\delta}\,\underline{\delta}-\underline{\delta}^3\right)}{\left(1-\underline{\delta}\,\overline{\delta}\right)^2}.$$

The key difference will be the fact that now player 2 may be constrained to ask for not more than all of issue Y for himself (i.e., make an offer of $\underline{y}=0$ to player 1.) In that case player 1 will keep him indifferent while 2 provides too much utility to 1. That is, 1 solves the analogue of (3) which is

$$(1-\overline{y})+\underline{\delta}\left(1-\frac{\overline{\delta}\,(1-\underline{\delta})}{1-\underline{\delta}\,\overline{\delta}}\right) = \overline{\delta}\,(1-0)+\underline{\delta}^2\left(1-\frac{(1-\underline{\delta})}{1-\underline{\delta}\,\overline{\delta}}\right)$$

which leads to

$$\overline{y} = \frac{1-\overline{\delta}+\underline{\delta}-2\,\overline{\delta}\,\underline{\delta}+\overline{\delta}^2\,\underline{\delta}-\underline{\delta}^3+\overline{\delta}\,\underline{\delta}^3}{1-\underline{\delta}\,\overline{\delta}}.$$

Proof of Proposition 1: The proposition is easily proved by comparison of the player's payoffs. Suppose the unconstrained case obtains. Player 1 now obtains a total utility of

$$\frac{1-\overline{\delta}^3\,(1-\underline{\delta})-\underline{\delta}+\overline{\delta}^2\,\underline{\delta}^3+\underline{\delta}^4+\overline{\delta}\,\left(1-2\,\underline{\delta}+\underline{\delta}^2-\underline{\delta}^3-\underline{\delta}^4\right)}{\left(1-\underline{\delta}\,\overline{\delta}\,\right)^2} \tag{8}$$

if issue X is first, and

$$\frac{1+\overline{\delta}^4\,(1-\underline{\delta})+\underline{\delta}+\overline{\delta}^2\,\underline{\delta}-\overline{\delta}^3\,(1-\underline{\delta})\,\underline{\delta}-\underline{\delta}^3-\overline{\delta}\,\left(1+2\,\underline{\delta}-\underline{\delta}^3\right)}{\left(1-\underline{\delta}\,\overline{\delta}\,\right)^2} \tag{9}$$

if issue Y is first. Computing (8)-(9) we get

$$\frac{\left(1-\overline{\delta}\right)\,(1-\underline{\delta})\,\left(\overline{\delta}^3+\overline{\delta}^2\,(2-\underline{\delta})+\overline{\delta}\,\left(2+\underline{\delta}^2\right)-\underline{\delta}\,\left(2+2\,\underline{\delta}+\underline{\delta}^2\right)\right)}{\left(1-\underline{\delta}\,\overline{\delta}\right)^2} \tag{10}$$

the sign of which is determined by the second term in the numerator, that is

$$\begin{aligned}\overline{\delta}^3 + \overline{\delta}^2\,(2-\underline{\delta}) + \overline{\delta}\,\left(2+\underline{\delta}^2\right) - \underline{\delta}\,\left(2+2\,\underline{\delta}+\underline{\delta}^2\right) = \\ \left(\overline{\delta}^3 - \underline{\delta}^3\right) + \left(\overline{\delta}^2 - \underline{\delta}^2\right) + \left(2 - \underline{\delta}\,\overline{\delta}\,\left(\overline{\delta}-\underline{\delta}\right)\right).\end{aligned}$$

This is clearly positive, indicating that player 1 obtains a higher payoff if X, his easy issue, is dealt with first.

Open order issue-by-issue agendas:
In order to determine the equilibrium allocation we require the reservation utility level each player is willing to accept. Suppose player 1 has received an offer by player 2 and now has to choose if to reject it or not. He will reject anything that does not give him at least the payoff he can get from rejecting and making his best counter-offer. A similar argument applies to player 2, of course. Let player 2's reservation level of utility be denoted by u_2. Then player 1's reservation level u_1 is determined as

$$u_1 = \max \begin{cases} \overline{\delta}x + \underline{\delta}^2\,\dfrac{\underline{\delta}(1-\overline{\delta})}{1-\underline{\delta}\,\overline{\delta}} & \text{s.t.} \quad (1-x) + \overline{\delta}\,\dfrac{1-\underline{\delta}}{1-\underline{\delta}\,\overline{\delta}} = u_2 \\ \underline{\delta}y + \overline{\delta}^2\,\dfrac{\overline{\delta}\,(1-\underline{\delta})}{1-\underline{\delta}\,\overline{\delta}} & \text{s.t.} \quad (1-y) + \underline{\delta}\,\dfrac{1-\overline{\delta}}{1-\underline{\delta}\,\overline{\delta}} = u_2 \end{cases}$$

