6. Nonrenewable resource use I: the theory of depletion

For non-renewable resources we are concerned with the production rate over time and the date of resource exhaustion.

We will consider optimal extraction for:
- an individual mine operator
- a competitive mining industry,

under different assumptions of costs and the nature of the mineral deposit.

A. Basic intuition about the mining problem

- mine owner chooses output level with the goal of maximizing the present value of profits over time

- How does the mine owner's problem differ from the typical firm's profit maximizing problem when no non-renewable resource is involved (reproducible goods)?

1. Adjustment to the profit maximizing rule

p (=MR) = MC

p (=MR) = MC + op. cost of depletion

opportunity cost of depletion = cost of using up the fixed resource stock

2. Value of resource rent over time

- in a perfectly competitive environment with no uncertainty, in market equilibrium, all assets must have the same return

Why?

- What is the return to a non-renewable resource? – the resource rent.

- The rental value of a mineral must grow at the same rate as alternative assets – or there would be no mineral extraction.

3. The stock constraint

B. Extraction from a mine facing a constant price

B.1 The Efficient Extraction Path

- a giant block of pure copper
- price of copper is constant forever
- the marginal cost of cutting off a piece of copper rises with the size of the piece removed
- a mining company owns a known stock S_0

$$S_t - S_{t+1} = q_t \tag{6.1}$$

We want to maximize profits in all periods:

$$\pi = pq_0 - C(q_0) + \left(\frac{1}{(1+r)}\right)[pq_1 - C(q_1)] + \left(\frac{1}{(1+r)}\right)^2 [pq_2 - C(q_2)] + \ldots + \left(\frac{1}{(1+r)}\right)^T [pq_T - C(q_T)] \tag{6.2}$$

Maximization requires that the present value marginal profit be the same in all periods.

This simplifies to the *r* percent rule of extraction or Hotelling's rule.

The profit maximizing firm will choose an extraction path q_t, q_{t+1}, .. so that p-MC(q_t) is increasing at r% per year.

p-MC(q_t) is called rent, user cost, royalty, dynamic rent, or Hotelling rent.
For a constant price and if MC increases with q, this implies that q must be falling over time.

The optimal extraction program must also satisfy:

$$q_0 + q_1 + ... + q_T \leq S_0 \tag{6.3}$$

How do we determine the correct initial level of extraction? The marginal undiscounted profit in period T must be as large as possible, and then working backwards we can determine the optimal q_0.

B.2 The value of the mining firm

The value of a firm with S_t tons remaining:

$$V_t = pq_t^* - C(q_t^*) + \left(\frac{1}{(1+r)}\right)[pq_{t+1}^* - C(q_{t+1}^*)] + \ldots + \left(\frac{1}{(1+r)}\right)^{T-t}[pq_T^* - C(q_T^*)] \tag{6.4}$$

In the next period the value of the firm is reduced because q_t of the stock of the resource has been used up.

$$V_{t+1} = pq_{t+1}^* - C(q_{t+1}^*) + \left(\frac{1}{(1+r)}\right)[pq_{t+2} - C(q_{t+2}^*)] + \ldots + \left(\frac{1}{(1+r)}\right)^{T-t+1}[pq_T^* - C(q_T^*)]$$

We can show that:

$$V_t - V_{t+1} = [pq_t^* - C(q_t^*)](1+r) - rV_t \tag{6.5}$$

The rate of return to the mine owner from mining q_t^* is:

It can be shown that the decline in value of the firm from extracting q_t^* (economic depreciation) is equal to the rent associated with q_t^*.

Before taxes are assessed on a mining firm, the firm should be given a depletion allowance equal to V_t-V_{t+1}. If this is done then a tax on profits will be neutral – i.e. there will be no impact on the optimal extraction path for the firm.

C. Extraction by a competitive mineral industry

C.1 A two period model for the competitive industry

- for the extractive industry as a whole, price should rise as the stock of the resource is gradually used up.

- Problem for the industry – maximize profits over the two periods – implies maximizing economic rent by choosing output in each of the two periods, without exceeding the total stock available.

$$Max_{\{q_0, q_1\}} R = (B(q_0) - cq_0) + \frac{(B(q_1) - cq_1)}{(1+r)} \qquad (6.6)$$

$$\text{subject to } q_0 + q_1 = S_0.$$

B(q) is the area under the demand curve.
A constrained optimization problem. The solution yields the following:

These conditions imply that a mine-owner will be indifferent to extracting a unit of ore and selling it today, or extracting and selling it next period.

This is called a flow equilibrium condition – market forces of supply and demand will ensure that it is met.

What is the intuition of this flow equilibrium condition?

Suppose most mine owners prefer to delay extraction:

Now suppose most mine owners prefer to sell their ore today:

Our constrained optimization problem implies that the present value of rents must be the same in each period.