The first of these corresponds to an initial offer on X, while the second corresponds to an initial offer of Y. Simplification yields that player 1's reservation level of utility is

$$u_1 = \max \begin{cases} \dfrac{\overline{\delta}\,(1+\overline{\delta}) + \underline{\delta}^3\,(1-\overline{\delta}) - 2\,\underline{\delta}\,\overline{\delta}^2}{1-\underline{\delta}\,\overline{\delta}} - \overline{\delta}u_2 \\ \dfrac{\underline{\delta}\,(1+\underline{\delta}) + \overline{\delta}^3\,(1-\underline{\delta}) - 2\,\underline{\delta}^2\,\overline{\delta}}{1-\underline{\delta}\,\overline{\delta}} - \underline{\delta}u_2 \end{cases} \tag{11}$$

In effect these two equations define two utility possibility frontiers for player 1, dependent on which issue he chooses to offer on first in the next period. As is readily apparent they are both straight lines. However, the top equation in (11) has a higher intercept and lower slope, given that $\underline{\delta} < \overline{\delta}$. To see this note that the numerators of the intercept term are identical if $\overline{\delta} = \underline{\delta}$, and that their difference can be expressed as

$$(\overline{\delta}-\underline{\delta}) + \underline{\delta}\,\overline{\delta}(\overline{\delta}^2 - \underline{\delta}^2) + (\overline{\delta}^2 - \underline{\delta}^2) - (\overline{\delta}^3 - \underline{\delta}^3)$$

which is positive. The lines intersect when $\overline{\delta} > \underline{\delta}$ and

$$u_2 = \frac{(\overline{\delta}-\underline{\delta}) + (\overline{\delta}^2 - \underline{\delta}^2) - (\overline{\delta}^3 - \underline{\delta}^3) + \overline{\delta}^3\,\underline{\delta} - \overline{\delta}\,\underline{\delta}^3 - 2\,\overline{\delta}^2\,\underline{\delta} + 2\,\overline{\delta}\,\underline{\delta}^2}{(\overline{\delta}-\underline{\delta})(1-\underline{\delta}\,\overline{\delta})}.$$

Similar computations can be made for player 2. They lead to a reservation utility level

$$u_2 = \max \begin{cases} \dfrac{\underline{\delta}\,(1+\underline{\delta}) + \overline{\delta}^3\,(1-\underline{\delta}) - 2\,\underline{\delta}^2\,\overline{\delta}}{1-\underline{\delta}\,\overline{\delta}} - \underline{\delta}u_1 \\ \dfrac{\overline{\delta}\,(1+\overline{\delta}) + \underline{\delta}^3\,(1-\overline{\delta}) - 2\,\underline{\delta}\,\overline{\delta}^2}{1-\underline{\delta}\,\overline{\delta}} - \overline{\delta}u_1 \end{cases} \tag{12}$$

where the first line is computed assuming an offer on X first, and the second line assuming an offer on Y first. Note that the lines in (11) and (12) are simply switched due to symmetry. The fact that the reservation levels for a player are chosen from different offer structures depending on the reservation utility level of the opponent can, in principle, lead to a multiplicity of equilibria as demonstrated in Lang and Rosenthal (2001) and exploited in In and Serrano (2000) in a related setting. However, here this does not occur. It is easily verified that this is the case by the following arguments. Suppose that both players in equilibrium choose to make offers on X first. Then u_1 is determined according to the first line in (11) while u_2 is determined according to the first line in (12). Solving the two equations we can determine the reservation levels under this assumption and then we can verify if they are such that the first lines of (11) and (12) are indeed the respective maximal values. Computing the difference between the first line in (12) and the second and substituting in the computed value for u_1 we obtain

$$\frac{-(1-\underline{\delta})\,(1-\overline{\delta})^2\,(\overline{\delta}(1+\overline{\delta})^2-\underline{\delta}(1+\underline{\delta})^2)}{1-\underline{\delta}\,\overline{\delta}}$$

which is clearly negative. But that indicates that the first line of (12) is not the maximal value of u_2, for the given value of u_1.

Repeating this exercise for all combinations one finds that the only constellation is for player 1 to offer on X first while player 2 offers on Y first. u_1 is therefore determined by the first line in (11) while u_2 is determined by the second line in (12). Solving these two equations simultaneously we obtain the equilibrium reservation levels of utility, from which we can back out the equilibrium offers, which are presented in the main body of the text.

Proof of Proposition 3: Let $(\overline{x},\overline{y})$ denote the offers by player 1 and $(\underline{x},\underline{y})$ denote the offers by player 2. The two equations defining the Rubinstein bargaining solution for joint offers are:

$$(1-\overline{y})+(1-\overline{x}) \geq \underline{\delta}(1-\underline{x})+\overline{\delta}(1-\underline{y}) \tag{13}$$

$$\underline{x}+\underline{y} \geq \overline{\delta}\,\overline{x}+\underline{\delta}\,\overline{y}. \tag{14}$$

Since $\overline{x}$ and $\overline{y}$ are perfect substitutes for player 2, but player 1 has a higher discount factor on X, player 1 will choose to maximize $\overline{x}$ while reducing $\overline{y}$ correspondingly. Similarly, $\underline{x}$ and $\underline{y}$ are perfect substitutes for player 1, but player 2 has higher delay cost on X. Therefore player 2 will choose to reduce $\underline{y}$ as much as possible. Indeed, both players will choose a corner solution for their offers, where $\overline{x}=1$ and $\underline{y}=0$. Hence (13) and (14) simplify to

$$(1-\overline{y}) \geq \underline{\delta}(1-\underline{x})+\overline{\delta} \tag{15}$$

$$\underline{x} \geq \overline{\delta}+\underline{\delta}\,\overline{y}. \tag{16}$$

Therefore the equilibrium offers are

$$(\overline{x},\overline{y})=\left(1,\frac{1-\overline{\delta}}{1+\underline{\delta}}\right) \quad \text{and} \quad (\underline{y},\underline{x})=\left(\frac{\overline{\delta}+\underline{\delta}}{1+\underline{\delta}},0\right).$$

Risk of Breakdown:
Fixed Order: We need to solve

$$(1-\overline{x})+(1-\underline{y}) \geq \underline{\rho}\,((1-\underline{x})+(1-\overline{y})) \tag{17}$$

$$\underline{x}+\overline{y} \geq \overline{\rho}\,(\overline{x}+\underline{y}) \tag{18}$$

where

$$\overline{y} = \frac{1-\overline{\rho}}{1-\overline{\rho}\underline{\rho}} \qquad \text{and} \qquad \underline{y} = \underline{\rho}\frac{1-\overline{\rho}}{1-\overline{\rho}\underline{\rho}}. \tag{19}$$

Here an $\bar{}$ denotes an offer by player 1 while an $\underline{\ }$ denotes an offer by player 2. Assuming both (17) and (18) hold as equalities and solving for $\overline{x}$ we obtain

$$\overline{x} = \frac{2-(3-\overline{\rho})\underline{\rho}}{1-\overline{\rho}\underline{\rho}}. \tag{20}$$

Since this magnitude can exceed 1, which is not a feasible offer, we again have the possibility of a corner solution where $\overline{x} = 1$ and only inequality (18) holds as an equality, while (17) remains a strict inequality.

The analogous computations for the Y then X agenda order are

$$(1-\overline{y}) + (1-\underline{x}) \geq \overline{\rho}\left((1-\underline{y}) + (1-\overline{x})\right) \tag{21}$$

$$\underline{y} + \overline{x} \geq \underline{\rho}\,(\overline{y} + \underline{x}) \tag{22}$$

where

$$\overline{x} = \frac{1-\underline{\rho}}{1-\overline{\rho}\underline{\rho}} \qquad \text{and} \qquad \underline{x} = \overline{\rho}\frac{1-\underline{\rho}}{1-\overline{\rho}\underline{\rho}}. \tag{23}$$

If (21) and (22) hold as equalities

$$\overline{y} = \frac{2-(3-\underline{\rho})\overline{\rho}}{1-\overline{\rho}\underline{\rho}}.$$

However, $\underline{y} < 0$ is not a feasible offer. In that case (21) holds as equality but (22) will remain a strict inequality. We then get $\overline{y} = 2(1-\overline{\rho})$.