C2. A two period example

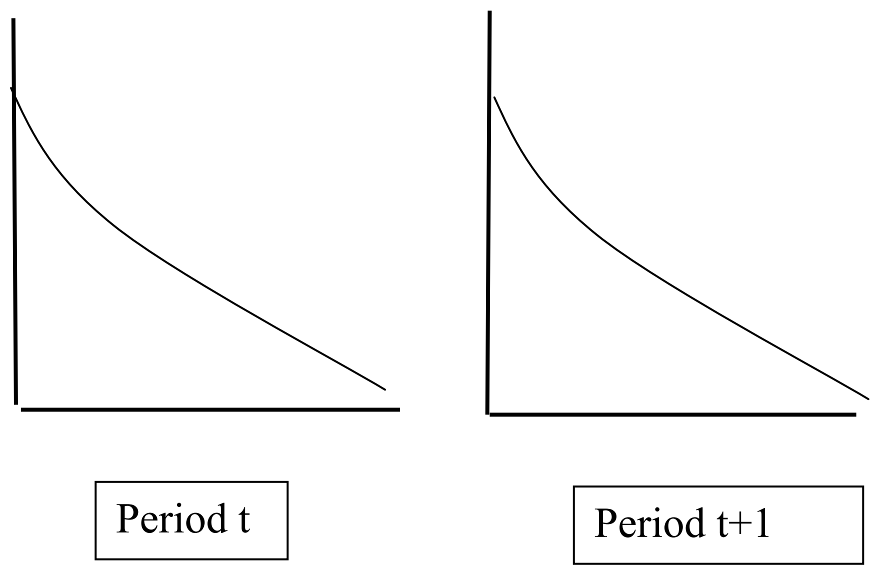
Suppose demand is given by: P=10-0.2Q and the marginal extraction cost is constant at $2 per unit of ore extracted. Suppose the resource stock is fixed at 60 units of ore. If we are only concerned with two periods, determine the optimal extraction level in each of these periods; price in each period and the present value of resource rent.

Note that resource rent grows at the rate of interest. What about price?

What happens to the quantity extracted over time?

What happens to the present value of resource rent over time?

C.3 Multiperiod problem
- These results extend easily to the many period case.



- In a competitive extractive industry with identical firms, each with a unit of the resource:
    - Industry price net of extraction costs rises at r% per year
    - Each firm makes a profit of $p(Q_0)$-c in present value terms
    - Extraction ends when the stock S_0 is used up and $p(Q_T)$ is at its largest value
    - In the final period $Q_T=0$

Social surplus will be maximized with this extraction path.

Implication: Free markets will deplete resources at the optimal rate

Why might we not believe this to hold in practice?

Figure 8.6 in the text shows the optimal time path for price and extraction for a multiperiod problem.

D. An empirical test of the r percent rule

Current market value of an extraction firm is the present value of future net profits:

$$V^i = (p_0 - c^i)q_0^i + \left(\frac{1}{(1+r)}\right)(p_1 - c^i)q_1^i + \ldots +$$
$$\left(\frac{1}{(1+r)}\right)^T (p_T - c^i)q_T^i$$

Miller and Upton (1985) – if (p_0-c^i) rises at r% then V^i can be written as:

$$V^i = (p_0 - c^i)(q_0^i + q_1^i + \ldots + q_T^i)$$
$$= (p_0 - c^i)S_0^i$$

Miller and Upton regress V_i / S_0^i on $(p_0 - c^i)$ for a group of firms and find reasonable confirmation of r% rule for oil extractive firms.

Other empirical tests have been less successful.

Note this cost structure is too simplistic. Ignores the fact that extraction costs usually increase as a deposit is depleted.

E. Quality variation in a deposit

If extraction costs rise with extraction our r% rule needs to be adjusted.

It will now include a term that reflects the increased cost that results when a unit of ore is extracted.

The cost function:

$$C = C(q,S)$$

where C increases as q increases or as S decreases.

Modified r% rule:

$$\frac{P(t) - C(t)_q - [P(t+1) - C(t+1)_q - C(t+1)_S]}{P(t) - C(t)_q} = r \tag{6.7}$$

$C(t)_q$ represents the increase in cost when the extraction rate increases by one unit in period t.

$C(t+1)_S$ represents the change in extraction cost if the stock is increased by on unit – it is negative because costs and stock level are negatively related.

F. Deposits of Differing Quality

Suppose we have two types of deposits, one with costs per unit of extraction of c1 and the other with c2. c2>c1.

Discuss the extraction profile of the two deposits over time.

What would happen if c2 rises?

G. Exhaustion and Backstops

When the price of a non-renewable resource rises high enough we might expect demand to go to zero as another cheaper substitute is available. This is called a backstop.

The expected price of a backstop affects the price of the ore being used today and its optimal extraction path.