The payoffs to player 1 under the two agenda orders are

$$\frac{2(1-\underline{\rho})}{1-\overline{\rho}\underline{\rho}} \qquad \text{and} \qquad \frac{2(1-\overline{\rho})}{1-\overline{\rho}\underline{\rho}}$$

if both players are unconstrained. Clearly the first of these is larger. In the constrained case player 1 obtains a payoff of

$$1 + \frac{\underline{\rho}(1-\overline{\rho})}{1-\overline{\rho}\underline{\rho}} \qquad \text{or} \qquad 2(1-\overline{\rho}) + \frac{\overline{\rho}(1-\underline{\rho})}{1-\overline{\rho}\underline{\rho}}.$$

As long as $2\overline{\rho} - 1 > (\overline{\rho} - \underline{\rho})/(1-\overline{\rho}\underline{\rho})$ player 1 prefers the X then Y agenda in this case too.

Proof Proposition 5: Suppose that player 2 can achieve u_2 from the continuation equilibrium path after he rejects an offer by player 1. The probability with which he discounts this utility level depends on the offer which player 1 has made. Hence player 1's maximal utility is

$$u_1 = \max \begin{cases} x + \dfrac{\underline{\rho}(1-\overline{\rho})}{1-\underline{\rho}\,\overline{\rho}} & \text{s.t.} \quad (1-x) + \dfrac{1-\underline{\rho}}{1-\underline{\rho}\,\overline{\rho}} = \underline{\rho}u_2 \\ y + \dfrac{\overline{\rho}\,(1-\underline{\rho})}{1-\underline{\rho}\,\overline{\rho}} & \text{s.t.} \quad (1-y) + \dfrac{1-\overline{\rho}}{1-\underline{\rho}\,\overline{\rho}} = \overline{\rho}u_2 \end{cases}$$

The first of these corresponds to an initial offer on X, while the second corresponds to an initial offer on Y. Simplification yields that player 1's reservation level of utility is

$$u_1 = \max \begin{cases} 2 - \underline{\rho} u_2 \\ 2 - \overline{\rho} u_2 \end{cases} \tag{24}$$

Clearly player 1 can achieve the higher utility by making his initial offer on X. Similarly, player 2 can, for any given utility that player 1 has to achieve, obtain his highest payoff from making an initial Y offer. Therefore we need to solve

$$(1-x) + \frac{1-\underline{\rho}}{1-\overline{\rho}\underline{\rho}} \geq \underline{\rho}\left((1-y) + \underline{\rho}\frac{1-\overline{\rho}}{1-\overline{\rho}\underline{\rho}}\right) \tag{25}$$

$$y + \frac{1-\underline{\rho}}{1-\underline{\rho}\overline{\rho}} \geq \underline{\rho}\left(x + \underline{\rho}\frac{1-\overline{\rho}}{1-\underline{\rho}\overline{\rho}}\right). \tag{26}$$

Assuming that both hold with equality we obtain

$$x = \frac{1}{1+\underline{\rho}} + \frac{1-\underline{\rho}-\underline{\rho}^2+\overline{\rho}\underline{\rho}^2}{(1-\underline{\rho})(1-\overline{\rho}\underline{\rho})} \tag{27}$$

$$y = \frac{\underline{\rho}}{1+\underline{\rho}} - \frac{1-\underline{\rho}-\underline{\rho}^2+\overline{\rho}\underline{\rho}^2}{(1-\underline{\rho})(1-\overline{\rho}\underline{\rho})}. \tag{28}$$

Both of these may lie outside the feasible set $[0,1]$ however, and indeed do so for the same set of parameters. The equilibrium values of x and y are then 1 and 0 respectively.

Proof of Proposition 6: Both players now may make offers of either x, y, or (x,y). First suppose that player 2 can guarantee himself a level of u_2 in any continuation path after he rejects player 1's offer. In order to be accepted player 1 must make an offer that gives player 2 at least as much, discounted by the applicable continuation probability. Note that we have already shown that player 1 is better off making an offer on X than on Y. How does a joint offer compare to a X only offer? Any such joint offer must satisfy

$$1-x+1-y \geq \underline{\rho} u_2 \quad \longrightarrow \quad u_1 = x+y \leq 2-\underline{\rho}$$

and hence is equivalent to an offer on X only. (See (24) above.) Since there is no risk of breakdown "between" issues, a joint offer has no advantage over an agenda offer, and both are possible. The equilibrium utilities are uniquely determined, however, and correspond to the open agenda case.

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